

# **ROSETTA MARS EXPRESS VENUS EXPRESS**

## **Radio Science Experiments RSI / MaRS / VeRa**

### **IFMS Doppler Processing and Calibration Software: Level 1a to Level 2 Software Design Specifications**

**Issue: 4**  
**Revision: 2**  
**Date: 24.01.2005**  
**Document: MEX-MRS-IGM-DS-3035**

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## ACRONYMS

A/D	Analog/Digital
AGC	Automatic Gain Control
AGVTP	Archive Generation, Validation and Transfer Plan
AOL	Amplitude Open Loop
ATDF	Archival Tracking Data Format
CD-ROM	Compact Disk - Read Only Memory
CL	Closed-Loop
DDS	Data Delivery System
DSN	Deep Space Network
DVD	Digital Versatile Disk
ESA	European Space Agency
ESOC	European Space Operation Center
ESTEC	European Space Technology Center
FOL	Frequency Open Loop
G/S	Ground Station
HGA	High Gain Antenna
IFMS	Intermediate Frequency Modulation System
JPL	Jet Propulsion Laboratory
LCP	Left Circular Polarization
LGA	Low Gain Antenna
LOS	Line Of Sight
MaRS	Mars Express Radio Science Experiment
MGA	Medium Gain Antenna
MGS	Mars Global Surveyor
NASA	National Aeronautics and Space Administration
ODR	Original Data Record
OL	Open-Loop
ONED	one-way dual-frequency mode
ONES	One-way single-frequency mode
PDS	Planetary Data System
POL	Polarization Open Loop
RCP	Right Circular Polarization
RSR	Radio Science Receiver
RX	Receiver
S/C	Spacecraft
SIS	Software Interface Specification
S-TX	S-Band Transmitter

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SPICE	Space Planet Instrument C-Matrix Events
TBC	To Be Confirmed
TBD	To Be Determined
TWOD	Two-way dual-frequency mode
TWOS	Two-way single-frequency mode
USO	Ultra Stable Oszillator
X-TX	X-band Transmitter



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## 1 INTRODUCTION

### 1.1 SCOPE

This document specifies the requirements for the development of the IFMS processing software, transferring Level 1a IFMS data towards Level 2. The software shall analyze radio Doppler tracking data recorded at the IFMS receiving systems of the ground stations New Norcia (NNO). AGC and meteo data are handled via the IFMS (tbd) software.

### 1.2 REFERENCED DOCUMENTS

	Reference Number	Title	Issue Number	Date
[1]	MEX-MRS-IGM-IS-3016	Radio Science File naming Convention	9.6	22.10.2004
[2]	IFMS_OCCFTP_10_3_1	IFMS-to-OCC	10.3.1	15.09.2004
[3]	MEX-MRS-IGM-DS-3039	Radio Science Predicted and Reconstructed Orbit Data: Specifications	1.0	07.01.2004

### 1.3 DOCUMENT OVERVIEW

Section 2 defines the design specifications:

2.1 the input file names and used constants

2.2 defines MODULE PREDICT

2.3 defines MODULE DOPPLER

Section 3 gives an overview of the output file name definitions.

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## 2 SOFTWARE DESIGN SPECIFICATIONS

### 2.1 MAIN PROGRAM SPECIFICATIONS

The MAIN program shall read the IFMS level 1a Doppler data both at S-band and X-band, compute  $\frac{V_{LOS}}{c}$  and the sky frequencies. A detailed explanation of the computation is given in section 2.2. After reading the data the MAIN program shall correct the Doppler data for the contribution by the propagation through the plasma and the Earth troposphere. That step will be done in Module M\_CALIBRATION. Doppler residuals will be computed from the predicted or reconstructed Doppler provided by M\_PREDICT. The output data files shall be produced via M\_DOPPLER\_OUTPUT.

#### 2.1.1 Modules

The MAIN program uses a number of modules:

1. M\_READ\_INPUT\_DATA
2. M\_PREDICT
3. M\_DOPPLER\_SHIFT
4. M\_CALIBRATION
5. M\_IONO\_CALIB
6. M\_DOPPLER\_OUTPUT
7. M\_DIFFERENTIAL DOPPLER
8. M\_GLOBAL\_VAR

and some general modules, wherein shared subroutines and functions are provided

9. M\_FILE\_UTILITIES
10. M\_READ\_HEADER
11. M\_ERROR
12. M\_UTILITIES
13. M\_FILE\_NAMING\_CONVENTION
14. M\_LABELNAMEIFMS
15. M\_LABEL
16. M\_INTERPOL
17. M\_SEARCH

The flow diagram is shown in section 2.1.4

## 2.1.2 Input files

### 2.1.2.1 *Data file types*

**IFMS-SPEC-1210:** the following table defines the input file types and the logical file names used in this specification and within the program. Klobuchar coefficients are only needed if X-band and S-band Doppler files have no overlapping timestamp or the kind of data processing is occultation:

File type	Logical name within program
IFMS level 1a Doppler X-band S-band	IFMS_DOPPLER_X IFMS_DOPPLER_S
predicted Doppler file	PREDICT_FILE
IFMS_Meteo file level 1a	IFMS_METEO
IFMS AGC file level 1a	IFMS_AGC_X IFMS_AGC_S
Klobuchar coefficients for Earth ionosphere calibration	ION_COEFF
Orbit SPICE Kernels	N/A

### 2.1.2.2 *File names*

**IFMS-SPEC-1220:** File names are defined in [1] section 4.1

### 2.1.2.3 *File formats*

**IFMS-SPEC-1230:** File formats are defined in [1] in section 5.2, section 8 and section 9

### 2.1.3 Definition of constants

**IFMS-DEF-1010: ASTRONOMICAL UNIT (AU)**

$$1 \text{ AU} = 149,597,870 \text{ kilometers}$$

**IFMS-DEF-1020: SPEED OF LIGHT**

$$c = 299,792,458 \text{ m/s}$$

**IFMS-DEF-1025: PHYSICAL CONSTANTS**

Constant		Value	SI units
Electron charge	e	$1.6022 \cdot 10^{-19}$	A s
Electron mass	$m_e$	$9.1094 \cdot 10^{-31}$	kg
Electric field constant	$\epsilon_0$	$8.8542 \cdot 10^{-12}$	$\text{s}^4 \text{ A}^2 \text{ m}^{-3} \text{ kg}^{-1}$
Plasma constant	$\frac{1}{2} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0}$	40.30924	$\text{m}^3 \text{ s}^{-2}$

**IFMS-DEF-1030: CARRIER FREQUENCIES Mars Express (nominal)**

**Mars Express:**

frequency band	uplink	downlink
S-band	2114.676 MHz	2296.482 MHz
X-band	7116.936 MHz	8420.432 MHz

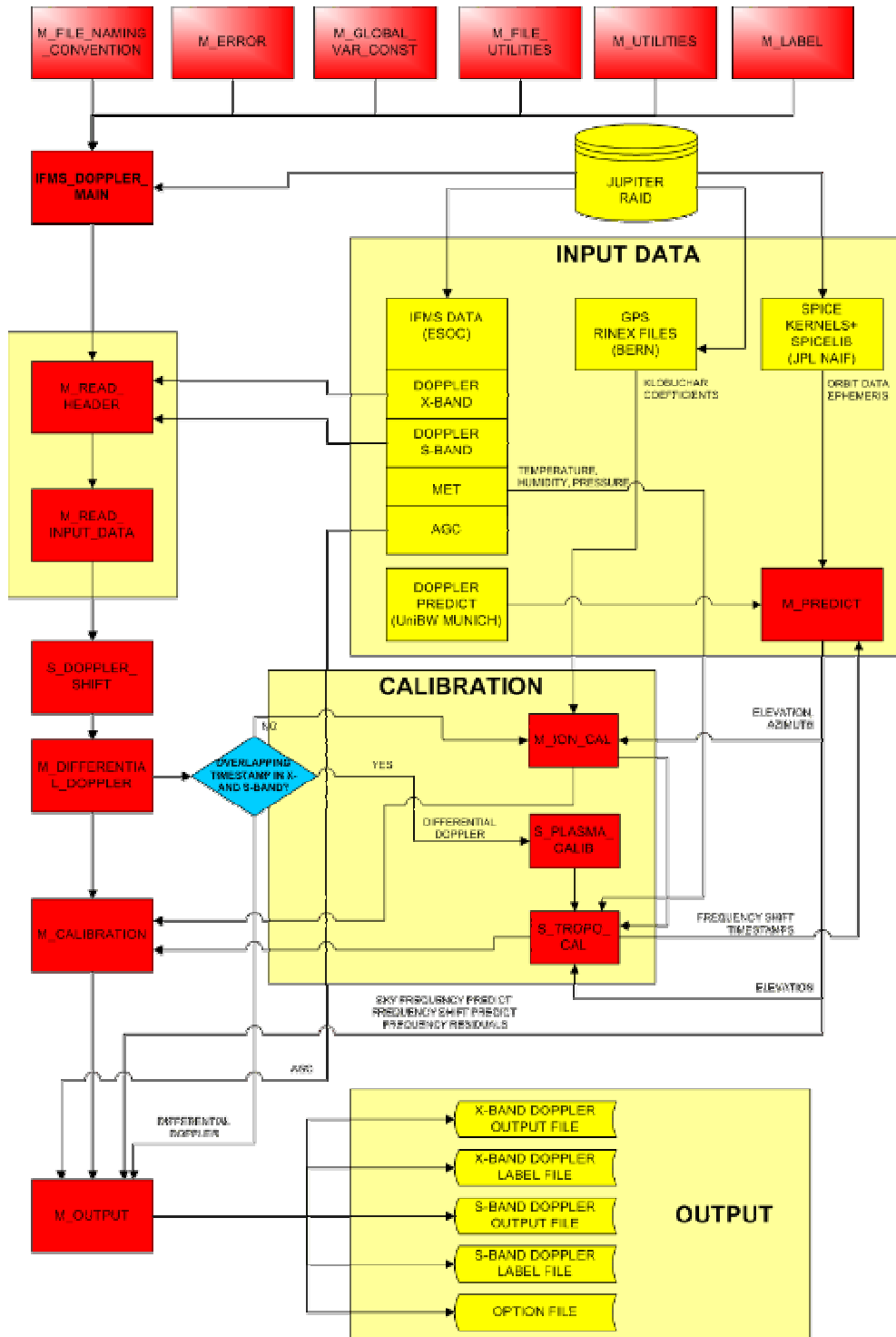
Actual transmitted frequencies (up and downlink) may vary according to expected Doppler shift (approx. 10 – 100 kHz).

**IFMS-DEF-1031: Transponder constants and ratios**

**Mars Express:**

frequency band uplink	transponder ratios downlink/uplink	
	S-band	X-band
S-band	240/211	880/211
X-band	240/749	880/749

### 2.1.4 Flow Diagram



Figur 1: Flowchart for evaluation software for the IFMS Doppler data



## 2.2 MODULE M\_READ\_INPUT\_DATA AND M\_READ\_HEADER

M\_READ\_HEADER reads the header and the active table from IFMS\_DOPPLER\_X, IFMS\_DOPPLER\_S, IFMS\_METEO, IFMS\_AGC\_X and IFMS\_AGC\_S.

M\_READ\_INPUT\_DATA reads IFMS level 1a Doppler data at X-band and S-band, the meteorological data, the AGC data at X-band and S-band, respectively from IFMS\_DOPPLER\_X, IFMS\_DOPPLER\_S, IFMS\_METEO, IFMS\_AGC\_X and IFMS\_AGC\_S.

**IFMS-SPEC-2305:** M\_READ\_HEADER accepts the information contained in the header and active table of IFMS\_DOPPLER\_X, IFMS\_DOPPLER\_S, IFMS\_METEO, IFMS\_AGC\_X and IFMS\_AGC\_S.

**IFMS-SPEC-2310:** M\_READ\_INPUT\_DATA accepts Doppler data from IFMS\_DOPPLER\_X, IFMS\_DOPPLER\_S, IFMS\_METEO, IFMS\_AGC\_X and IFMS\_AGC\_S.

**IFMS-SPEC-2315:** The file name formats are defined according to [1] section 5.2.

**IFMS-SPEC-2316:** The file formats are defined according to [2] and [1] section 5.2.

**IFMS-SPEC-2320:** IFMS\_DOPPLER\_X, IFMS\_DOPPLER\_S, IFMS\_METEO, IFMS\_AGC\_X and IFMS\_AGC\_S file names will be accepted via a Windows interface.

**IFMS-SPEC-2321:** The kind of data processing for (a) occultation entry, (b) occultation exit, (c) gravity, and (d) solar corona is selected via an option file.

## 2.3 SUBROUTINE S\_DOPPLER\_SHIFT

IFMS-DEF-2400: Definitions:

Acronyms as specified in [2]	Symbol	Explanation	SI units
Count	count( $t_i$ )	count at a given timestamp $t_i$	
DeltaCount	$\Delta$ count( $t_i$ )	Variation of the counts of a numerical clock between two timestamps $t_i$ and $t_{i-1}$ , i.e nodes of the 17.5 MHz signal	
DeltaTime	$\Delta$ time	Time interval between two timestamps $t_i$ and $t_{i-1}$	sec
Phase	phase( $t_i$ )	measured phase in cycles	cycles
DeltaPhase	$\Delta$ phase	Variation in phase between two timestamps $t_i$ and $t_{i-1}$	cycles
ActualCarrierFreqOffset	$f_{\text{offset,up}}$	Expected uplink Doppler shift <b>Contained in ActiveTable</b>	Hz
InputCarrierFreqOffset	$f_{\text{offset,down}}$	expected uplink Doppler shift multiplied by the transponder ratio factor $k$ defined in IFMS-DEF-1031; subtracted from expected nominal uplink frequency received at spacecraft	Hz
ZeroDopplerDeltaPhase	$\Delta$ phase <sub>Dop=0</sub>	Phase change in $\Delta$ time assuming that Doppler shift is zero	cycles
DeltaPhaseDoppler	$\Delta$ phase <sub>Dop</sub>	Phase change in $\Delta$ time for the true Doppler shift	cycles
UlmCarFrSel	$f_{\text{inter}}$	Intermediate frequency after downconversion Ca. 70 MHz or 230 MHz <b>Contained in ActiveTable</b>	Hz
XxxUplkConv	$f_{\text{LO}}$	Ground station local oscillator frequency in order to generate true uplink frequency Ca. 7100 MHz or 2100 MHz <b>Contained in ActiveTable</b>	Hz
UplinkCarrierFreq	$f_{\text{up}}$	transmitted uplink carrier frequency	Hz

**IFMS-SPEC-2401:** computation of antenna frequency fantenna

$$\Delta count(t_i) = count(t_i) - count(t_{i-1})$$

$$\Delta time = \frac{\Delta count}{17.5 \times 10^6}$$

$$\Delta phase(t_i) = phase(t_i) - phase(t_{i-1})$$

$$f_{offset,down} = k \cdot f_{offset,up}$$

$$\Delta phase_{Dop=0} = \Delta time \cdot f_{offset,down}$$

$$\Delta phase_{Dop}(t_i) = \Delta phase(t_i) - \Delta phase_{Dop=0}$$

$$f_{up} = f_{offset,up} + f_{inter} + f_{LO}$$

$$f_{antenna}(t_i) = k \cdot f_{up} + \frac{\Delta phase_{Dop}(t_i)}{\Delta time(t_i)}$$

For detailed information see in [2] section 6.3.

**IFMS-SPEC-2402:** cancelled

**IFMS-SPEC-2403:** The uplink frequency  $f_{up}$  (result from IFMS-SPEC-2401) is transferred to M\_DOPPLER\_OUTPUT and stored in column 7.

**IFMS-SPEC-2404:** Column 8 in M\_DOPPLER\_OUTPUT (uplink frequency ramp rate) is set to zero.

**IFMS-SPEC-2345:** The antenna frequency  $f_{antenna}$  (result from IFMS-SPEC-2401) is transferred to M\_DOPPLER\_OUTPUT and stored in column 9.

## 2.4 MODULE M\_CALIBRATION

M\_CALIBRATION corrects for the contribution by the propagation through the plasma and the neutral Earth troposphere via the subroutines S\_PLASMA\_CALIB and S\_TROPO\_CALIB.

Plasma media correction can only be performed if two downlink frequencies have been recorded and is done only for gravity observations. If only one frequency is available or for occultation observations, the Earth ionosphere is corrected via the Klobuchar model.

### IFMS-SPEC-2360: Tropospheric calibration

The path delay (unit is meter) of the dry and wet component of the Earth troposphere is (Hofmann-Wellenhoff et al., Global Positioning System, 4<sup>th</sup> Ed.):

$$\Delta_{dry}(E) = \frac{10^{-6}}{5} \frac{77.64 \frac{p}{T}}{\sin(\sqrt{E^2 + 6.25})} [40136 + 148.72(T - 273.16)] \quad (1.1)$$

$$\Delta_{wet}(E) = \frac{10^{-6}}{5} \frac{-12.96T + 3.718 \cdot 10^5}{\sin(\sqrt{E^2 + 2.25})} \frac{e}{T^2} 11000$$

where  $p$ ,  $T$  and  $e$  are the atmospheric pressure, Temperature and partial water vapour pressure, respectively, as observed at the ground station site.

These values are given in the IFMS\_METEO file. The elevation angle  $E$  (unit in degrees) is provided by M\_PREDICT.

The following transformations have to be applied:

	equation (20)	IFMS_METEO	M_PREDICT
<b>pressure <math>p</math></b>	mbar	hPascal	-
<b>Temperature <math>T</math></b>	Kelvin	°Celsius	-
<b>Water vapour partial pressure <math>e</math></b>	hPascal	-	-
<b>humidity <math>h</math></b>	-	% humidity	-
<b>elevation <math>E</math></b>	degrees	-	radian

The relation between the water vapour partial pressure and the humidity given in IFMS\_METEO is:

$$e = 6.108 \cdot 10^{-2} \cdot \text{humidity} [\%] \cdot \exp \left\{ \frac{17.393(T - 272.15)}{T - 33.95} \right\} \quad (1.2)$$

The total tropospheric calibration expressed as delay time in seconds is:

$$\tau_{tropo} = \frac{2}{c} \{ \Delta_{dry}(E) + \Delta_{wet}(E) \} \quad (1.3)$$

for the two-way radio link where  $c$  is the speed of light with definition given in IFMS-DEF-1020 and

$$\tau_{tropo} = \frac{1}{c} \{ \Delta_{dry}(E) + \Delta_{wet}(E) \}$$

for the one-way radio link.

**IFMS-SPEC-2361:** The correction for the Earth troposphere is then for one-way radio link:

$$m_{ONE} = \tau_{tropo} \cdot f_{down} \quad (1.4)$$

and for the two-way radio link:

$$m_{TWO} = \tau_{tropo} \cdot (f_{down} + f_{up}) \quad (1.5)$$

where  $m$  is the cycle advance and the shift in frequency is:

$$\Delta f_{ONE,tropo} = \frac{dm_{ONE}}{dt} \quad (1.6)$$

and for the two-way radio link:

$$\Delta f_{TWO,tropo} = \frac{dm_{TWO}}{dt} \quad (1.7)$$

This is done for each frequency band.

**IFMS-SPEC-2362:** The result from IFMS-SPEC-2361 is transferred to M\_DOPPLER\_OUTPUT and stored in column 12.

### **IFMS-SPEC-2363: Tropospheric Correction**

The Doppler corrected for the troposphere is then

$$f_{antenna}|_{tropo\_corrected} = f_{antenna} - \Delta f_{carrier,tropo} \quad (1.8)$$

This is done for each frequency band.

**IFMS-SPEC-2364:** The result from IFMS-SPEC-2361 is transferred to M\_DOPPLER\_OUTPUT and stored in column 10.

### **IFMS-SPEC-2365: Differential Doppler**

The result from IFMS-SPEC-2363 is taken to compute the differential Doppler

$$\delta f = f_{S,antenna}|_{tropo\_corrected} - \frac{3}{11} f_{X,antenna}|_{tropo\_corrected} \quad (1.9)$$

### **IFMS-SPEC-2366: Plasma calibration**

Derive the temporal change in electron content from the differential Doppler and computes the dispersive frequency shift for each frequency band.  $f_s$  and  $f_x$  are downlink carrier frequencies and  $c$  is the speed of light, all defined in section 1.

$$\delta f = -\frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \left\{ \frac{1}{f_S^2} - \frac{1}{f_X^2} \right\} f_S \frac{dl}{dt}$$

$$\Rightarrow \frac{dl}{dt} = -\left\{ \frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \right\}^{-1} \frac{\delta f}{f_S} \left\{ \frac{1}{f_S^2} - \frac{1}{f_X^2} \right\}^{-1} \quad (1.10)$$

**IFMS-SPEC-2367: plasma correction**

The temporal change in electron content will be used to correct for the downlink plasma propagation for gravity observations only:

$$f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} + \frac{40.31}{c} \frac{1}{f_S} \frac{dl}{dt}$$

$$f_{X,antenna,cal} = f_{X,antenna}|_{tropo\_corrected} + \frac{40.31}{c} \frac{1}{f_X} \frac{dl}{dt} \quad (1.11)$$

If equation (1.10) is applied to equation (1.11), the plasma correction is than

$$f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} - \delta f \frac{121}{112}$$

$$f_{X,antenna,cal} = f_{X,antenna}|_{tropo\_corrected} - \delta f \frac{33}{112} \quad (1.12)$$

**FOR FURTHER DETAILS SEE APPENDIX B**

**IFMS-SPEC-2368:** The result from IFMS-SPEC-2367 is transferred to M\_DOPPLER\_OUTPUT and stored in column 10.

## 2.5 MODULE M\_IONO\_CAL

If only one frequency is available, the Earth ionosphere plasma has to be modeled. Module M\_IONO\_CAL models the electron content of the Earth ionosphere at any local time and pointing direction of the ground station antenna and determines the path delay. This is done using the Klobuchar model introducing the Klobuchar coefficients from GPS measurements.

The module is currently only valid for the NNO ground station.

**IFMS-SPEC-2610:** Module M\_IONO\_CAL accepts the actual needed Klobuchar coefficients from input file ION\_COEFF. The input file can be downloaded from

***<ftp.unibe.ch/aiub/CODE/>***

$$\tau = \frac{40.31}{c} \left( \frac{1}{f_s^2} - \frac{1}{f_x^2} \right) \cdot I$$
$$\Rightarrow I = \frac{c}{40.31} \left( \frac{1}{f_s^2} - \frac{1}{f_x^2} \right)^{-1} \cdot \tau$$

Berechnung der Ableitung von I nach der sample time und daraus wie bei Calibration mit differentielltem Doppler rechnen.

## 2.6 MODULE M\_PREDICT

M\_PREDICT accepts a Doppler predict file: the predict file PREDICT\_FILE considers all possible perturbing forces as the best known gravity field and solar and albedo radiation pressure. For more details about the PREDICT\_FILE see document [3]. M\_PREDICT interpolates for a given time stamp between the predicted parameters and returns an estimated parameter for each observed time stamp. This is done for each frequency band.

**IFMS-SPEC-2210:** M\_PREDICT accepts input data from PREDICT\_FILE with the file name format defined in [1] section 8.1 or in [1] section 8.2 for the predicted orbit or the reconstructed orbit file, respectively. PREDICT\_FILE is either a two-way or an one-way file.

**IFMS-SPEC-2220:** M\_PREDICT accepts predicted Doppler data from PREDICT\_FILE (file name specified in IFMS-SPEC-2210) formatted as defined in [1] section 8.1 or or in [1] section 8.2 for the predicted orbit or the reconstructed orbit file, respectively.

**IFMS-SPEC-2225:** M\_PREDICT\_FILE covers a time period of data as defined in [3]. Time stamps defined in [3].

**IFMS-SPEC-2226:** Subroutine S\_DOP\_PRED reads predicted Doppler data from PREDICT\_FILE and computes for each frequency band the predicted shift in frequency  $\Delta f_{pred}$  received at a given ground station via

$$\Delta f_{pred} = k \cdot f_{up} (P_{up} + P_{down} + P_{up} \cdot P_{down})$$

where  $P_{up} = \frac{\Delta f_{up}}{f_{up}} = \frac{v_{r,up}}{c}$  and  $P_{down}$  is the predicted Doppler of the uplink and the downlink path, respectively. The result is stored in the array DOPPLER\_PREDICT. For more details about the computation see Appendix A.

**IFMS-SPEC-2230:** Subroutine S\_DOP\_PRED accepts from M\_READ\_INPUT\_DATA the array TIME\_DOPPLER representing the observed Doppler time stamps. S\_DOP\_PRED interpolates between each predicted parameter of DOPPLER\_PREDICT for each observed time stamp given as TIME\_DOPPLER. This is done for each frequency band.



**IFMS-SPEC-2236:** Subroutine S\_DOP\_PRED reads predicted Doppler data from PREDICT\_FILE and computes for each frequency band the predicted antenna frequency  $f_{pred,antenna}$  received at a given ground station via

$$f_{pred,antenna} = k \cdot f_{up} (1 + P_{up} + P_{down} + P_{up} \cdot P_{down})$$

where  $P_{up} = \frac{\Delta f_{up}}{f_{up}} = \frac{v_{r,up}}{c}$  and  $P_{down}$  is the predicted Doppler of the uplink and the downlink path, respectively. The result is stored in the array DOPPLER\_PREDICT\_SKY. For more details about the computation see Appendix A.

**IFMS-SPEC-2240:** The interpolated results will be provided as the arrays DOPPLER\_PREDICT\_INT in subroutine S\_DOP\_PRED. The arrays are transferred to the subroutine M\_DOPPLER\_OUTPUT and are stored in column 11 of the output file, respectively. This is done for each frequency band.

**IFMS-SPEC-2250:** Subroutine S\_DOP\_PRED computes for each frequency band the frequency residuals  $\Delta f_{res}$  by subtracting the interpolated predicted antenna frequency  $\Delta f_{pred,antenna}$  stored in the array DOPPLER\_PREDICT\_INT from the measured and calibrated antenna frequency  $f_{antenna,cal}$  stored in the array DOPPLER\_CALIB.

$$\Delta f_{res} = f_{antenna,cal} - f_{antenna,pred}$$

**IFMS-SPEC-2251:** The result from IFMS-SPEC-2250 is transferred to M\_DOPPLER\_OUTPUT and stored in column 14.

## 2.7 MODULE M\_DOPPLER\_OUTPUT

### 2.7.1 Data files

**IFMS-SPEC-2396:** The DOPPLER\_OUTPUT file names are defined as

**rggIFMSL02\_sss\_yyddhhmm\_qq.TAB**

The definitions are given in Table 0-1.

**Table 0-1:** DOPPLER\_OUTPUT file name Definition

placeholder	description	example
r	spacecraft name M = MEX R = Rosetta V = VEX	M
gg	ground station xx = ESA Cerbreros 32 = ESA New Norcia	32
IFMS	Data source IFMS = IFMS file	IFMS
L02	Data level L02	L02
sss	File type D1X = X-band Doppler file channel 1 D1S = S-band Doppler file channel 1 D2X = X-band Doppler file channel 2 D2S = S-band Doppler file channel 2	D1X
yy	year	03
ddd	day of year	180
hhmm	start time of data in hour, minute	2345
qq	not used	00
TAB	Extension .TAB data file	TAB

**IFMS-SPEC-2397:** The format of the DOPPLER\_OUTPUT\_X file is defined in Table 2-2. The format of the DOPPLER\_OUTPUT\_S file is defined in Table 2-3.

**IFMS-SPEC-2398:** if only X-band Doppler data exist, the differential Doppler cannot be computed and is set to -99999.999. if only S-band Doppler data exist, the differential Doppler cannot be computed and is set to -99999.999.

**Table 0-2** Definition of DOPPLER\_OUTPUT\_X file format

column	description	unit	resolution
1	Sample number		
2	Ground received time <i>as UTC in ISO format</i>		
3	Ground received time <i>as UTC in fractions of day of year starting with the first day of the year the data was recorded at 00:00.000</i>	days	10 <sup>-10</sup> days
4	Ground received time <i>as elapsed terrestrial barycentric dynamic time (TDB) time since noon of the first calendar day of year 2000 (12:00 1 January 2000 TDB)</i>	sec	10 <sup>-6</sup> sec
5	Geometric impact parameter <i><u>Propagation experiments:</u> approximate value of the closest approach of a downlink geometric ray path to the center of the reference body (Sun, planet, minor object). When two-way, the value is approximate average of uplink and downlink rays</i> <i><u>Gravity observations:</u> geometric distance of the s/c from the center of mass of referenced body</i>	km	10 <sup>-3</sup> m
6	Transmit frequency ramp reference time <i>UTC in ISO format</i> <i>The time (t<sub>0</sub>) at which the transmitted frequency would have been f<sub>0</sub> using the coefficients f<sub>0</sub> (column 7) and df (column 8). At any time t within the interval when those coefficients are valid, the transmitted frequency f<sub>t</sub> may be calculated from</i> $f_t = f_0 + df \cdot (t - t_0)$ <i><u>For DSN two-way measurements:</u></i> <i>f<sub>t</sub> is the uplink frequency of the ground transmitter; the f<sub>t</sub> photon will reach the receiver one RTLT later.</i> <i><u>For DSN one-way measurements:</u></i> <i>f<sub>t</sub> is the downlink frequency of the spacecraft transmitter; the f<sub>t</sub> photon will reach the receiver OWLT later. In both cases, f<sub>0</sub> and df may change; but f<sub>t</sub> is always continuous, and changes in the coefficients occur only on integer seconds.</i> <i><u>For IFMS measurements:</u></i> $f_t = f_0$ <i>because df=0.</i>		

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7	<p>Transmit frequency corresponding to time in column 6</p> <p><u>Two-way coherent modes:</u>  <i>Uplink frequency of ground station</i>  <i>S-band order of 2100 MHz</i>  <i>X-band order of 7100 MHz</i></p> <p><u>One-way mode:</u>  <i>S/C transmission frequency</i>  <i>X-band order of 8400 MHz</i>  <i>S-band order of 2300 MHz</i></p>	Hz	$10^{-6}$ Hz
8	<p>Uplink frequency ramp rate</p> <p><u>DSN two-way coherent:</u>  <i>Time derivative of uplink frequency in column 7</i></p> <p><u>DSN one-way downlink mode:</u>  <i>Value of spacecraft frequency drift, if known and/or meaningful; -99999.999999</i></p> <p><u>IFMS measurements:</u>  <i>Ramp rate is always zero; df=0</i></p>	Hz/sec	$10^{-6}$ Hz/sec
9	<p>Observed X-band antenna frequency</p> <p><i>Frequency of the signal at the terminals of the receiving antenna structure at UTC TIME columns 2 to 4 (<math>t_r</math>). Set to -9999999999.999999 for missing or corrupted data.</i></p>	Hz	$10^{-6}$ Hz
10	<p>Predicted X-band antenna frequency</p> <p><i>Based on the ESOC reconstructed orbit file or SPICE kernels</i></p> <p><i>Expected frequency of the signal at the terminals of the receiving antenna structure at UTC TIME in columns 2 to 4 (<math>t_r</math>). The calculation includes geometrical effects (relative positions and motions of ground station and spacecraft, including Earth rotation and light time adjustments), tuning of both the transmitter and receiver and a model-based correction for one- or two-way (as appropriate) propagation through the Earth's atmosphere.</i></p>	Hz	$10^{-6}$ Hz
11	<p>Correction of Earth atmosphere propagation</p> <p>Correction term for the propagation of the signal in the Earth atmosphere, based on meteorological data observed at the ground station site (MET-files)</p>	Hz	$10^{-6}$ Hz
12	<p>Residual calibrated X-band frequency shift</p> <p><i>column 9 minus 10</i></p>	Hz	$10^{-6}$ Hz
13	<p>Received signal level</p> <p><u>Closed-loop data:</u>  <i>Signal level from AGC in decibels relative</i></p>	dBm / dB	0.1 dB

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	<p>to one milliwatt (dBm).  <u>Open-loop (RSR):</u>  Signal level in decibels (dB) relative to an arbitrary reference.</p>		
14	<p>Differential Doppler</p> $f_s - \frac{3}{11} f_x$ <p>Where <math>f_s</math> and <math>f_x</math> are the received S-band and X-band frequencies  If BAND_NAME = X (from the label file), <math>f_x</math> comes from column 9 in this table and <math>f_s</math> comes from column 9 in the file identified by SOURCE_ID (from the label file).   If BAND_NAME = S (from the label file), <math>f_s</math> comes from column 9 in this table and <math>f_x</math> comes from column 9 in the file identified by SOURCE_ID (from the label file).  if either band is not available, this column is set "-99999.999"</p>	Hz	$10^{-6}$ Hz
15	<p>standard deviation of the observed antenna frequency X-band in column 9 (open-loop only)  for closed-loop this value is set "-99999.999"</p>	Hz	$10^{-6}$ Hz
16	<p>Received X-band signal quality (open-loop only)  Ratio of observed received signal strength to the statistical standard deviation of the measurement, column 15 divided by column 19  For closed-loop this is value is set "-999.9"</p>	dB	0.1 dB
17	<p>standard deviation of received signal level at X-band (open-loop)  A statistical measure of the error in determining SIGNAL LEVEL (column 15) based on fit of a data spectrum to a sinc function. Uses the same arbitrary scale factor as column 15; units of dB.  for closed-loop this is set "-999.9"</p>	dB	0.1 dB

Table 0-3: Definition of DOPPLER\_OUTPUT\_S file format

column	description	unit	resolution
1	Sample number		
2	Ground received time <i>as UTC in ISO format</i>		
3	Ground received time <i>as UTC in fractions of day of year starting with the first day of the year the data was recorded at 00:00.000</i>	days	10 <sup>-10</sup> days
4	Ground received time <i>as elapsed terrestrial barycentric dynamic time (TDB) time since noon of the first calendar day of year 2000 (12:00 1 January 2000 TDB)</i>	sec	10 <sup>-6</sup> sec
5	Geometric impact parameter <i><u>Propagation experiments:</u> approximate value of the closest approach of a downlink geometric ray path to the center of the reference body (Sun, planet, minor object). When two-way, the value is approximate average of uplink and downlink rays</i> <i><u>Gravity observations:</u> geometric distance of the s/c from the center of mass of referenced body</i>	km	10 <sup>-3</sup> m
6	Transmit frequency ramp reference time <i>UTC in ISO format</i> <i>The time (t<sub>0</sub>) at which the transmitted frequency would have been f<sub>0</sub> using the coefficients f<sub>0</sub> (column 7) and df (column 8). At any time t within the interval when those coefficients are valid, the transmitted frequency f<sub>t</sub> may be calculated from</i> $f_t = f_0 + df \cdot (t - t_0)$ <i><u>For DSN two-way measurements:</u></i> <i>f<sub>t</sub> is the uplink frequency of the ground transmitter; the f<sub>t</sub> photon will reach the receiver one RTLT later.</i> <i><u>For DSN one-way measurements:</u></i> <i>f<sub>t</sub> is the downlink frequency of the spacecraft transmitter; the f<sub>t</sub> photon will reach the receiver OWLT later. In both cases, f<sub>0</sub> and df may change; but f<sub>t</sub> is always continuous, and changes in the coefficients occur only on integer seconds.</i> <i><u>For IFMS measurements:</u></i> $f_t = f_0$ <i>because df=0.</i>		

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7	<p>Transmitted frequency corresponding to time in column 6</p> <p><u>Two-way coherent modes:</u>  <i>Uplink frequency of ground station</i>  <i>S-band order of 2100 MHz</i>  <i>X-band order of 7100 MHz</i></p> <p><u>One-way mode:</u>  <i>S/C transmission frequency</i>  <i>X-band order of 8400 MHz</i>  <i>S-band order of 2300 MHz</i></p>	Hz	$10^{-6}$ Hz
8	<p>Uplink frequency ramp rate</p> <p><u>DSN two-way coherent:</u>  <i>Time derivative of uplink frequency in column 7</i></p> <p><u>DSN one-way downlink mode:</u>  <i>Value of spacecraft frequency drift, if known and/or meaningful; -99999.999999</i></p> <p><u>IFMS measurements:</u>  <i>Ramp rate is always zero; df=0</i></p>	Hz/sec	$10^{-6}$ Hz/sec
9	<p>Observed S-band antenna frequency</p> <p><i>Frequency of the signal at the terminals of the receiving antenna structure at UTC TIME columns 2 to 4 (<math>t_r</math>). Set to -9999999999.999999 for missing or corrupted data.</i></p>	Hz	$10^{-6}$ Hz
10	<p>Predicted S-band antenna frequency</p> <p><i>Based on the ESOC reconstructed orbit file or SPICE kernels</i></p> <p><i>Expected frequency of the signal at the terminals of the receiving antenna structure at UTC TIME in columns 2 to 4 (<math>t_r</math>). The calculation includes geometrical effects (relative positions and motions of ground station and spacecraft, including Earth rotation and light time adjustments), tuning of both the transmitter and receiver and a model-based correction for one- or two-way (as appropriate) propagation through the Earth's atmosphere.</i></p>	Hz	$10^{-6}$ Hz
11	<p>Correction of Earth atmosphere propagation</p> <p><i>Correction term for the propagation of the signal in the Earth atmosphere and ionosphere, based on meteorological data observed at the ground station site (MET-files)</i></p>	Hz	$10^{-6}$ Hz
12	<p>Residual calibrated X-band frequency shift</p> <p><i>column 9 minus 10</i></p>	Hz	$10^{-6}$ Hz
13	<p>Received S-band signal level</p>	dBm /	0.1 dB

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	<p><u>Closed-loop data:</u>  Signal level from AGC in decibels relative to one milliwatt (dBm).</p> <p><u>Open-loop (RSR):</u>  Signal level in decibels (dB) relative to an arbitrary reference.</p>	dB	
14	<p>Differential Doppler</p> $f_s - \frac{3}{11} f_x$ <p>Where <math>f_s</math> and <math>f_x</math> are the received S-band and X-band frequencies  If BAND_NAME = X (from the label file), <math>f_x</math> comes from column 9 in this table and <math>f_s</math> comes from column 9 in the file identified by SOURCE_ID (from the label file).   If BAND_NAME = S (from the label file), <math>f_s</math> comes from column 9 in this table and <math>f_x</math> comes from column 9 in the file identified by SOURCE_ID (from the label file).  if either band is not available, this column is set "-99999.999"</p>	Hz	$10^{-6}$ Hz
15	<p>standard deviation of the observed antenna frequency S-band in column 9 (open-loop only)  for closed-loop this value is set "-99999.999"</p>	Hz	$10^{-6}$ Hz
16	<p>Received S-band signal quality (open-loop only)  Ratio of observed received signal strength to the statistical standard deviation of the measurement, column 15 divided by column 19  For closed-loop this is value is set "-999.9"</p>	dB	0.1 dB
17	<p>standard deviation of received signal level at S-band (open-loop)  A statistical measure of the error in determining SIGNAL LEVEL (column 15) based on fit of a data spectrum to a sinc function. Uses the same arbitrary scale factor as column 15; units of dB.  for closed-loop this is set "-999.9"</p>	dB	0.1 dB



## 2.7.2 Additional Files

The Module M\_OUTPUT generates an additional output file a so called option file. This file contains the processing mode, the whole path of all input files, additional information like downlink and uplink frequency in Hz, the sample rate in samples per seconds and error messages. The option file will not be distributed and is only intended for internal use. Therefore the filename of the option file is not complying with [1]. The file name is defined as

option\_vv.log

where vv stands for a succeeding number. An example of an option file is shown in Figure 0-1.

```
OC
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\NN13_MEX1_2004_093_
OP_D1_110549_0002
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\NN11_MEX1_2004_093_
OP_D1_110358_0002
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\NN11_MEX1_2004_093_
OP_ME_110224_0000
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\CGIM0930.04N
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\predict_093.txt
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\NN13_MEX1_2004_093_
OP_G1_110608_0002
Z:\processed_temp\mex\Orbit\DOY_093_MEX\NN13_NN11\D1\NN11_MEX1_2004_093_
OP_G1_110419_0002

CONFIGURATION INFO:
-----
UPLINK-FREQUENCY X-BAND: 7167299449.9976720809936523
DOWNLINK-FREQUENCY X-BAND: 8420859166.8864507675170898
UPLINK-FREQUENCY S-BAND: 7167299449.9976720809936523
DOWNLINK-FREQUENCY S-BAND: 2296597954.6053957939147949
SAMPLE RATE X-BAND: 1.0000
SAMPLE RATE S-BAND: 1.0000

ERRORS:
-----
```

**Figure 0-1: Example of an option file**

### **2.7.3 Label files**

See [1] for more information.

## APPENDIX A

### Computation of the sky frequency received at ground station from doppler predicts

#### Acronyms:

$f_{gs}$  = frequency emitted from ground station  
 $f_{sc}$  = frequency emitted from spacecraft  
 $f_{r_{sc}}$  = frequency received at spacecraft  
 $f_{r_{gs}}$  = frequency received at ground station

$\Delta f_{sc}$  = frequency shift received at spacecraft in the uplink signal emitted from groundstation

$\Delta f_{gs}$  = frequency shift received at groundstation in the downlink signal emitted from the spacecraft

K = transponder conversion ratio

$P_{UL}$  = doppler predict of the uplink signal independent from frequency

$P_{DL}$  = doppler predict of the downlink signal independent from frequency

#### General relations:

$$P_{UL} = \frac{\Delta f_{sc}}{f_{s_{gs}}}$$

$$P_{DL} = \frac{\Delta f_{gs}}{f_{s_{sc}}}$$

### One-way case

$$\Delta f_{gs} = f_{s_{sc}} \cdot P_{DL}$$

it is needed

$$f_{r_{gs}} = \Delta f_{gs} + f_{s_{sc}}$$

therefore the sky frequency is

$$f_{r_{gs}} = f_{s_{sc}} \cdot P_{DL} + f_{s_{sc}}$$

or

$$f_{r_{gs}} = f_{s_{sc}} \cdot (P_{DL} + 1)$$

Two-way case:

$$\Delta f_{sc} = f_{s_{gs}} \cdot P_{UL}$$

$$\Delta f_{gs} = f_{s_{sc}} \cdot P_{DL}$$

needed is

$$f_{r_{gs}} = \Delta f_{gs} + f_{s_{sc}}$$

therefore

$$f_{r_{gs}} = f_{s_{sc}} \cdot P_{DL} + f_{s_{sc}}$$

or

$$f_{r_{gs}} = f_{s_{sc}} \cdot (P_{DL} + 1)$$

with

$$f_{s_{sc}} = K \cdot f_{r_{sc}}$$

$$\Rightarrow f_{r_{gs}} = K \cdot f_{r_{sc}} \cdot (P_{DL} + 1)$$

$$\Rightarrow f_{r_{gs}} = K \cdot (f_{s_{gs}} + \Delta f_{sc}) \cdot (P_{DL} + 1)$$

$$\Rightarrow f_{r_{gs}} = K \cdot f_{s_{gs}} \left( 1 + \frac{\Delta f_{sc}}{f_{s_{gs}}} \right) \cdot (P_{DL} + 1)$$

$$\Rightarrow f_{r_{gs}} = K \cdot f_{s_{gs}} (1 + P_{UL}) \cdot (1 + P_{DL})$$

and therefore the sky frequency is

$$f_{r_{gs}} = K \cdot f_{s_{gs}} (1 + P_{UL} + P_{DL} + P_{UL} \cdot P_{DL})$$

## APPENDIX B

### Computation of the plasma correction using the differential doppler

The differential doppler is computed via

$$\delta f = f_{S,antenna}|_{tropo\_corrected} - \frac{3}{11} f_{X,antenna}|_{tropo\_corrected} \quad (1.13)$$

or

$$\delta f = -\frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \left\{ \frac{1}{f_S^2} - \frac{1}{f_X^2} \right\} f_S \frac{dl}{dt} \quad (1.14)$$

therefore the temporal change in electron content is

$$\frac{dl}{dt} = -\left\{ \frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \right\}^{-1} \frac{\delta f}{f_S} \left\{ \frac{1}{f_S^2} - \frac{1}{f_X^2} \right\}^{-1} \quad (1.15)$$

the plasma correction for S-Band is

$$f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} + \frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \frac{1}{f_S} \frac{dl}{dt} \quad (1.16)$$

and for X-Band

$$f_{X,antenna,cal} = f_{X,antenna}|_{tropo\_corrected} + \frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \frac{1}{f_X} \frac{dl}{dt} \quad (1.17)$$

If equation (1.15) is inserted into (1.16)

$$f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} + \frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \frac{1}{f_S} \left( -\left\{ \frac{1}{2c} \frac{1}{4\pi^2} \frac{e^2}{m_e \epsilon_0} \right\}^{-1} \frac{\delta f}{f_S} \left\{ \frac{1}{f_S^2} - \frac{1}{f_X^2} \right\}^{-1} \right)$$

$$\Rightarrow f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} - \delta f \left\{ \frac{f_S^2}{f_S^2} - \frac{f_S^2}{f_X^2} \right\}^{-1}$$

and with the general relations

$$f_x = \frac{11}{3} f_s \Leftrightarrow \frac{f_x}{f_s} = \frac{11}{3} \Leftrightarrow \frac{f_s}{f_x} = \frac{3}{11} \quad (1.18)$$

follows than

$$f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} - \delta f \left\{ 1 - \frac{9}{121} \right\}^{-1}.$$

Therefore equation (1.16) can be written as

$$f_{S,antenna,cal} = f_{S,antenna}|_{tropo\_corrected} - \delta f \frac{121}{112} \quad (1.19).$$

A similar computation can be done for equation (1.17).

$$\Rightarrow f_{X,antenna,cal} = f_{X,antenna}|_{tropo\_corrected} - \delta f \left\{ \frac{f_x f_s}{f_s^2} - \frac{f_x f_s}{f_x^2} \right\}^{-1}$$

Using equation (1.18)

$$\Rightarrow f_{X,antenna,cal} = f_{X,antenna}|_{tropo\_corrected} - \delta f \left\{ \frac{11}{3} - \frac{3}{11} \right\}^{-1},$$

therefore equation (1.17) can be written as

$$f_{X,antenna,cal} = f_{X,antenna}|_{tropo\_corrected} - \delta f \frac{33}{112} \quad (1.20)$$