Conversion of Mar Express MARSIS Active sounder ionosphere traces to electron density profiles

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1. Introduction

Since its commissioning in August of 2005, the Active Ionospheric Sounding mode of the MARSIS radar sounder aboard ESA Mars Express has collected more than 40,000 ionograms showing an ionospheric trace. Many of these ionograms are of sufficient quality to attempt conversion to an electron density profile. This operation has been accomplished for more than 13,000 such traces, and the results are being used to generate a comprensive picture of the dayside martian ionosphere. In the following pages, we shall summarize the procedures used to convert the raw data of a MARSIS ionogram to an electron density profile.

A sample ionogram is shown in Fig. 1. An ionogram is a plot of delay time against sounding frequency. Under the assumption that the sounding wave travels at the vacuum speed of light c, an "apparent range," *i. e.,* apparent distance from the spacecraft, can be calculated directly from the delay time by

$$z_{\mathrm{app}} = ct_{\mathrm{delay}}/2 \tag{1}$$

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where z_{app} is the apparent range, t_{delay} the delay time, and c the vacuum speed of light. Assuming that propagation is along the nadir direction, the apparent range can be converted to an apparent altitude by subtraction from the spacecraft altitude. The apparent range is displayed as the right-hand vertical axis.

Because the sounding wave reflects where the plasma frequency equals the sounding wave frequency, the plasma frequency can be directly converted to plasma density by equation 2:

$$n_{\mathrm{e}}\ (\mathrm{cm}^{-3}) \approx [f_{\mathrm{s}}\ (\mathrm{Hz})/8.98 \times 10^3]^2 \tag{2}$$

(see, *e. g. Gurnett and Bhattacharjee* [2005], p. 11, equation 2.3.5).

Two features of this ionogram contribute to electron density profiles: the vertical stripes in the upper left-hand corner labeled "f_{pe} distortion harmonics" and the horizontal bright band labeled "Ionospheric trace". In Section 2 we shall briefly summarize the measurement procedure for using the f_{pe} harmonics to determine the spacecraft-local plasma frequency and hence the plasma density. In Section 3 we shall explain the procedure for converting the local plasma frequency and the ionospheric trace into an electron density profile.

2. *In situ* plasma density

A feature of the data that is essential for the conversion of sounding data to electron density profiles is illustrated in the upper left corner of Fig. 1. The vertical stripes labeled "f_{pe} instrument distortion harmonics" represent harmonics of the plasma frequency local to the spacecraft. The cause of this disturbance is that the sounding wave pulse excites intense electron plasma oscillations at the local plasma frequency in the vicinity of the spacecraft. These oscillations then couple to the antenna and drive the receiver at a high enough intensity that the signal is distorted and harmonics of the plasma frequency are detected in the receiver. This allows determination of the local plasma frequency when it exceeds 30 kHz. The plasma frequency at the spacecraft, determined using these harmonics, provides the starting point for converting the delay time to corrected range.

In order to determine the plasma frequency, an adjustable electronic ruler has been programmed to appear superimposed on a selected ionogram. The ticks of this ruler are then adjusted to match the frequency spacing between the distortion harmonic lines. The spacing is read from the screen electronically and stored to a file for processing. The frequency spacing is then converted to a plasma density by equation 2.

3. Ionospheric trace inversion

This section is concerned with the reflection of the sounding wave from the topside ionosphere, seen in Fig. 1 as the mostly horizontal high-intensity band. We shall give a step-by-step explanation of our procedure for converting the ionospheric trace into an electron density profile.

3.1. Digitization of the ionospheric trace

The ionospheric trace is converted to a numerical function of delay time versus sounding frequency by a semi-automated process in which the person doing the tracing draws an electronic box around the ionospheric trace on a computer screen. The program then chooses the minimum-delay-time pixel for each frequency within the box at which a selected threshold intensity is reached. This threshold has been empirically set at

1×10^{-15}(V/m)^2/Hz. Because the data are noisy, the person doing the tracing must then examine the chosen points and delete those due to noise. This person must also measure the spacecraft-local plasma frequency by means of an on-screen adjustable scale used to measure the spacing between the distortion harmonics as discussed in Section 2. These results are stored in a file for further processing. The solid white line in Fig. 1 shows the result of digitizing the ionospheric trace. (The dashed white curve indicates our estimate of the electron plasma frequency profile between the spacecraft-local plasma frequency and the lowest-frequency sounding point.)

In Fig. 2 the sounding frequencies have been converted to plasma densities using equation 2 and the apparent range has been converted to apparent altitude by subtraction from the altitude. The dots correspond to measurements from the ionogram. The ionospheric maximum electron density is immediately seen to be $n(\mathrm{Peak}) = 1.45 \times 10^5$ cm^{-3}.

3.2. Smoothing the ionospheric trace

The delay time has a resolution of 91.4 μs, equivalent to an uncertainty of approximately ± 6.8 km in apparent range. This resolution leads to a stairstep appearance in the ionospheric trace, as seen in the solid white trace in Fig. 1 and the black dots in 2. A two-step process is used to smooth the trace. The points are culled so that only upper corners of the stairstep pattern are kept. The delay times from the remaining points are then used to linearly interpolate to intermediate instrument frequencies. The interpolated values therefore represent the smallest possible delay times based on the original trace. We choose this method because the smallest possible delay time should represent a raypath that lies along the nadir direction.

The result of the culling and interpolation process is shown in Fig. 3 as a blue-green colored curve connecting the upper corners of the original trace. The resulting trace is used as input to the inversion process.

3.3. The inversion procedure: conversion of the smoothed ionospheric trace to an electron density profile

The desired result from a topside sounder, such as MARSIS, is the electron density profile along the nadir direction. The ionospheric trace from an ionogram is a tabulation of the measured delay time as a function of sounding frequency. We have seen that the apparent range can be computed from the delay time under the assumption that the sounding wave travels through a vacuum away from the reflection point.

However, the sounding wave travels through the plasma medium of the ionosphere, and it is therefore necessary to correct Equation 1 by using the dispersion relation of an electromagnetic wave in a plasma:

$$v_{\text{phase}} = \frac{c}{[1 - (f_{\text{pe}}/f_{\text{s}})^2]^{1/2}} \tag{3}$$

which implies a group velocity of

$$v_{\text{group}} = c[1 - (f_{\text{pe}}/f_{\text{s}})^2]^{1/2} \tag{4}$$

where v_{phase} is the phase velocity of the sounding wave, f_{pe} the plasma frequency along the ray path, f_{s} the sounding wave frequency, and v_{group} the group velocity of the sounding wave. The unmagnetized dispersion relation is appropriate because the magnetic fields at Mars make only a small difference in the index of refraction at applicable frequencies. The time between the launching and reception of the sounding pulse, the delay time, is therefore given by

$$t_{\text{delay}} = 2\int_0^{z_{\text{refl}}} \frac{dz}{v_{\text{group}}} = \frac{2}{c}\int_0^{z_{\text{refl}}} \frac{dz}{[1-(f_{\text{pe}}/f)^2]^{1/2}} \tag{5}$$

where z is the distance from the spacecraft along the sounding raypath and z_{refl} is z at the reflection point; z_{refl} is called the *range*. Equation 5 can be applied along any ray path; however, in this application, the simplifying assumption is made that the ionosphere is horizontally stratified and that the sounding wave and its reflection travel along the spacecraft-nadir line.

Note that the value of the integral is a *measured* quantity, whereas the value to be calculated, the range, is the upper limit of the integral. Solving this equation is referred to as *inversion*.

The solution to equation 5 has been derived by several authors. *Budden* [1961] derives an analytic solution using the fact that equation 5 is a form of Abel's integral equation. The MARSIS team uses a method lending itself to irregular discrete data rather than a continuous function. *Jackson* [1969] outlines the basic method, which he refers to as the *lamination* method, so called because the ionosphere is assumed to be horizontally stratified. [A similar method is used by *Nielsen et al.* [2006].] First, in order to get a unique solution, it is necessary to assume that the final result, z as a function of f_{pe}, is monotonic. Second, it is assumed that the magnetic field is insignificant for purposes of ray tracing. The subscript j is assigned to each sounding frequency and its corresponding delay time. For each $t_{\text{delay},j}$ corresponding to sounding frequency $f_{\text{s},j} = f_{\text{pe}}(z_j)$, we wish to compute range z_j. In this context, the subscript 0 is reserved for values at the position of the spacecraft, so that $z_0 = 0$ and $f_{\text{pe}}(z_0)$ is measured directly from the ionogram as described in Section 2. It is necessary to choose an integrable functional form that can be applied to each interval between sounding points. *Jackson* [1969] has assumed a second-order logarithmic polynomial. For work on MARSIS, a simple logarithmic form is found to be convenient and to give reasonable results. For each sounding frequency interval, we make the substitution

$$z = \frac{1}{\alpha_j} \ln \left[\frac{f_{\rm pe}(z)}{f_{\rm pe}(z_{j-1})} \right] = \frac{1}{\alpha_j} \ln \left[\frac{f_{\rm pe}(z)}{f_{{\rm s},j-1}} \right] \tag{6}$$

where the range of integration has been divided at the data points (designated by the subscript j), z_{j-1} is the start point of the range of the jth interval, and α_j is the exponential constant for the jth interval. Changing variables in Equation 5 from z to $f_{\rm pe}$ according to equation 6 yields the time delay of the jth sounding wave:

$$t_{{\rm delay},j} = \sum_{i=0}^{j} \Delta t_{{\rm delay},i,j} = \frac{2}{c} \sum_{i=0}^{j} \int_{f_{{\rm pe},i-1}}^{f_{{\rm pe},i}} \frac{df_{\rm pe}}{\alpha_i f_{\rm pe}[1-(f_{\rm pe}/f_{{\rm s},j})^2]^{1/2}} \tag{7}$$

where $\Delta t_{{\rm delay},i,j}$ is the delay time for the jth sounding wave frequency over the ith ray path segment, and $0 \leq i \leq j$. The integrals on the right-hand side of equation 7 can now be performed by elementary methods, giving the result

$$t_{{\rm delay},j} = \sum_{i=0}^{j} \Delta t_{{\rm delay},i,j} = \sum_{i=0}^{j} \frac{1}{c\alpha_i} \ln \left[\frac{1-\sqrt{1-(f_{{\rm s},i}/f_{{\rm s},j})}}{1+\sqrt{1-(f_{{\rm s},i}/f_{{\rm s},j})}} \cdot \frac{1+\sqrt{1-(f_{{\rm s},i-1}/f_{{\rm s},j})}}{1-\sqrt{1-(f_{{\rm s},i-1}/f_{{\rm s},j})}} \right] \tag{8}$$

With the plasma frequency local to the spacecraft $f_{{\rm pe},0}$, given directly by the spectrogram as described in Section 2, equation 8 constitutes a system of j equations in j unknowns, the α_is. Solving for the α_is gives a complete description of the electron density profile.

In Fig. 4, the red curve is the apparent altitude corrected for dispersion according to Equation 8. This process yields a maximum electron density n(Peak) of 1.45×10^5 cm^{-3} at an altitude h(Peak) of 134.8 km. These are reasonable values for these quantities (see, *e. g., Bougher et al.* [2001]).

References

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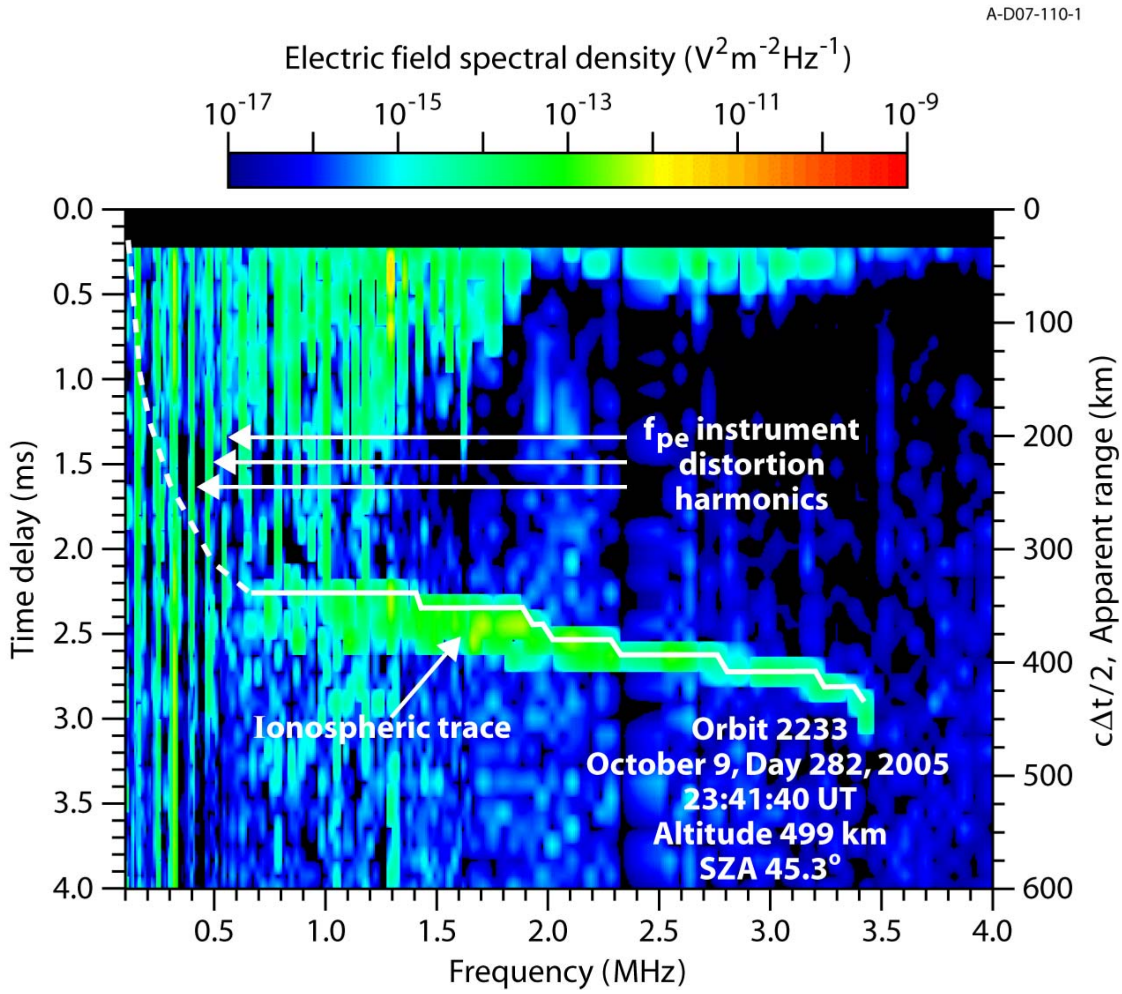

Figure 1. An ionogram: color coded received intensity plotted as a function of sounding frequency (abscissa) and delay time (ordinate). The apparent range, shown on the right-hand vertical axis, is given by equation 1. The ionospheric trace and electron plasma frequency harmonics are labeled. The solid white line shows the trace selected by the procedure given in Subsection 3.1. The dashed line shows where the low frequency trace would be based on the inversion solution , given in Subsection 3.3.

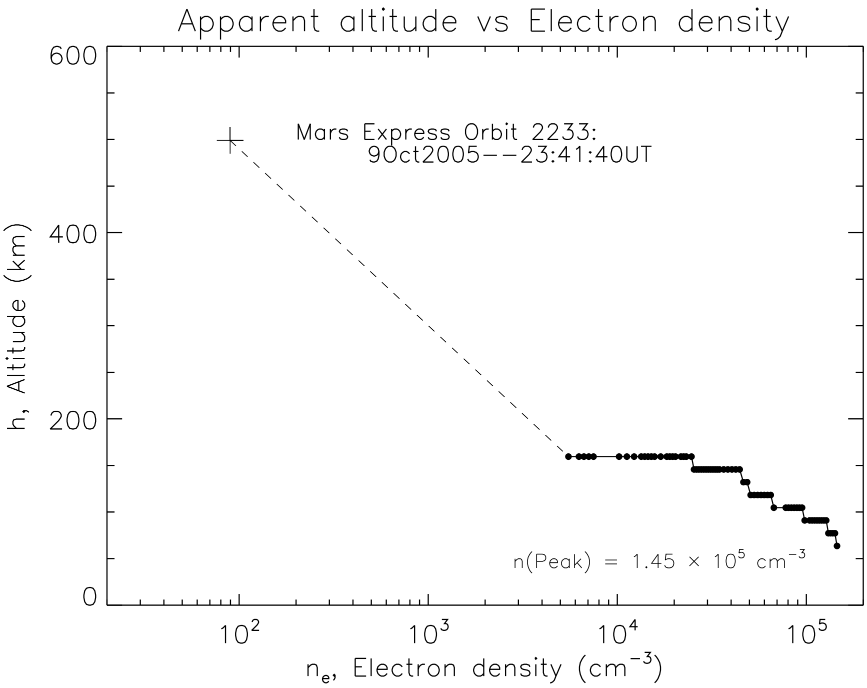

Figure 2. Apparent altitude vs electron density converted from the white curve of apparent range vs plasma frequency in Fig. 1. Apparent altitude is obtained by subtracting apparent range from the spacecraft altitude. Electron density is obtained from plasma frequency by equation 2. The black dots represent data points written to file from the original ionogram. The large plus sign represents the altitude and electron density at the spacecraft.

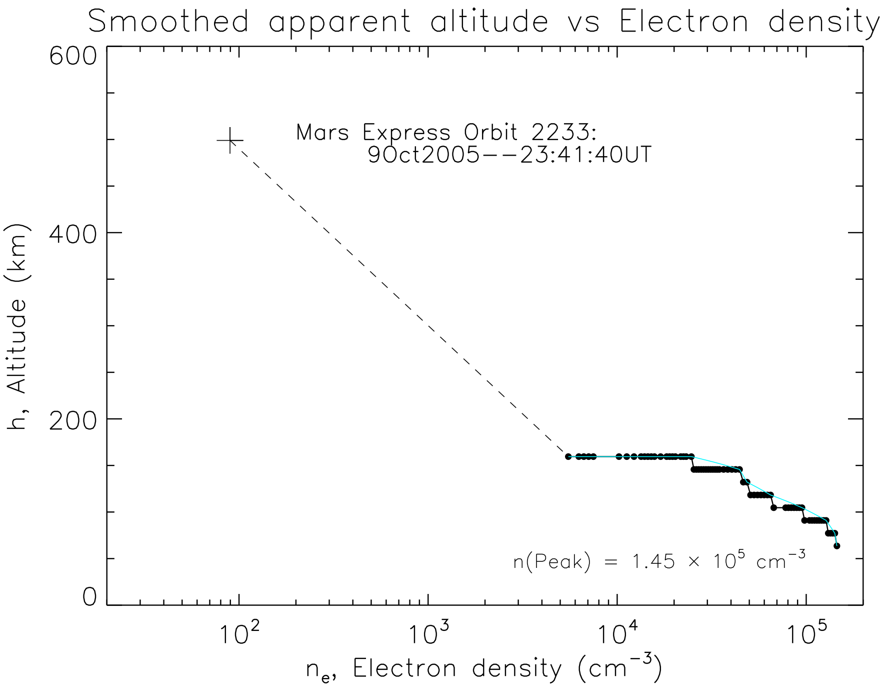

Figure 3. Identical to Fig. 2 except that the culled and interpolated ionospheric trace is plotted in blue-green over the trace converted from the original data.

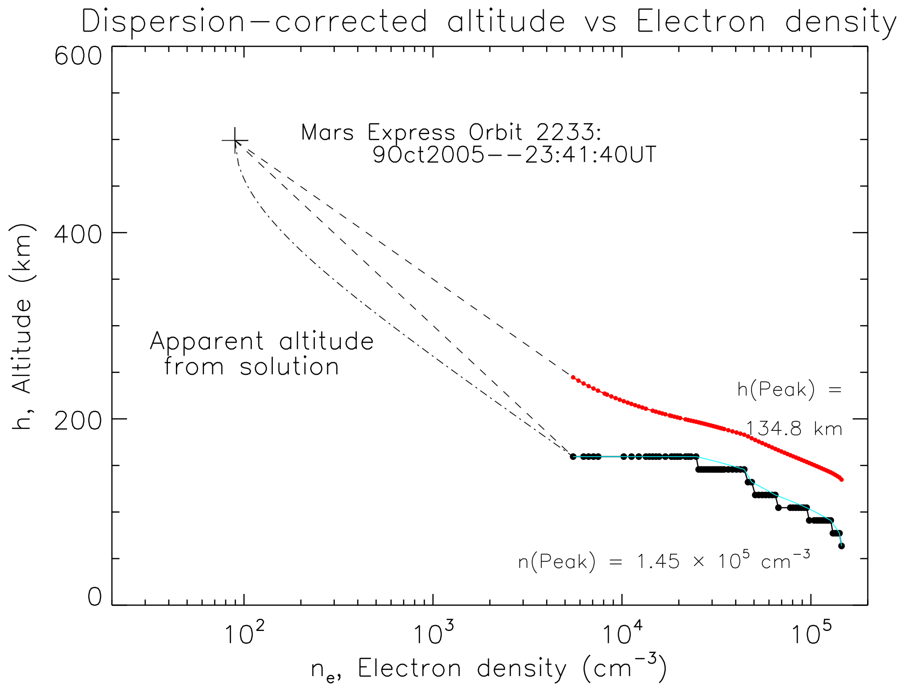

Figure 4. Identical to Fig. 3 except that the corrected altitude, according to equation 8 is now shown in red. The dot-dashed curve is the apparent altitude as a function of electron density inferred from the inversion solution. The ionospheric density peak is computed to occur at 134.8 km altitude.