1 Real gas equations

In the context of the PPI experiment, there are two entities that must be modified because of deviations from an ideal gas case: the equation of state and the heat capacity. If we consider a mixture of real gases with given mole fractions u_i, $\sum_i u_i = 1$, the equation of state can be written as

$$p = cRT(1 + Z(c,T)) \tag{1}$$

where $R = 8.3143\,\mathrm{J\,/\,mol\,K}$ is the universal gas constant, T temperature and $p = \sum_i p_i$, $c = \sum_i c_i$ and $Z(c,T) = \sum_i u_i Z_i(c_i,T)$ are the pressure, concentration ($c = \rho/m$, where ρ is the density and $m = \sum_i u_i m_i$ the mean molecular mass) and virial expansion of the mixture as defined by their respective partial quantities $p_i = u_i p$ and $c_i = u_i c$. For minor species like Argon and Methane, it is sufficient to approximate the virial expansion by the second virial coefficient

$$Z(c,T) = B(T)c = c\sum_{i=0}^{n-1} a_i\left(\frac{T_0}{T} - 1\right)^i \tag{2}$$

where the coefficients a_i are given in Table 1 and the reference temperature $T_0 = 298.15\,\mathrm{K}$. For Nitrogen, the major species in the atmosphere of Titan, a more detailed expansion can be written as

$$\begin{aligned} Z(c,T) = \;& (a_0 + \tfrac{a_1}{\sqrt{T}} + \tfrac{a_2}{T} + \tfrac{a_3}{T^2} + \tfrac{a_4}{T^3})c + \tfrac{a_5}{T^3}c^2 \\ & + (\tfrac{a_6}{T^3} + \tfrac{a_7}{T^4})e^{-\gamma c^2}c^2 + \tfrac{a_8}{T^5}e^{-\gamma c^2}c^4 \end{aligned} \tag{3}$$

where the coefficients a_i are given in Table 1 and the constant $\gamma = 5.6 \cdot 10^{-6}\,\mathrm{m^3/mol}$. The virial expansion can be used to derive functional forms for the following quantities: volume expansivity α_p

$$\alpha_p = -\frac{1}{c}\left(\frac{\partial c}{\partial T}\right)_p = \frac{1}{T}\frac{1 + \frac{T}{1+Z}\left(\frac{\partial Z}{\partial T}\right)_c}{1 + \frac{c}{1+Z}\left(\frac{\partial Z}{\partial c}\right)_T} \tag{4}$$

and the isothermal compressibility κ_T

$$\kappa_T = \frac{1}{c}\left(\frac{\partial c}{\partial p}\right)_T = \frac{1}{p}\left(1 + \frac{c}{1+Z}\left(\frac{\partial Z}{\partial c}\right)_T\right)^{-1} \tag{5}$$

Turning our attention now to the heat capacity, an adequate approximation for the specific isochoric heat capacity for an ideal gas can be expressed as

$$c_V^i(T) = \frac{R}{m_i}\left(\frac{n_i}{2} + \sum_j b_{ij}\frac{x_{ij}^2 e^{x_{ij}}}{(e^{x_{ij}} - 1)^2}\right) \tag{6}$$

where the mean molar masses m_i, degrees of freedom n_i and relevant vibrational constants b_{ij} and temperatures $x_{ij}(T) = T_{ij}/T$ are given in Table 2 for the different species. The isochoric heat capacity of the mixture is then

$$c_V(c,T) = \frac{1}{m}\sum_i u_i m_i c_V^i(T) + \frac{R}{m}T^2\alpha_p\left(\frac{\partial Z}{\partial T}\right)_c \tag{7}$$

and the isobaric heat capacity is

$$c_p = c_V + \frac{T\alpha_p^2}{mc\kappa_T} \tag{8}$$

Now the relation between the ambient atmospheric pressure and the total pressure measured by PPI can be written for subsonic flow conditions as

$$p = p_t\left(1 + \frac{v^2}{2c_pT}\right)^{-mc_p/R(1+Z)} \tag{9}$$

where v is the velocity of the probe relative to the atmosphere.

Assuming a hydrostatic equilibrium, the relation between changes in pressure and height can be written

$$-\frac{dp}{p} = \frac{mg}{RT(1+Z)}dz \tag{10}$$

where g is the local gravitational acceleration. In addition to that, the equation of motion for the probe can be written as

$$\boldsymbol{a} = -g\hat{\boldsymbol{e}}_r - \rho v\boldsymbol{v}\frac{SC_D}{2M} \tag{11}$$

where a, S, C_D and M are the acceleration, reference area, drag coefficient and mass of the probe, respectively. The descent profile of the probe can now be reconstructed by first using Eq. 10 from ground up to make a rough estimate of the profile, hence giving $g(t)$, and then iteratively using Eq. 11 by alternating between determining the quantity $SC_D/2M$ for different probe configurations, and integrating the profile from the beginning of the measuring sequence to the ground.

		a_i	
i	Ar	CH_4	N_2
0	$-1.6 \cdot 10^{-5}$	$-4.3 \cdot 10^{-5}$	$1.66015 \cdot 10^{-5}$
1	$-6.0 \cdot 10^{-5}$	$-1.14 \cdot 10^{-4}$	$1.30439 \cdot 10^{-3}$
2	$-1.0 \cdot 10^{-5}$	$-1.9 \cdot 10^{-5}$	-0.0297238
3		$-7.0 \cdot 10^{-6}$	0.415580
4			-51.4741
5			0.225089
6			-0.206832
7			-1.45900
8			$6.75930 \cdot 10^{-5}$

Table 1: Coefficients for virial expansions of Argon, Methane and Nitrogen as given by Eqs. 2 and 3 for concentration in mol / m^3 and temperature in Kelvin.

	Ar	CH_4	N_2
m	0.03995	0.01604	0.02802
n	3	6	5
b_0		0.008449	1.0054
b_1		4.6942	
b_2		3.4865	
T_0		648	3353
T_1		1957	
T_2		3895	

Table 2: Molar masses m (kg / mol), degrees of freedom n, vibrational coefficients b_j and temperatures T_j (K) for different species as used in Eq. 6.

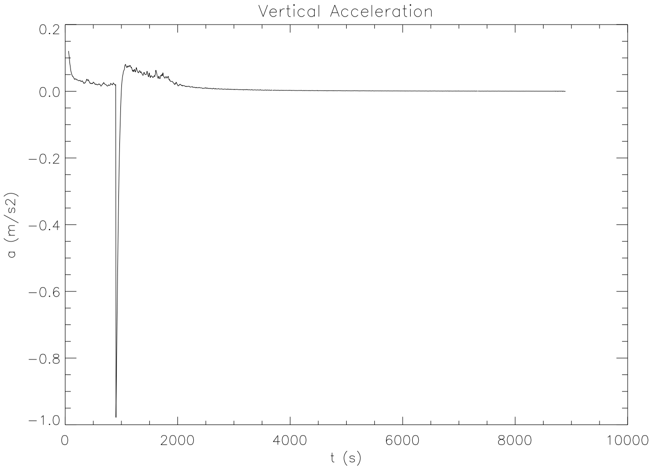

Figure 1: Vertical acceleration derived from the equation of motion (11) of the probe.

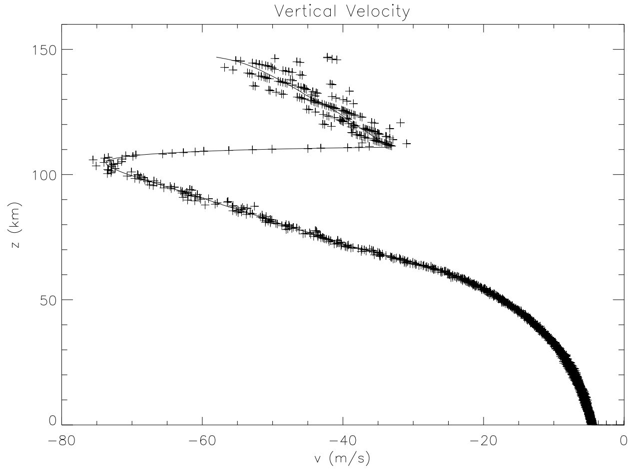

Figure 2: Vertical velocity derived from Eq. 11 (solid line) and from pressure differential (crosses).

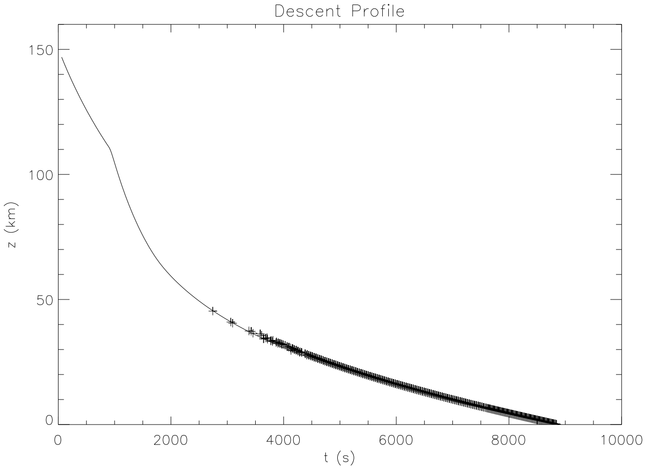

Figure 3: Altitude of the probe measured by PPI (solid line) and radar altimeter (crosses).

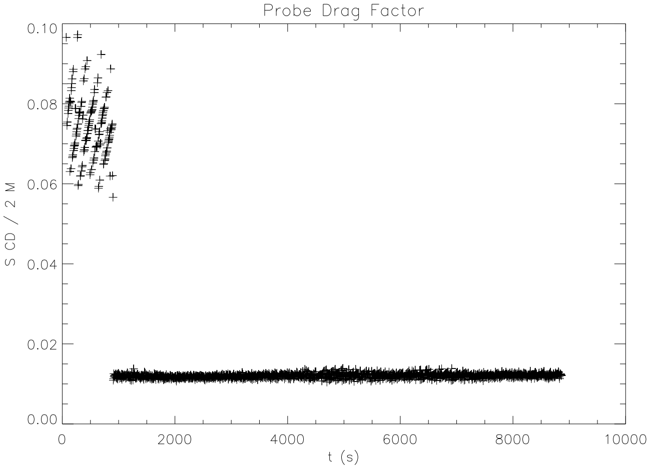

Figure 4: Drag factor $SC_D/2M$ derived from PPI data.

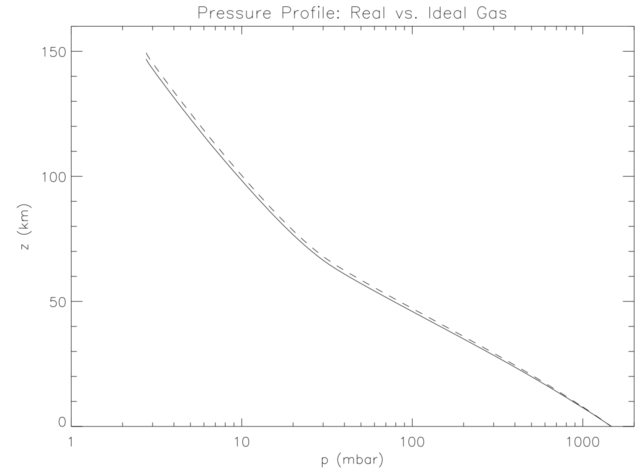

Figure 5: Ambient pressure as a function of altitude derived from real (solid line) and ideal (dashed line) gas equations.

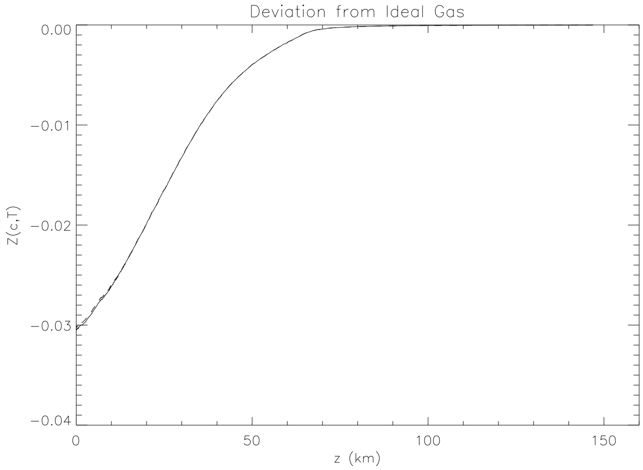

Figure 6: The total virial expansion $Z(c, T)$ as a function of altitude (solid line) and contribution from Nitrogen (dashed line).

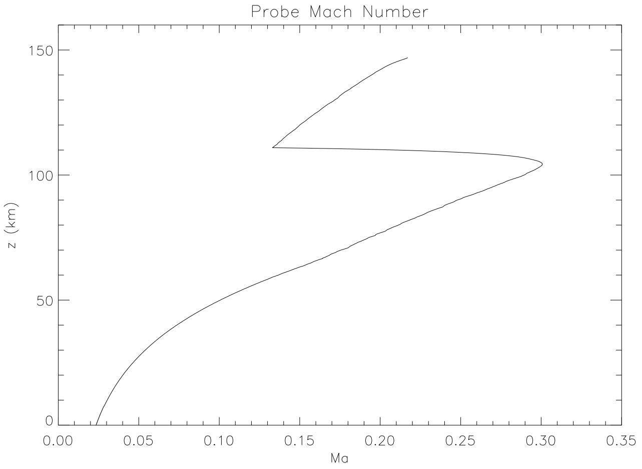

Figure 7: Mach number v/c_s of the probe as a function of altitude.

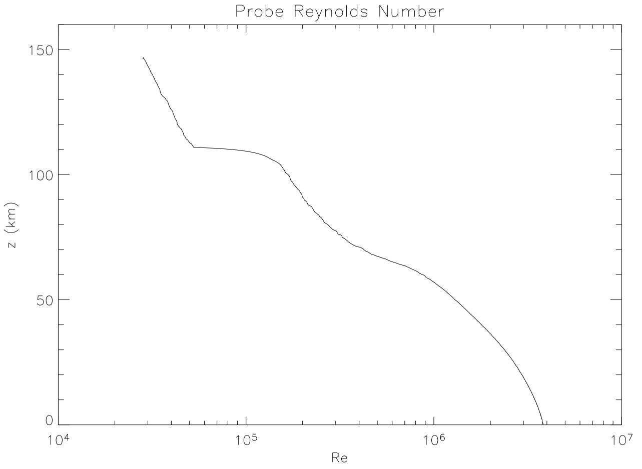

Figure 8: Reynolds number $\rho l v/\eta$ of the probe as a function of altitude.