

Dynamic error correction of a thermometer for atmospheric measurements

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Abstract

This work deals with the dynamic correction of a thermometer for atmospheric measurements. The sensor analysed is the one launched onboard the Huygens probe of the Cassini mission. The specific aspects of the measurement are related to the strong changes in the fluid-dynamic conditions during the operative phase. This peculiarity does not allow the use of one of the various correction techniques based on the knowledge of the dynamic characteristics of the sensor. The thermometer, however, has the peculiarity of being composed of two independent platinum sensors, having very different dynamic properties. The study, starting from a previously conceived dynamic scheme of the sensor, assesses the possibility of determining continuously, from measurement data, the current values of the four time constants required for dynamic characterisation. Lastly one procedure for the dynamic correction of the measurements is presented, along with some verification of its effectiveness and, of the impact on the overall measurement uncertainty. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

This study is focused on a temperature sensor, named TEM, that has been devised for the temperature profile retrieval during the descent of the Huygens probe, in the Titan atmosphere. TEM measurement starts at an altitude of about 180 km and ends at the surface of Titan. The expected environmental parameters are dramatically varying: probe velocity from 140 to 2 m/s, density from 5 to 5 kg/m³, temperature from 200 to 100 K.

The sensor is therefore facing conditions radically

different from the beginning to the end. Various papers [1–3] present the different features of this sensor; however, in the following the characteristics relevant to our analysis will be briefly recalled.

Two independent platinum resistance elements ('fine' and 'coarse') compose the sensor, the scheme of which is shown in Fig. 1. The 'fine' sensor is a thin platinum wire wound on a Pt–Ro cage-like tubular frame. The 'coarse' sensor is also a platinum wire but imbedded in a layer of glass, deposited on the tubular structure. The original reason for this doubling of the sensor was mainly for the reliability of the measurement. The fine sensor is rather delicate and could easily break because of an impact with any small particle, during the high velocity descent. The

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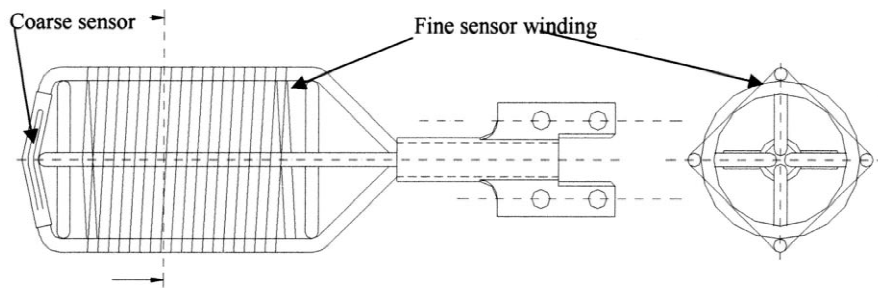


Fig. 1. Scheme of the sensor.

coarse sensor is much tougher. The dynamic properties of the two sensors are very different. The response time of the coarse much longer than that of the fine; moreover, both change dramatically from beginning to end of the measurement.

The evaluation of the frequency response of the sensor, at various altitudes, based on the 'nominal' atmosphere of Titan and descent velocity, has shown the need of a dynamic correction to use the whole frequency bandwidth allowed by the sampling frequency (at least for the upper part of the descent) [1]. As pointed out the dynamic properties of the sensor change continuously during measurement, so the conventional dynamic correction techniques [4,5], relying on the assumption of time invariant systems, are not applicable.

2. Dynamic model

If a platinum wire temperature sensor is modelled neglecting all heat exchanges, except for the convection of the wire with the fluid, the well-known first-order linear equation is obtained [6]. The coarse sensor dynamic behaviour can be represented quite accurately by a first-order model, therefore, a single parameter τ_c is enough for its characterisation.

However, the accuracy of the first order model, depends on the validity of the mentioned assumptions. So for the fine sensor, it depends on the length/diameter ratio and convective index. The first-order model has shown its limitations when used to represent the fine sensor and leads to an overestimation of the frequency bandwidth if the time constant is derived from the response time. An effective dynamic model of the fine sensor, has proven to be that of a second-order system, defined

by three time constants [2], expressed by the following differential equation:

$$\tau_1 \tau_2 \frac{d^2 T}{dt^2} + 2(\tau_1 + \tau_2) \frac{dT}{dt} + T = T_a + \tau_3 \frac{dT_a}{dt} \quad (1)$$

The reduction of the dynamic performance of the sensor, with respect to those expected for the wire alone, comes from the heat exchange between the latter and the tubular structure supporting it; the achievable thermal de-coupling, is limited because of strength requirements.

3. Dynamic correction

Expression (1) is obtained by combining the two differential equations expressing the thermal balance of the cage structure and of the fine sensor filament; for the latter the following can be written:

$$c_f \frac{dT_f}{dt} + \frac{T_f}{R_\alpha} + \frac{T_f}{R_i} = \frac{T_s}{R_i} + \frac{T_a}{R_\alpha} \quad \text{or} \quad c_f \frac{dT_f}{dt} + \frac{T_f}{R_\alpha} + \frac{T_f}{R_i} - \frac{T_s}{R_i} = \frac{T_a}{R_\alpha} \quad (2)$$

Where c_f is the thermal capacity of the wire, T_f its average temperature (i.e., the temperature measured by the fine sensor). T_s is the temperature of the structure (i.e., the temperature measured by the coarse sensor). T_a is the atmospheric temperature. R_α is the thermal resistance due to the convective heat exchange between wire and atmosphere. R_i the thermal resistance between wire and structure. Eq. (2) has the well-known shape of first-order systems, where the input is the weighted sum of two temperatures. As for every first-order instrument the deriva-

tive part can be neglected when the input band frequency is below the cutting frequency, i.e., $f_c = (2\pi c_i R_e)^{-1}$, where R_e is the equivalent resistance of the parallel of R_i and R_α . Under this hypothesis Eq. (2) can be written:

$$\frac{T_\alpha}{R_\alpha} = \frac{T_f}{R_\alpha} + \frac{T_f}{R_i} - \frac{T_s}{R_i} \Rightarrow T_\alpha = T_f + \frac{R_\alpha}{R_i}(T_f - T_s) \quad (3)$$

The above expression shows that the temperature of the atmosphere can be derived from the readings of the two sensors, allowing improved frequency bandwidth of the measurement up to the f_c value. It has to be noted that, the high frequency content of the fictitious variable of the second member of (2), depends only on the second addendum, being the first dependent from the latter through a transfer function having a 'low pass filter' characteristic.

The knowledge of the ratio R_α/R_i is required to apply the correction scheme expressed by (3), unfortunately it depends on the convective exchange factor which changes during measurement; the following paragraph shows a procedure allowing calculation from the measurements.

4. Determination of the correction factor

The availability of two measurements, made with sensors having different dynamic properties (the fine and coarse) allows derivation of some information

about the environment they are facing. If the two were a first-order system, the convective index could be directly calculated and, a dynamic compensation applied [7]. Our case is slightly different because of the more complex transfer function of the sensor. However, the four dynamic parameters are correlated. Therefore, if one of them is determined, the remaining can be derived from it. The diagram of Fig. 2, where the four time constants are plotted against altitude from Titan surface, suggests (by similarity of the curves), a quasi-linear relationship between the parameters. This has been verified by expressing the three time constants of fine sensor as function of the coarse one. A linear relationship led to a correlation factor of about 0.99, where using second-order polynomials gave values exceeding 0.999. The instrument then can be fully characterised if only one time constant (or a correlated parameter) is derived from the measurements.

The adopted scheme determines the time constant of the coarse sensor from the transfer function between its readings and those of the fine. This procedure rests on the assumption that the temperature spectrum contains meaningful components above the coarse sensor cutting frequency, if that was not the case, however, the dynamic correction would be meaningless too.

For verification, this method has been applied to data obtained during a balloon flight in the earth's atmosphere [3]. Fig. 3 shows the ratio between the Fourier transforms of the coarse and fine measure-

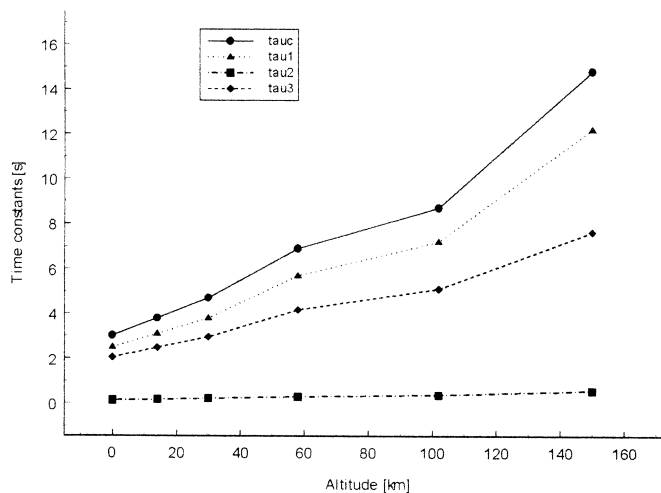


Fig. 2. Time constants of the sensor(s) predicted at various altitudes.

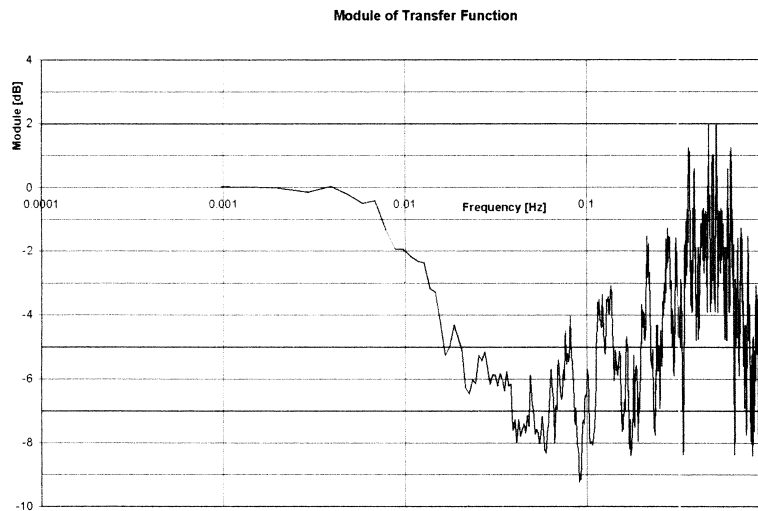


Fig. 3. Transfer function between coarse and fine sensors during measurements in Earth's atmosphere.

ments. Time constants are derived using a least-squares approach, with the caution of excluding the high-frequency part of the spectra, which derives from the quantization noise.

Once the coarse sensor time constant, τ_c has been calculated, the three dynamic parameters of the fine, τ_1 , τ_2 , τ_3 , are derived from it. The last step is the calculation of the correction factor from them.

The correction factor R_a/R_i of Eq. (3) can be written as function of the three time constants. Its expression is rather long so it has been included in Appendix A.

The block diagram of Fig. 4 summarises the whole correction procedure.

Although the proposed dynamic correction method has been tailored to the specific situation foreseen during the Huygens probe measurement, the concept can be applied to any thermometer whose dynamic behaviour is influenced by the supporting structure, provided an additional sensing element is fitted on it.

In a more generic application, not suffering of strict limitations in the sampling frequency (as in our case), different procedures could be profitably adopted to determine the dynamic parameters. For instance a method based on the time lag between the two sensors readings is applicable.

5. Application of the method

The complete correction procedure has been experimentally verified, at first, by application to a 'reference condition', i.e., to measurements made during a thermal shock test, consisting of a sudden exposure to a flow of nitrogen vapours. Fig. 5 shows the relative measurement errors computed for the measured temperatures and the corrected ones, assuming the input temperature was an ideal step. Correcting the temperature does not enable an exact match for the ideal behaviour of the first-order

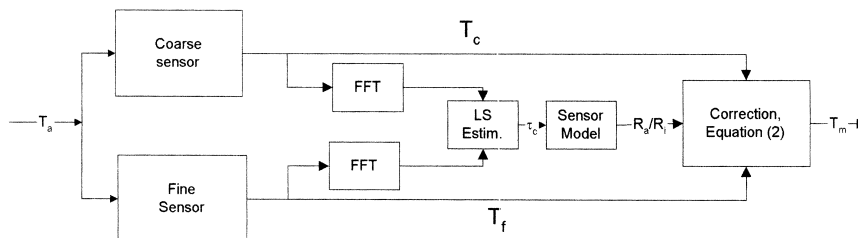


Fig. 4. Block diagram of the correction procedure.

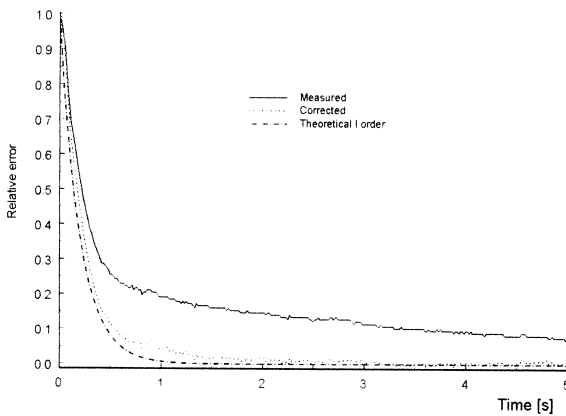


Fig. 5. Comparison between the corrected and uncorrected measurement in case of step input.

system. Nevertheless, the dynamic error after 1.5 s is reduced by a factor 10.

To test the method on data similar to those expected during the instrument operative phase, a simulation has been performed using the nominal Titan atmospheric temperature profile and the same acquisition parameters (A/D resolution and sampling frequency) foreseen during the descent in it. The ‘measured’ temperature of fine and coarse sensors have been obtained using a numerical thermal model (based on ESATAN software) of the sensor and its holding stem. A transient thermal analysis has been performed assuming (as a boundary condition) the nominal Titan’s atmospheric temperature profile ‘enriched’ with a white noise having zero mean and 3°C r.m.s. The added noise has been band limited to the measurement Nyquist frequency in order not to excessively slow the numerical integration.

The data have been used, first of all, to verify the capability of the method to determine the coarse time constant using the ratio between the spectra of the two sensor readings. Fig. 5 shows the computed time constant along with the theoretical (already shown in Fig. 2). It has to be noted that a trade-off, involving the number of points to use in the FFT, has to be performed. Using a large number of points leads to higher spectral resolution and so to a more accurate determination of the interpolating function, this however implies averaging the sensor characteristics in a longer time, losing the association between the determined dynamic parameter and a specific time.

Being constrained to the FFT algorithm power of 2 law, 32 to 512 points have been tried. Using the r.m.s. of the deviation between the computed time constants and the theoretical ones as cost index the optimum has been obtained with spectra based on 64 points, but with few differences for 128 (Fig. 6).

The result of the simulation is shown in Fig. 7: the corrected temperature is plotted along with the fine measured values and the ‘actual’ profile used in the simulation.

The comparison between the r.m.s. error for the measurements (with and without the application of proposed correction strategy) shows that the method always gives a reduction of the total error. The magnitude of the error reduction varies from 5 to 40% depending on the ambient conditions but the average on the whole profile is 20%.

A correction technique based on the manipulation of the frequency spectra of the fine sensor measurement, as described in Ref. [7] has been also evaluated; the spectra obtained by FFT of the measured temperatures have been divided by the harmonic transfer function, then inverse-Fourier transformed. The comparison between the signal corrected in such a way and the actual temperature has shown in some cases an increase of the error because of the amplification of high-frequency noise components. This method has not been investigated further, although a low pass filtering of the data would have solved the inconvenience, because it offered no advantage with respect to the adopted one.

6. Uncertainty analysis

Any correction technique brings a contribution to the measurement uncertainty¹ that has to be compared with the reduction of the dynamic error. The uncertainty related to the correction scheme of (3), can be easily derived using the expression for combined uncertainty [8]:

$$i_{T_a} = \sqrt{i_{T_f}^2 (1 + R_a/R_i)^2 + (R_a R_i)^2 i_{T_c}^2 + i_{R_a/R_c}^2 (T_f - T_c)} \quad (4)$$

¹In the analysis all uncertainty considered have the meaning of standard uncertainty according to Ref. [8].

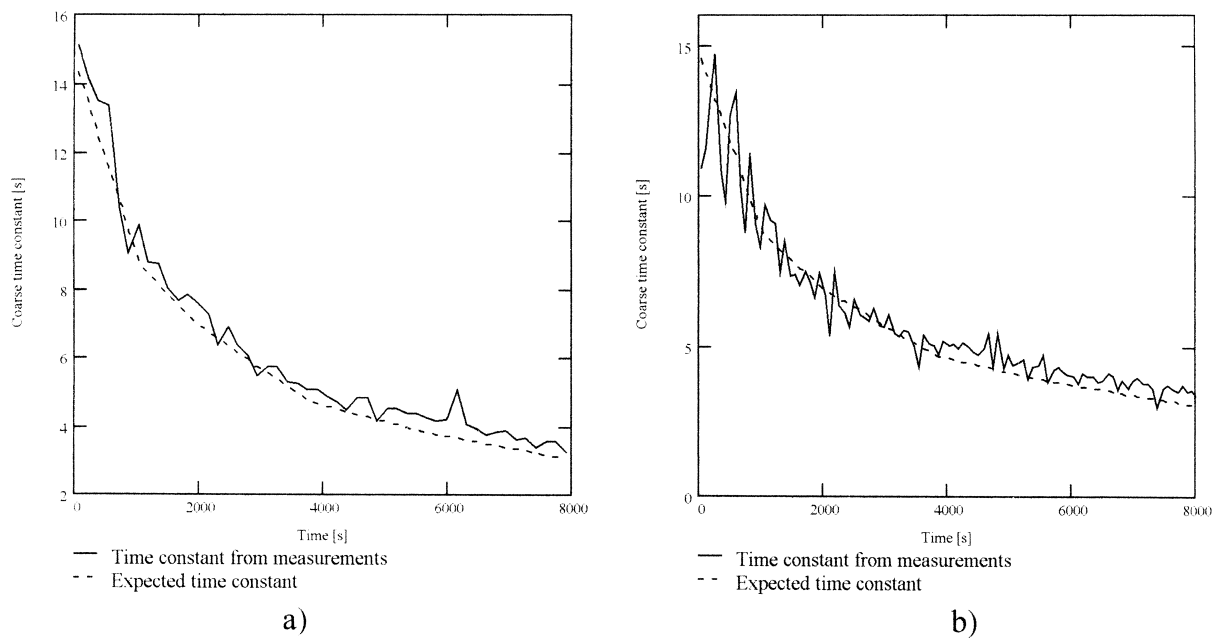


Fig. 6. Coarse sensor time constant derived from comparison the two sensor readings FFT, (a) 128 points used, (b) 64 points used.

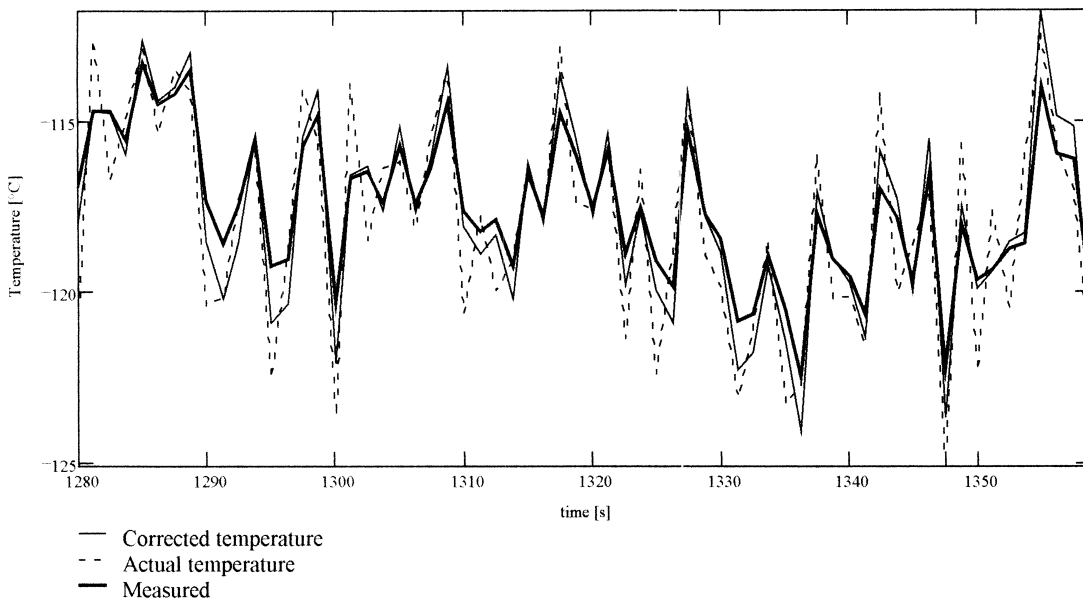


Fig. 7. Comparison of the actual, measured and corrected temperature.

Relation (4) clearly shows that whenever the correction algorithm is applied, even if $T_f = T_c$ i.e., in a quasi-static condition, there is an increase in measurement uncertainty. The decision whether or not correcting the measurements will have to be made by looking at the frequency content of the measured temperature. The value of the correction factor R_a/R_i is in the range 0.2–0.5 with uncertainty 0.04, the uncertainty of T_f is 0.02°C and that of T_c 0.1°C , therefore the correction shall not be applied when the expected dynamic error is lower than 0.06°C .

For instance during measurement at an altitude of 50-km components of T_f frequency spectrum above 0.05 Hz will be attenuated by more than 20% if no correction is applied; the presence of components in such frequency band having amplitude higher than 0.3°C would make convenient the dynamic correction.

7. Conclusions

A method to improve the dynamic performances of a dual sensor thermometer has been proposed and validated using both experimental and simulated data. The method derives the dynamic properties of the thermometer in the specific environmental condition from the transfer function of the two sensors readings, and by using this information corrects the reading of the faster sensor for the effect of the structure thermal inertia, allowing the reduction of dynamic errors. The relevance of the accuracy improvement depends on the temperature profile, in the case of step-like input the maximum reduction in the error is in the order of 20% of the temperature step.

The application of the method, however, will lead to an increase in measurement uncertainty of about 0.06°C therefore correction will be made only if the dynamic error is above that value.

A simulation of the measurement expected for this specific instrument, the temperature profile of Titan's atmosphere, with the worst assumption of a wide spectrum temperature with bandwidth up to the Nyquist limit, has proven the method capable of

reducing the measurement error by up to 40% with an average of 20%.

Appendix A. Relationship between time constants and the correction factor

Referring to the lumped scheme of Fig. A.1 [2] both fine and coarse sensors can be described by a differential equation of the kind:

$$\tau_1 \tau_2 \frac{d^2 T}{dt^2} + 2(\tau_1 + \tau_2) \frac{dT}{dt} + T = T_a + \tau_3 \frac{dT_a}{dt} \quad (\text{A.1})$$

Where T is the measured temperature (T_f for fine sensor and T_c for the coarse one), T_a is the atmospheric temperature and t the time. Time constants, τ , are obviously different for coarse and fine sensor but all can be expressed as functions of the physical parameters of the scheme, C_f , thermal capacity of the wire, C_s thermal capacity of the structure, R_i thermal resistance between wire and structure, R_s thermal resistance due to the convective heat exchange between structure and atmosphere, R_a , thermal resistance due to the convective heat exchange between wire and atmosphere; for instance τ_1 of the coarse is given by relation (A.2).

In case of the coarse sensor, τ_3 is almost coincident with τ_2 and much smaller than τ_1 so that the behaviour can be approximated by a first order equation, so that τ_1 (indicated in the following as τ_{c1} to avoid mismatch with the corresponding parameter of the fine sensor) is sufficient for its characterisation.

Being all the four time constants depending on the five parameters of the lumped scheme, the ratio

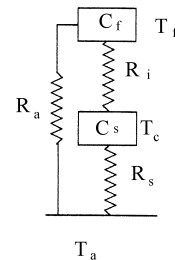


Fig. A.1. Thermometer lumped scheme.

