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On the Phase Relationship Between Electron Density Variations in the Comet Interaction Region and Associated Magnetic Field Magnitude Oscillations

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1 Motivation

The Rosetta Magnetometer RPCMAG is a very sensitive fluxgate magnetometer. As any relative measuring magnetometer the RPCMAG measurements are subject to significant offset errors. In case of a spinning spacecraft offset corrections of the magnetic field components in the spin-plane of the spacecraft can easily be corrected for. Further tools exist for proper correction of the spin axis component. However, Rosetta is a non-spinning spacecraft, a three-axis stabilized spacecraft. This implies major challenges with respect to the correction of magnetometer offsets. The temperature dependance of the offsets introduces another complicating aspects of the calibration task, necessary to provide magnetic field measurements in a scientific archive, useful for further science exploitation.

Several methods have been developed in the past to handle the offset determination problem. For example, structural properties of magnetic field fluctuations can be used to estimate the offsets. The most well-known method is the Hedgecock method (Hedgecock, P.C., A correlation technique for magnetometer zero level determination, Space Science Instrumentation,1,8390,1975), using the fact that Alfven waves oscillations are purely transverse oscillations. More recently, the Plaschke method (Plaschke, F., Narita, Y., On determining fluxgate magnetometer spin axis offsets from mirror mode observations, Ann. Geophys., 34, 759766, doi:10.5194/angeo-34-759-2016, 2016; Frühauff, D., Plaschke, F., Glassmeier, K.H., Spin axis offset calibration on THEMIS using mirror modes, Ann. Geophys., 35, 117-121, doi:10.5194/angeo-35-117-2017, 2017) was proposed using mirror mode structures, magnetic variations of typical changes in the magnetic field magnitude. Both methods are regarded as suitable tools to determine offsets.

Another possibility to determine any offset problem may be a detailed analysis of phase relations between the various electro-magnetic and mechanical parameters of the coupled electromagnetic-hydrodynamic fields of the plasma. These are, for example, magnetic and electric field oscillations or plasma density variations. In a generalized terminology these relations between different parameters are called cross-polarisations. Of particular interest is a study of the phase relation between magnetic field magnitude and electron density oscillations. If this phase relation depends on the magnitude of the background magnetic field, any offset error will impact the observed phase with respect to the theoretically expected phase. Thus, the phase relation might be disturbed if significant offset errors exist. This motivates the current report. It aims at a more detailed analysis of the cross polarisation between field magnitude and electron density.

2 Multi-Fluid Plasma Waves and Cross Polarisation

2.1 MHD Plasma

To illustrate our approach we will start with the MHD equations for which the phase-relation between the density perturbation and the pertubation in the magnetic field is well known and understood. The equations governing a plasma in the MHD approximation consist of:

$$\begin{aligned}
\rho\frac{\partial \underline{u}}{\partial t} &= -\nabla\cdot p_e + \underline{j}\times\underline{B} \\
\frac{\partial \rho}{\partial t} &= -\nabla\cdot(\rho\underline{u}) \\
\nabla\times\underline{B} &= \mu_0\underline{j} \\
\nabla\times\underline{E} &= -\frac{\partial \underline{B}}{\partial t}.
\end{aligned}$$

Hereby ρ denotes the mass density, $\underline{u}$ the bulk velocity of the fluid and $\underline{B}_0$ the background magnetic field. As a closure for the above system of equations we use the adiabatic energy equation

$$p\cdot\rho^{\gamma} = \text{const.} \tag{1}$$

With this and the ideal gas law

$$p = nk_BT = \frac{\rho}{m}k_BT, \tag{2}$$

where n denotes the number density and m the mass, the gradient of the pressure simplifies to

$$\nabla\cdot p = \gamma\frac{p}{\rho}\nabla\cdot\rho = \frac{\gamma k_BT}{m}\nabla\cdot\rho = v_s^2\nabla\cdot\rho. \tag{3}$$

Hereby $v_s = \sqrt{\frac{\gamma k_BT}{m}}$ is the sonic speed of the medium.

For small perturbations $|\delta B| \ll |B_0|$ we can use the linearised form of equations 1 to 1:

$$\begin{aligned}
\rho_0\frac{\partial \underline{u}_1}{\partial t} &= -v_s^2\nabla\cdot\rho_1 + \frac{1}{\mu_0}(\nabla\times\underline{B}_1)\times\underline{B}_0 \\
\frac{\partial \rho_1}{\partial t} &= -\rho_0\nabla\cdot\underline{u}_1 \\
\nabla\times\underline{B}_1 &= \mu_0\underline{j}_1 \\
\nabla\times\underline{E}_1 &= -\frac{\partial \underline{B}_1}{\partial t},
\end{aligned}$$

where we assumed a uniform background magnetic field $\underline{B_0} = (0, 0, B_0)$ and a plasma that is at rest $u_0 = 0$. For further simplification we assume that the perturbations of a quantity Φ can be explained by plane harmonic waves $\Phi(\underline{x}, t) = \Phi\exp\left(-i\left(\underline{xk} - \omega t\right)\right)$ and that wave propagation is restricted to the xz-plane $\underline{k} = (k_x, 0, k_z)$.

After some algebra we obtain from equation 1 and the above assumptions

$$\begin{aligned} u_x &= \frac{v_s^2 k_x}{\omega} \cdot \frac{\rho_1}{\rho_0} - \frac{B_0}{\mu_0 \rho_0 \omega} \left(k_z B_{x,1} - k_x B_{z,1}\right) \\ u_y &= -\frac{B_0 k_z}{\omega \mu_0} B_{y,1} \\ u_z &= \frac{v_s^2 k_z}{\omega} \frac{\rho_1}{\rho_0}. \end{aligned}$$

Inserting this into the continuity equation 1 and writing $\underline{k} = (k_x, 0, k_z) = k \cdot (\sin(\Theta), 0, \cos(\Theta))$, where Θ denotes the angle between the magnetic field and the propagation direction, yields:

$$\frac{\rho_1}{\rho_0}\left(\frac{\omega^2}{k^2} - v_s^2\right) = v_A^2 \left(\frac{B_{z,1}}{B_0}\sin(\Theta)^2 - \frac{B_{x,1}}{B_0}\sin(\Theta)\cos(\Theta)\right). \tag{4}$$

In order to analyse this equation information about the wave solution is needed. This can be obtained by the well known dispersion relation in a warm MHD plasma

$$\frac{\omega^2}{k^2} = v_{ph}^2 = \frac{1}{2}\left(v_A^2 + v_s^2 \pm \sqrt{\left(v_A^2 + v_s^2\right)^2 - 4 v_A^2 v_s^2 \cos(\Theta)^2}\right). \tag{5}$$

For the special case of wave propagation perpendicular to $\underline{B_0}$ ($\Theta = 90°$) we obtain from Equation 4 and Equation 5

$$\frac{\rho_1}{\rho_0} = \frac{B_{z,1}}{B_0} \tag{6}$$

which is expected for a fast mode.

In the case of wave propagation parallel to $\underline{B_0}$ ($\Theta = 0°$) the magnetic field perturbations decouple from the density perturbations and we obtain

$$\frac{\rho_1}{\rho_0}\left(\frac{\omega^2}{k^2} - v_s^2\right) = 0, \tag{7}$$

which describes a sonic wave with $v_{ph} = v_s$. For arbitrary propagation directions the sign of Equation 4 depends on the phase speed of the corresponding mode (Equation 5). The phase between the density and magnetic field perturbations always amounts to either 180° for the slow mode or 0° for the fast mode. An offset of the background magnetic field vector would not influence the phase for the MHD case. However, as the MHD model is based on severe restrictions, this has to be evaluated for other conditions using a more sophisticated model.

2.2 Multi-fluid Plasma

Contrary to the MHD model the multi-fluid approach allows for more than one particle species. This introduces effects based on the different masses of the particles, which in turn also influence the wave modes propagating in the medium. For our analysis we assume an electron-ion plasma where the ions are at rest $u_i \approx 0$. The set of Equations consists of

$$\begin{aligned} n_{e,0} m_e \frac{\partial \underline{u}_{e,1}}{\partial t} &= -e n_e \left(\underline{E_1} + \underline{u}_{e,1} \times \underline{B}_0 \right) - \nabla p_e \\ \frac{\partial n_{e,1}}{\partial t} &= -\nabla \cdot \left(n_{e,0} \underline{u}_{e,1} \right) \\ \nabla \times \underline{E}_1 &= -\frac{\partial \underline{B}_1}{\partial t} \\ \nabla \times \underline{B}_1 &= \mu_0 \underline{j}_1 + \frac{1}{c^2} \frac{\partial \underline{E}_1}{\partial t}. \end{aligned}$$

Analogously to the MHD approach we obtain from the momentum equation

$$\begin{aligned} u_{x,1} &= \frac{-ie}{\omega m_e} \left(E_{x,1} + u_y B_0 \right) - \frac{v_{se}^2 k_x}{\omega} \cdot \frac{n_{e,1}}{n_{e,0}} \\ u_{y,1} &= \frac{-ie}{\omega m_e} \left(E_{y,1} - u_x B_0 \right) \\ u_{z,1} &= \frac{-ie}{\omega m_e} E_{z,1} - \frac{v_{se}^2 k_z}{\omega} \cdot \frac{n_{e,1}}{n_{e,0}}. \end{aligned}$$

Unlike in the MHD case the equations for $u_{x,1}$ and $u_{y,1}$ are now coupled. Inserting the expression for $u_{y,1}$ inti that one for $u_{x,1}$ yields:

$$u_{x,1} = -\frac{i\Omega_e}{\omega - \frac{\Omega_e^2}{\omega}} \cdot \frac{E_{x,1}}{B_0} - \frac{\Omega_e^2}{\omega^2 - \Omega_e^2} \cdot \frac{E_{y,1}}{B_0} + \frac{v_{se}^2 k_x}{\omega - \frac{\Omega_e^2}{\omega}} \frac{n_{e,1}}{n_{e,0}}, \tag{8}$$

where $\Omega_e = eB_0/m_e$ is the electron gyro-frequency. With this we can relate the perturbations in electric field and the density:

$$\frac{n_{e,1}}{n_{e,0}} \left(1 - -\frac{v_s e^2 k_z^2}{\omega_e^2} - \frac{v_s e^2 k_x^2}{\omega^2 - \Omega_e^2} \right) = -\frac{i\Omega_e k_x}{\omega^2 - \Omega_e^2} \frac{E_{x,1}}{B_0} - \frac{\Omega_e^2 k_x}{\omega(\omega^2 - \Omega_e)} \frac{E_{y,1}}{B_0} - \frac{i k_z \Omega_e}{\omega^2} \frac{E_{z,1}}{B_0}. \tag{9}$$

In order to relate the electric field to the magnetic field through Faraday's Law (Equation 1) a relation between $E_{x,1}$ and $E_{y,1}$ is needed. The polarisation of the electric field vector for arbitrary propagation directions is given by

$$\frac{E_{x,1}}{E_{y,1}} = \frac{i(S-\eta^2)}{D}, \tag{10}$$

with $\eta = ck/\omega$, $\mathrm{S} = 1 - \omega_{\mathrm{pe}}^2/(\omega^2 - \Omega_{\mathrm{e}}^2)$, $\mathrm{D} = \omega_{\mathrm{pe}}^2\Omega_{\mathrm{e}}/(\omega(\omega^2 - \Omega_{\mathrm{e}}))$ and the electron plasma frequency $\omega_{pe}^2 = n_{e,0}e^2/\epsilon_0 m_e$. As previously noted we can assume $E_{z,1} \approx 0$ for the cometary interaction region. Inserting the polarisation and neglecting the terms with $E_{z,1}$ yields

$$\frac{n_{e,1}}{n_{e,0}}\left(1 - -\frac{v_{se}^2 k^2\cos(\Theta)}{\omega^2} - \frac{v_{se}^2 k^2 \sin(\Theta)}{\omega^2 - \Omega_e^2}\right) = \left(\frac{\Omega_e\omega}{\omega^2-\Omega_e^2}\frac{S-\eta^2}{D} - \frac{\Omega_e^2}{\omega^2-\Omega_e^2}\right)\frac{B_{z,1}}{B_0}. \tag{11}$$

The sign of the expression depends, similar to the MHD case, on the dispersion relation of the corresponding waves modes. Substituting ω and k according to the dispersion relation, which can be found in e.g. Bittencourt, Fundamentals of Plasma Physic (2004), yields a phase of either 180° or 0°, describing the slow mode and the fast mode, respectively. Moreover, variations in the background magnetic field due to offsets do not affect the phase between magnetic field and density perturbations.

In the special case of perpendicular propagation we obtain

$$\frac{n_{e,1}}{n_{e,0}}\left(1 - \frac{v_{se}^2 k_x^2}{\omega^2 - \Omega_e^2}\right) = \frac{B_{z,1}}{B_0}\left(\frac{\Omega_e^2(\Omega_e^2 + 2\omega_{pe}^2 - \omega^2)}{(\omega^2-\Omega_e^2)(\omega^2-\Omega_e^2-\omega_{pe}^2)}\right). \tag{12}$$

In the right hand side expression we can identify the known reflection point $\omega^2 = \Omega_e + 2\omega_{pe}^2$ and resonance points $\omega^2 = \Omega_e^2$, $\Omega^2 = \Omega_e^2 + \omega_{pe}^2$. For high frequencies $\omega \to \infty$ the term on the right hand side vanishes implying that the magnetic field oscillations decouple from the density oscillations. This is reasonable in the limit of an electromagnetic wave.

For a cold plasma $v_{se} \approx 0$ in the low frequency approximation $\omega \ll \Omega_e$ this reduces to

$$\frac{n_{e,1}}{n_{e,0}} \approx \frac{B_{z,1}}{B_0} \tag{13}$$

which is consistent with Equation 6.

3 Conclusion

Our theoretical study of wave propagation in a warm plasma has shown that the real phase between magnetic field and density perturbations is not affected by the background magnetic field and therefore not sensitive to any offsets. In conclusion, determination of the phase of cross polarisation between density and magnetic field magnitude is not a suitable tool to determine offset problems.