Apodizing SPIRE interferograms

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1. Scope

This note is about apodizing interferograms[1] from the SPIRE imaging Fourier Transform Spectrometer (FTS). At the SPIRE FTS workshop in Lethbridge in May 2007, apodization was recommended as a solution to two problems:

1. Reducing channel fringes at high Optical Path Difference (OPD).
2. Suppressing the ringing at unresolved spectral features from the sinc-like instrumental line shape (ILS).

This note studies how different apodizing functions will decrease the channel fringes and the side-lobes of the ILS and the detrimental effect apodization has on the ability to retrieve spectral information.

The SPIRE Data Processing system makes a range of apodizing functions available. The SPIRE instrument team has to decide which apodization function to use in the standard product generation pipeline (SPG). This note identifies apodization strategies that consider

a. loss of spectral resolution
b. effective reduction of the side-lobes of the ILS
c. effective reduction of the channel fringes
d. accurate retrieval of spectral information (line flux)
e. effort required to extract spectral information
f. filtering of random noise

2. Overview

Section 3 details the apodizing functions available within the SPIRE data processing environment with an explicit statement of the trade-off between side-lobe reduction and increase in FWHM.

Section 4 establishes the location of the channel fringes in the SPIRE spectrometer, makes a theoretical prediction on the effectiveness of the apodizing functions to reduce the channel fringes, and spot-checks the theoretical prediction with a dataset from PFM4.

Section 5 quantifies how an ILS, that has been distorted by apodization, is mis-measured when applying different methods (Gauss fit, directly from data, Sinc fit).

Section 6 identifies four apodization strategies depending on the goals of the user.

3. Apodizing functions

During an extensive study, Filler[2] devised a graphical method for comparing different apodizing functions and their corresponding ILSs. This method graphs the normalized height of the absolute largest secondary lobe of the ILS, relative to the height of the absolute largest secondary lobe of the sinc function, against the full width at half

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[1] S. Davis, M. Abrams, J. Brault: "Fourier Transform Spectrometry", Academic Press, (2001), chapter 11.4., gives a careful introduction to apodization in Fourier transform spectroscopy.

[2] A. S. Filler, "Apodization and Interpolation in Fourier-Transform Spectroscopy", J. Opt. Soc. Am. **54**, 762-767 (1964).

maximum (FWHM) of the ILS, again relative to the FWHM of the sinc function. In the Filler diagram, an imaginary line can be drawn that has been confirmed empirically as a constraint on the ability of apodization functions to reduce the amplitude of secondary maxima while preserving spectral resolution.

Many apodizing functions have been proposed. Norton and Beer[3] introduced three functions corresponding to weak, medium and strong apodization that are close to the optimal line in the Filler diagram and are simple to compute. Naylor and Tahic[4] extended this work to ten apodizing functions which correspond to a normalized FWHM of the ILS ranging from 1.1 to 2.0 in steps of 0.1. Figure 1 shows the locus of the ten apodizing functions on the Filler diagram (red circles) compared with the three original ones of Norton and Beer (blue circles) corresponding to FWHM of 1.2, 1.4 and 1.6. The solid line is the empirical fit that Norton and Beer found to represent the optimum boundary.

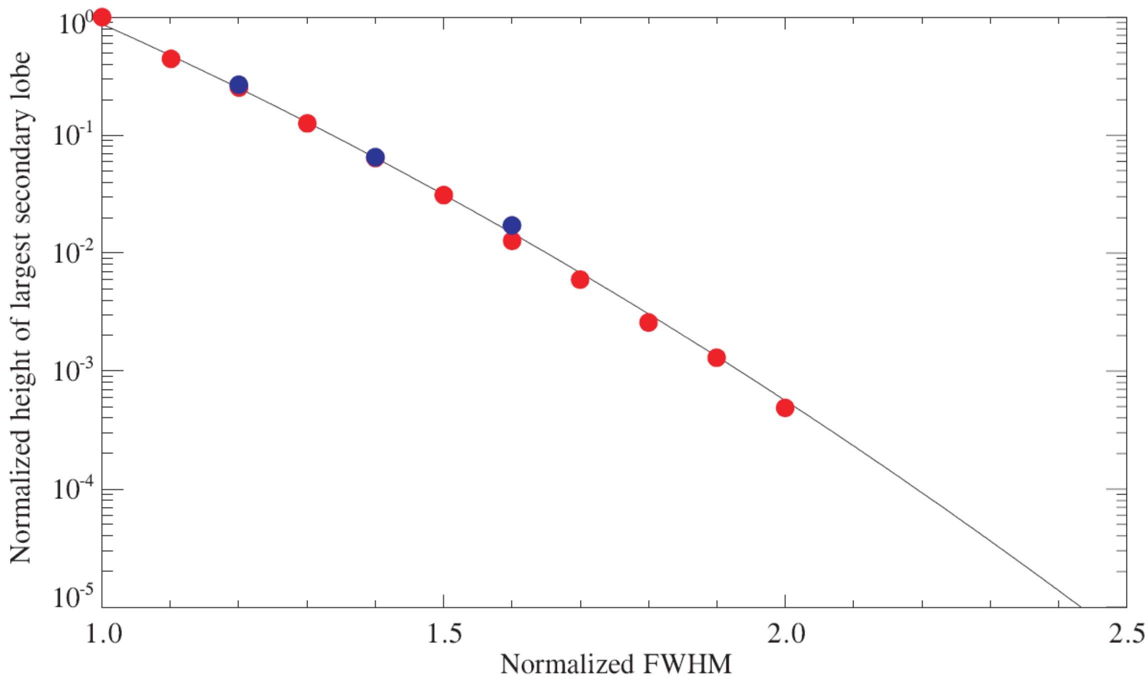

Figure 1: Filler diagram of the 10 apodizing functions of Naylor and Tahic (red circles) and the 3 original ones of Norton and Beer (blue circles).

The ten apodizing functions from Naylor and Tahic are available in the SPIRE data processing environment to apodize interferograms from the SPIRE FTS. By definition, apodizing with each one of these functions leads to a well defined increase in FWHM and a reduction of the amplitude of the largest side-lobe (see Table 1).

Apodizing function	Normalized FWHM	Amplitude of the secondary lobe over the center peak amplitude [%]	Normalized height of the largest secondary lobe [%]
None	1	-21.723	100.00
aNB_11	1.1	-9.631	44.34
aNB_12	1.2	-5.504	25.34

[3] R. H. Norton and R. Beer, "New Apodizing Functions for Fourier Spectrometry", J. Opt. Soc. Am. **66**, 259-264 (1976).
R. H. Norton and R. Beer, "Errata - New Apodizing Functions for Fourier Spectrometry", J. Opt. Soc. Am. **67**, 419 (1977).
[4] D. A. Naylor and M. K, Tahic, "Apodizing functions for Fourier transform spectroscopy", J. Opt. Soc. Am. A **24**, 3644-3648 (2007).

Apodizing function	Normalized FWHM	Amplitude of the secondary lobe over the center peak amplitude [%]	Normalized height of the largest secondary lobe [%]
aNB_13	1.3	-2.732	12.58
aNB_14	1.4	-1.389	6.39
aNB_15	1.5	-0.674	3.10
aNB_16	1.6	-0.275	1.27
aNB_17	1.7	-0.129	0.59
aNB_18	1.8	-0.031	0.14
aNB_19	1.9	-0.026	0.12
aNB_20	2.0	-0.010	0.05

Table 1: An overview of the apodizing functions proposed by Naylor and Tahic, the resulting increase in FWHM, and the achieved reduction of the sidelobes.

4. Channel fringes

The location of the channel fringes

Both detector arrays of the SPIRE spectrometer suffer from channel fringes. The channel fringe signatures in the interferograms occur at different locations for each detector. Table 2 details the lowest, average, and highest OPD values where the three recognizable channel fringe signatures start on average for five observations (30011720, 30011724, 30011728, 3001172B, 3001172F) from December 6, 2006. The observations were taken during the PFM4 test campaign, each containing 8 high resolution interferograms of SCal off and the cold black body at temperatures between 7 and 13 K. Another SLW channel fringe is predicted to be buried in the central burst, at an OPD between 0.34 - 0.70 cm.

	Lowest onset of channel fringe [cm OPD]	Average onset of channel fringe [cm OPD]	Highest onset of channel fringe [cm OPD]
SLW	10.13	10.51	10.79
SSW1	1.81	2.12	2.67
SSW2	6.43	6.71	7.23

Table 2: The lowest, average, and highest OPD where channel fringes signatures start for the average of five PFM4 observations

How much will apodizing functions deprecate SPIRE's channel fringes? This question is considered in the next two sections theoretically and empirically.

Theoretical prediction of the channel fringe reduction

Apodizing functions have values between 0 and 1. Their ability to deprecate a specific channel fringe is defined by the values of the apodizing function within the range where the channel fringe occurs. By definition, the shape of an apodizing function is independent of the OPD grid. The maximum OPD and the sampling interval define how often the apodizing function is evaluated. Figure 2 shows the apodizing function aNB_15 for the three spectral resolution modes of the SPIRE spectrometer.

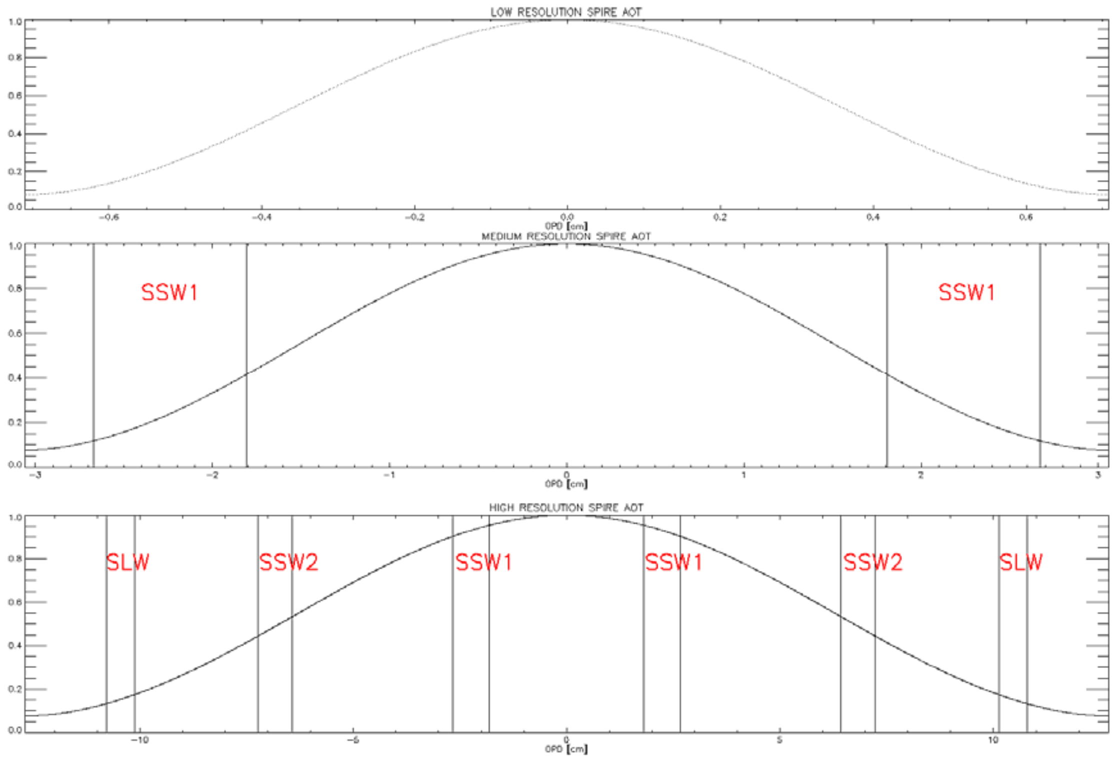

Figure 2: Apodizing function aNB_15 for SPIRE's low, medium, and high spectral resolution AOTs. The range of the location of the three channel fringes is also indicated.

It is therefore possible to make a theoretical prediction of the reduction of the channel fringes based on the location of the channel fringes, the definition of the AOTs, and the used apodizing function (see Table 3). For each of the ten available apodizing functions in column 1 the subsequent columns detail the expected reduction of the channel fringes in percent (NB: 100% reduction means that the channel fringe is completely removed):

Column 2: Reduction of the SLW channel fringe for high resolution.

Column 3: Reduction of the SSW1 channel fringe for medium resolution.

Column 4: Reduction of the SSW1 channel fringe for high resolution.

Column 5: Reduction of the SSW2 channel fringe for high resolution.

Columns 2 and 5 show that apodization is effective at reducing the channel fringes SLW and SSW2 at high spectral resolution. Similarly, column 3 shows that apodization is effective at reducing the SSW1 channel fringe at medium spectral resolution. Column 4 is a reminder that apodization will do little to deprecate the SSW1 channel fringe for a high resolution AOT.

Apodizing function	SLW - **High** resolution [%]	SSW1 - **Medium** resolution [%]	SSW1 - **High** resolution [%]	SSW2 - **High** resolution [%]
None	0	0	0	0
aNB_11	40.7 - 40.4	31.8 - 39.8	2.5 - 5.3	25.5 - 30.2
aNB_12	55.9 - 58.7	38.3 - 59.7	2.7 - 5.9	29.9 - 36.1
aNB_13	67.7 - 71.7	45.5 - 73.1	3.2 - 6.9	35.3 - 42.8

Apodizing function	SLW - **High** resolution [%]	SSW1 - **Medium** resolution [%]	SSW1 - **High** resolution [%]	SSW2 - **High** resolution [%]
aNB_14	76.7 - 80.9	52.1 - 82.4	3.7 - 7.9	40.5 - 49.1
aNB_15	82.6 - 86.7	58.6 - 88.2	4.6 - 9.7	46.7 - 55.6
aNB_16	87.3 - 91.1	64.5 - 92.4	5.5 - 11.6	52.4 - 61.5
aNB_17	90.9 - 94.0	69.9 - 95.1	6.3 - 13.2	57.7 - 66.9
aNB_18	93.6 - 96.0	74.9 - 96.9	7.2 - 15.0	62.8 - 72.0
aNB_19	95.6 - 97.4	79.1 - 98.0	8.0 - 16.8	67.4 - 76.4
aNB_20	97.0 - 98.4	82.8 - 98.8	9.1 - 18.8	71.7 - 80.3

Table 3: Reduction of a signal for the lowest and the highest onset of the channel fringes as reported in Table 2 - by apodizing function

Empirical verification of the channel fringe reduction

This theoretical result can be verified empirically with test data. The PFM4 observation 300113BB of the room and SCal4 at 80K was used to spot-check the validity of the results obtained above. The data from this observation present more prominent channel fringes than the cold black body observations used earlier. Those detectors were selected which presented the most challenging channel fringes in the sense of starting at the lowest OPD:

Channel fringe	Detector name	Start of the channel fringe [cm OPD]	End of the channel fringe [cm OPD]
SLW	SLW - D4	9.86	11.55
SSW1	SSW - D4	2.32	3.34
SSW2	SSW - A2	6.62	8.03

Table 4: Channel fringe regions for the detectors with the lowest onset of the three channel fringes for PFM4 observation 300113BB

Repeating the argument from the previous section, these channel fringe boundaries should lead to the following fringe reduction when applying the apodizing function aNB_15:

Apodizing function	SLW - **High** resolution [%]	SSW1 - **Medium** resolution [%]	SSW1 - **High** resolution [%]	SSW2 - **High** resolution [%]
aNB_15	81.00 – 90.19	79.04 – 92.29	7.19 – 14.55	48.63 – 63.80

Table 5: Expected reduction of a signal across the channel fringe ranges from Table 4 for aNB_15

In order to show specifically the spectral artefacts caused by the channel fringes without and with apodization, the data are processed as follows:

1. Compute the full spectrum for the high and medium resolution AOTs by performing the Fourier transform of the single-sided interferograms to a maximum OPD of 12.70 and 3.06 cm for high and medium spectral resolution.
2. Zero-pad the channel fringe regions of the interferogram as indicated in Table 4 (see the top part of Figure 3 for the resulting high resolution interferograms). Compute the zero-padded spectrum as the Fourier transform of the single-sided, partially zero-padded interferograms to a maximum OPD of 12.70 and 3.06 cm.

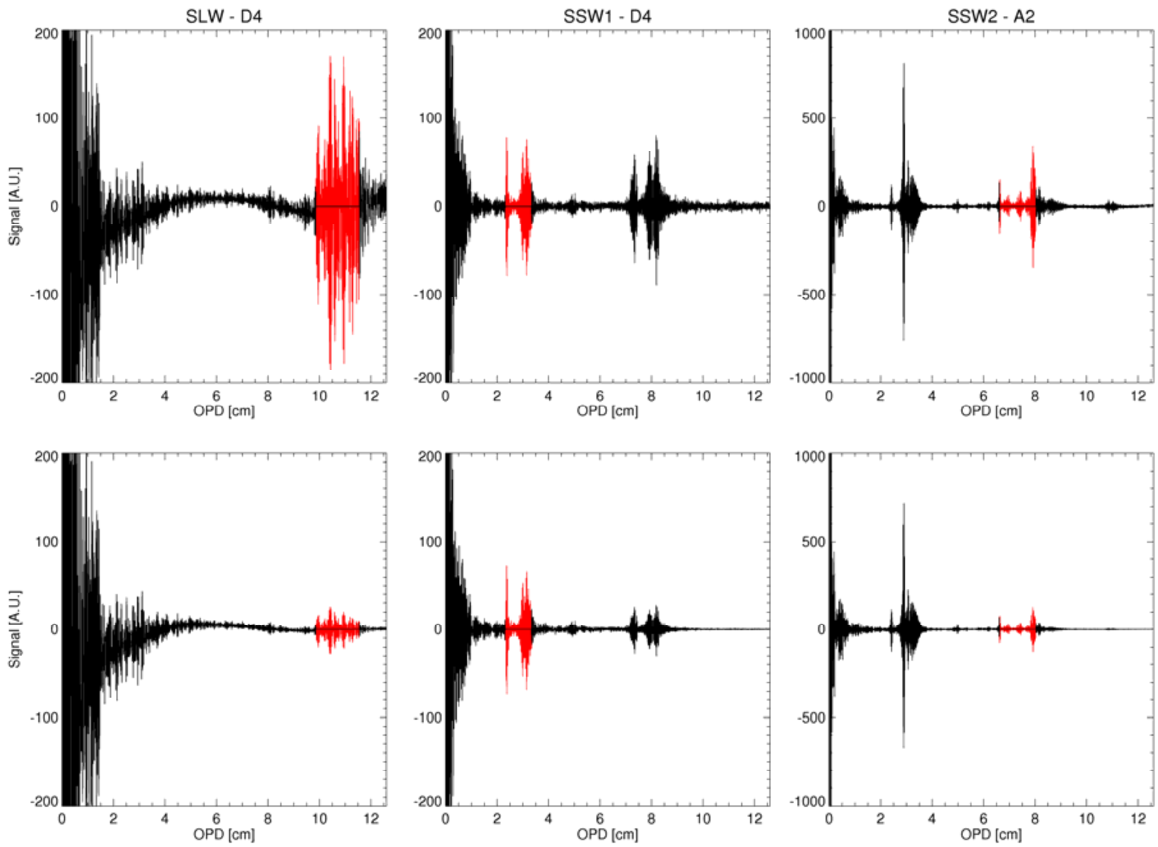

Figure 3: Full (red) and partially zero-padded (black) interferograms,
un-apodized at the top and **apodized** with aNB_15 at the bottom

3. Subtract the full spectrum from the zero-padded spectrum. The result is the “channel fringe spectrum”, i.e. the spectral artefacts introduced by the signal in the channel fringe regions defined in Table 4.
4. Apodize the interferogram with apodizing function aNB_15 and repeat steps 1-3. The bottom part of Figure 3 presents the resulting apodized and partially zero-padded high resolution interferograms.

Figure 4/Figure 5 present the un-apodized channel fringe spectra in the top row and channel fringe spectra from the apodized interferograms in the center and bottom rows for the case of high/medium spectral resolution.

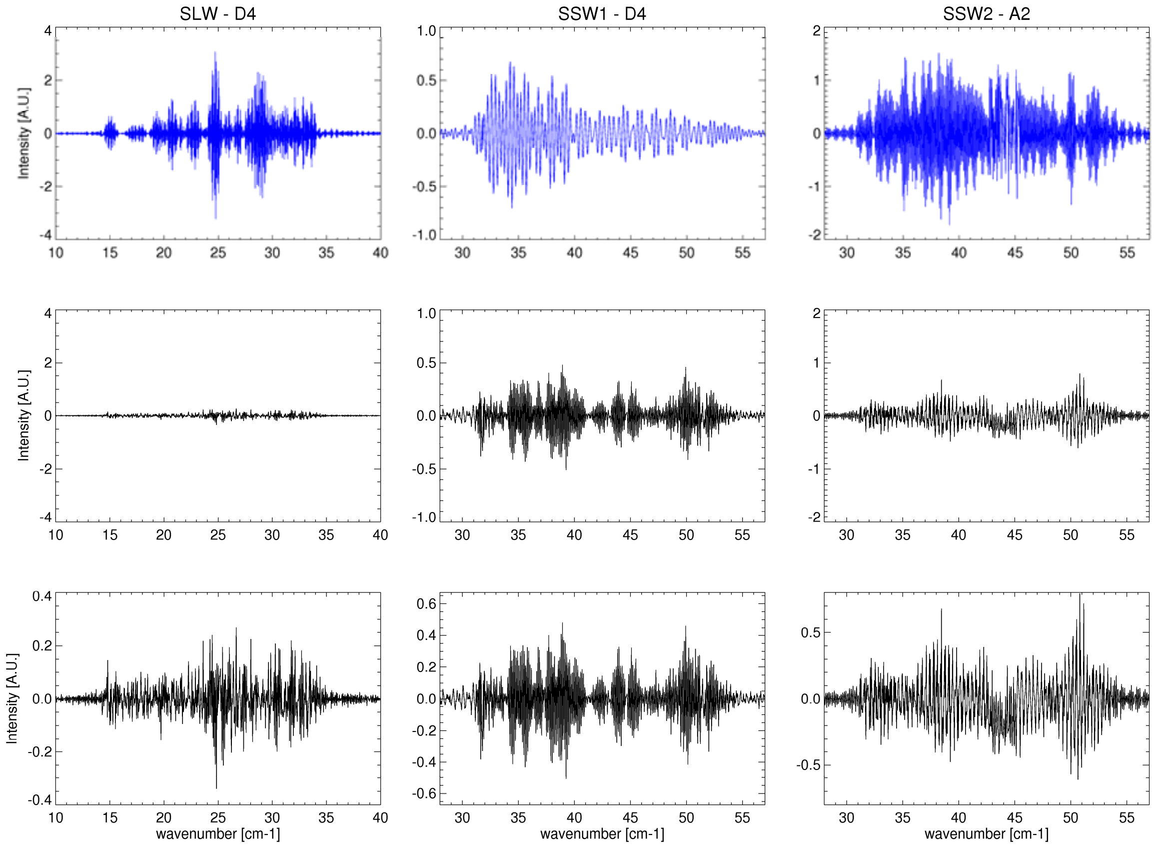

Figure 4: High resolution (OPD_{max}=12.7 cm) spectra of the channel fringes SLW, SSW1, and SSW2 without (blue) and with (black) apodization, using the apodizing function aNB_15. Data in the center and bottom row are identical. The center row is scaled exactly as the top row. The scales of the plots in the bottom row are reduced by 90%, 33%, 60% from left to right.

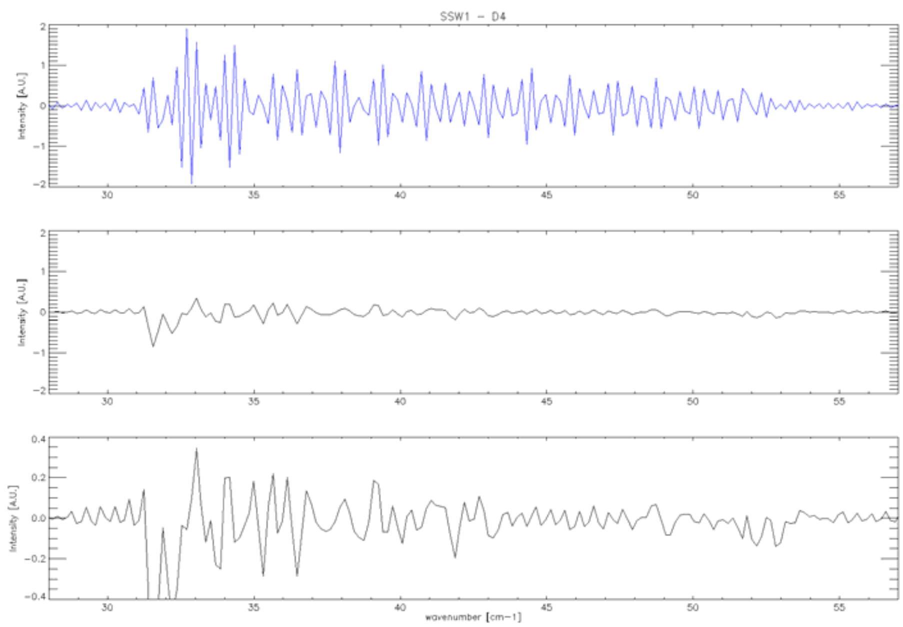

Figure 5: Medium resolution (OPD_{max}=3.06 cm) spectrum of the SSW1 channel fringe without (blue) and with (black) apodization, using the apodizing function aNB_15. Data in the center and bottom row are identical. The center row is scaled exactly as the top row. The scale of the bottom row is reduced by 80%.

Table 6 gives the root of the mean of the squares (RMS) of the channel fringe spectra in the spectral passbands (14 – 34 cm^{-1} for SLW and 33 – 55 cm^{-1} for SSW) without and with apodization:

	SLW – HR	SSW1 – MR	SSW1 – HR	SSW2 – HR
Un-apodized channel fringe spectrum	0.767	0.584	0.196	0.608
Apodized channel fringe spectrum	0.103	0.133	0.173	0.232
Reduction [%]	86.6	77.2	11.7	61.8

Table 6: RMS of the spectra of the channel fringes SLW, SSW1, SSW2 per AOT without and with apodization (aNB_15)

The theoretical and experimental results on the reduction of the channel fringe through apodization agree on the overall trend how the three channel fringes are affected by apodization: At high spectral resolution, the reduction works best for the SLW fringe (~90% reduction), to a limited degree for the SSW2 fringe (~60% reduction), and has the smallest impact for SSW1 (~10% reduction). At medium spectral resolution, the reduction of the SSW1 fringe is considerable (~80% reduction). The experimental data are within the theoretically predicted ranges.

When evaluating the reduction of the channel fringes by apodization, it is important to keep in mind that the amplitude of spectral lines will also be reduced in the process of apodization. For example, apodization with the function aNB_15 will reduce the amplitude of a spectral line by a factor of ~2 (see Table 8).

Summary

Based on theoretical and experimental results indicate that apodization can be an effective tool to remove two of the four instances where channel fringes contaminate data (SLW – high resolution; SSW1 – medium resolution). In the third case (SSW2 – high resolution), the channel fringe is somewhat reduced. In the fourth case (SSW1 – high resolution), however, apodization does not effectively remove the channel fringe signature. Apodization is also not effective at reducing the SLW channel which is buried in the central burst of the interferogram.

5. Retrieving spectral information

Continuum information

The ILS is not a concern for measurements of the spectral continuum if no spectral lines are present. The importance of reducing the channel fringe signature will then drive the apodization strategy. The three channel fringes mentioned above are all beyond the maximum OPD for low resolution observations. A medium or high resolution observation can be truncated to a lower OPD, e.g. 1.65 cm leading to more than 60 data points in SPIRE’s optical passbands at a medium spectral resolution of about 1.207 / (2 · 1.65 cm) = 0.37 cm^{-1}, to avoid the contamination of data by the three channel fringes identified above. This will remove the spectral signature of the channel fringes but possibly leave other unintended effects unresolved, such as the wavelength-dependent instrument efficiency due to the resonating cavities responsible for the channel fringes.

Extreme truncation is required to avoid the SLW channel fringe buried in the central burst, according to theory at an OPD between 0.34 and 0.70 cm. Truncation to an OPD of 0.33 cm will lead to between 15 and 20 spectral data points in SPIRE's optical passbands at a very low spectral resolution of about 1.207 / (2 · 0.33 cm) = 1.83 cm^{-1} which might, however, be sufficient to retrieve continuum information.

Line information

The main purpose of apodization is to enhance the ability to extract accurate information from unresolved line features. The following analysis estimates a limit of the accuracy with which line flux information can be recovered from a spectrum when applying apodizing functions. This study is limited to evaluate the detrimental effect of the distorted ILS and remains to be extended to cover the effect of the channel fringes.

This analysis is based on a noise-free, synthetic interferogram: A double-sided interferogram with a step size of 25µm and a maximum OPD of 12.69 cm containing 10,153 data points is created as a cosine function at a frequency of 25.00005 cm^{-1}, which is contained in the resulting wavescale grid (see Figure 6). The interferogram is then padded with 1,462,032 zeros on either side to finely interpolate the resulting spectrum and the Fourier transform is applied. The line flux represented by this interferogram is the integral across all frequencies of the Fourier transform of a cosine function. Its value of 0.5 was confirmed numerically by a multi-component sinc-fit to the spectrum (see the first row in Table 9).

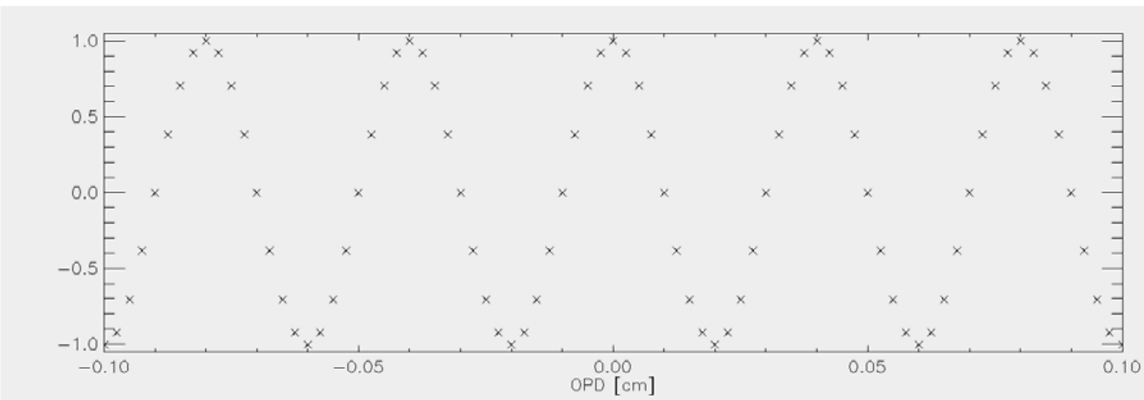

Figure 6: Zoom about the ZPD of the synthetic interferogram

The apodizing functions aNB_11 to aNB_20 are multiplied with the cosine function. The Fourier transform is then applied to calculate the spectrum. The flux of the line in the resulting spectrum is derived in three different ways:

1. Gauss fit (see Table 7).
2. Without fitting a curve, the flux is calculated as the product of the amplitude and the FWHM of the spectral data (see Table 8).
3. Sinc fit (see Table 9).

The appendix details how to relate the amplitude and FWHM of the fitted curves to the line flux. Figure 7 gives a sample plot of the fitting results, in this case for the un-apodized case.

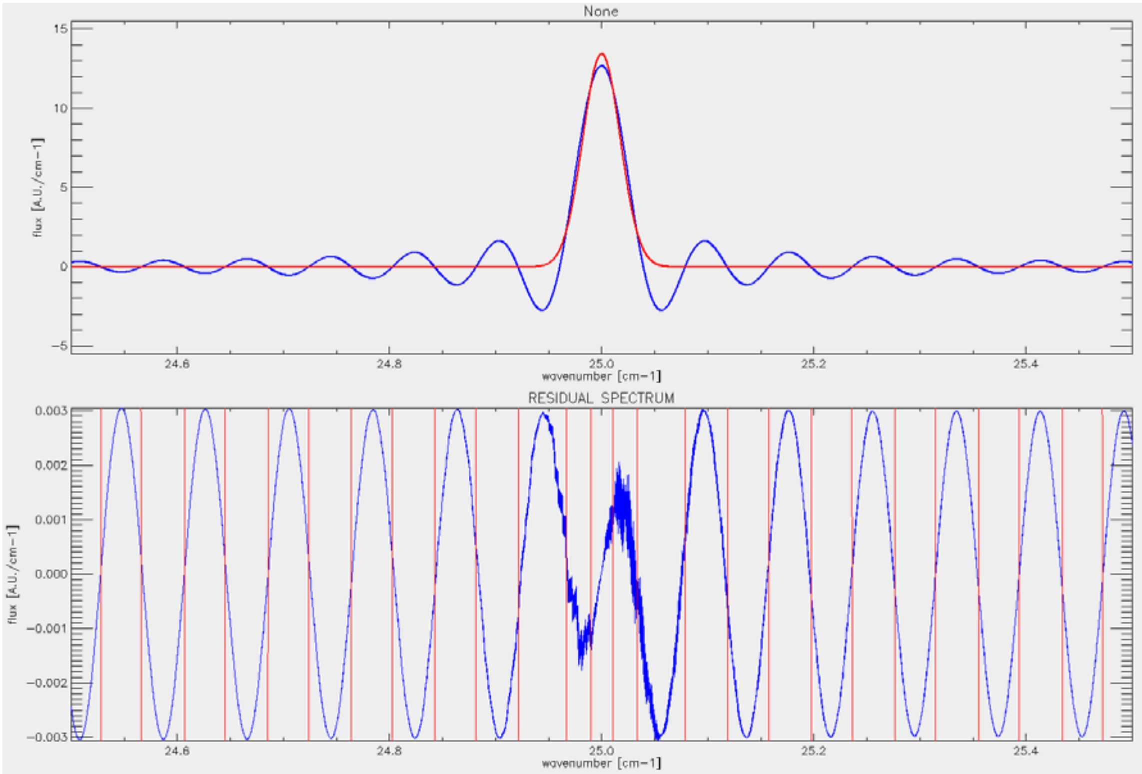

Figure 7: The Gauss (red) and Sinc (blue) fit to the un-apodized spectrum at the top and the differences of the spectrum and the fitted functions at the bottom.

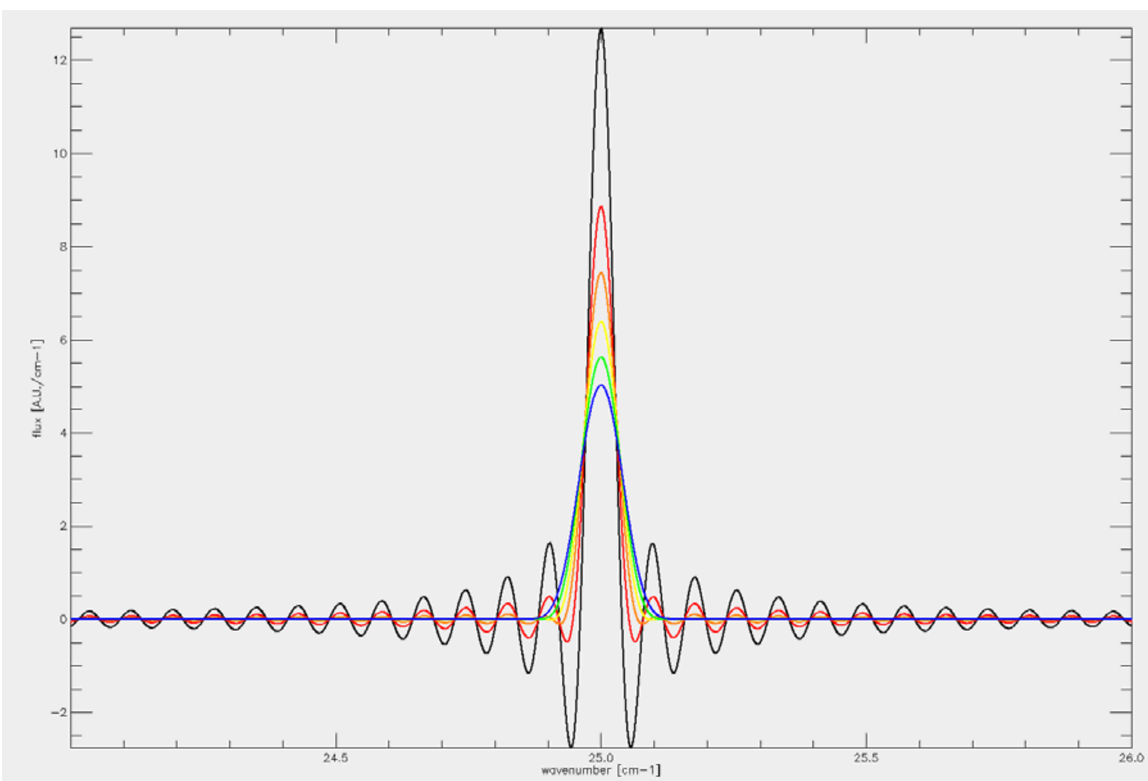

Figure 8: The un-apodized, ideal ILS (black) and after apodization with functions to increase the FWHM by 20% (red), 40% (orange), 60% (yellow), 80% (green), 100% (blue).

Apodizing function	FWHM	Amplitude	Flux	Relative Error
None	0.0416	13.4687	0.5959	19.2%
1.1	0.0475	10.2684	0.5189	3.8%
1.2	0.0528	9.1731	0.5152	3.0%
1.3	0.0582	8.2882	0.5131	2.6%
1.4	0.0634	7.6085	0.5133	2.7%
1.5	0.0685	7.0022	0.5103	2.1%
1.6	0.0735	6.4930	0.5078	1.6%
1.7	0.0784	6.0741	0.5070	1.4%
1.8	0.0834	5.6954	0.5059	1.2%
1.9	0.0883	5.3768	0.5055	1.1%
2	0.0933	5.0793	0.5044	0.9%

Table 7: The relative error of the derived line flux, the absolute value of the line flux, and the FWHM's and amplitudes, as derived from **fitting a Gaussian** to the spectra of an apodized cosine.

Apodizing function	FWHM	Amplitude	Flux	Relative Error
None	0.0474	12.6900	0.6020	20.4%
1.1	0.0521	9.8412	0.5125	2.5%
1.2	0.0570	8.8611	0.5049	1.0%
1.3	0.0616	8.0645	0.4969	-0.6%
1.4	0.0664	7.4409	0.4940	-1.2%
1.5	0.0712	6.8721	0.4890	-2.2%
1.6	0.0759	6.3897	0.4852	-3.0%
1.7	0.0807	5.9899	0.4834	-3.3%
1.8	0.0853	5.6288	0.4803	-3.9%
1.9	0.0902	5.3209	0.4802	-4.0%
2	0.0949	5.0340	0.4776	-4.5%

Table 8: The relative error of the derived line flux, the absolute value of the line flux, and the FWHM's and amplitudes, as derived from the spectra of an apodized cosine **without curve fitting**.

Apodizing function	FWHM	Amplitude	Flux	Relative Error
None	0.0476	12.6900	0.5000	0.0%
1.1	0.0476	9.8336	0.3879	-22.4%
1.2	0.0480	8.8343	0.3512	-29.8%
1.3	0.0653	7.0917	0.3837	-23.3%
1.4	0.0724	6.4620	0.3877	-22.5%
1.5	0.0774	5.9738	0.3832	-23.4%
1.6	0.0830	5.5389	0.3807	-23.9%
1.7	0.0889	5.1699	0.3806	-23.9%
1.8	0.0952	4.8296	0.3808	-23.8%
1.9	0.1005	4.5649	0.3801	-24.0%
2	0.1069	4.2965	0.3804	-23.9%

Table 9: The relative error of the derived line flux, the absolute value of the line flux, and the FWHM's and amplitudes, as derived from **fitting a sinc function** to the spectra of an apodized cosine.

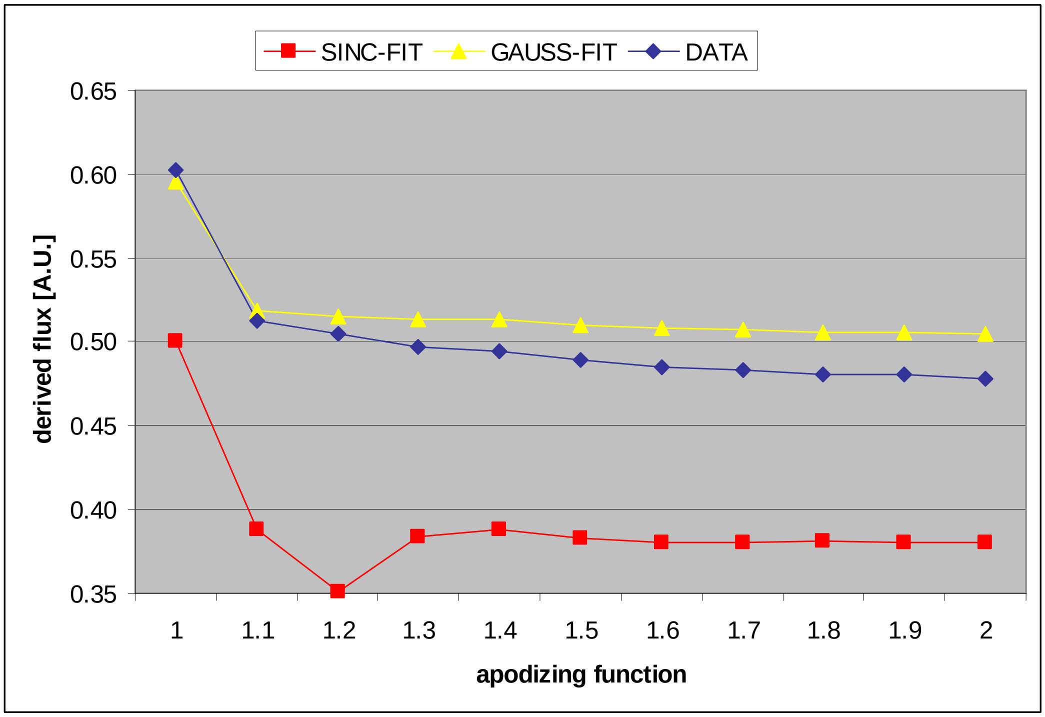

Figure 9: Comparison of the flux derived from a Sinc fit, a Gaussian fit, or directly from the data with and without apodization.

Figure 9 graphically compares the three methods of deriving the flux.

In the case of un-apodized data, the rather difficult multi-component sinc fit is the best method for retrieving line flux information. In this case, the two other methods, Gauss-fit and taking the FWHM and amplitude directly from the data directly, introduce significant error (19.2 and 20.4%). Note that the direct method without fitting a curve retrieves the line flux precisely if a factor of 1.207 is taken into account.

In the case of apodized data, the sinc-fit leads to large errors between -22.4 and -29.8%. The line flux error is between +0.9 and +3.8% when treating the apodized ILS as a Gaussian function. The line flux error is between -4.5 and +2.5% when using the product of FWHM and amplitude of the apodized spectra directly, leading to an uncertainty of 7.0% which may be acceptable in cases where a quick result is required and suitable data analysis tools are not available.

6. Apodization strategies

Apodization can help a user who is faced with the fully fledged problem of analyzing high resolution spectral data from SPIRE, which contain both, continuum and unresolved line features. Different apodization strategies for processing data from high resolution AOTs are available depending on user preferences.

Apodize for beauty – continuum

The premise of this scenario is a user who wants to extract information with minimal effort and good accuracy from a spectrum that is free of instrumental artefacts. This scenario assumes that the user is mainly interested in continuum features and therefore not overly concerned about the loss of spectral resolution.

Applying the apodizing function aNB_20 will significantly reduce various features of the spectrum in order to facilitate data interpretation:

- Reduce the secondary lobes of an unresolved spectral line to 0.01% of the center peak, effectively getting rid of the side-lobes altogether.
- Reduce the SLW channel fringe, which is at the highest OPD and therefore poses the greatest risk of confusing line information, by 97.0 – 98.4%.
- Reduce the SSW2 channel fringe, which is the second fringe in terms of OPD, by 71.7 – 80.3%.

The costs for the reduction of the artefacts masquerading the spectral information of the astronomical source are:

- Spectral resolution increases by a factor of 2 from 0.048 cm^{-1} to 0.096 cm^{-1}.
- The line amplitude is deprecated by 66.1% of the un-apodized line amplitude.
- The SSW1 channel fringe will be reduced only slightly, by 9.1 – 18.8%, i.e. it will become more prominent in comparison to the line feature.
- Extracting line flux information by fitting a Gaussian will lead to an error of 0.9%.
- Extracting line flux information directly from the data will lead to an error of 4.5%.

Apodize for beauty – lines

The premise of this scenario is a user who wants to extract information with minimal effort and good accuracy from a spectrum that is free of instrumental artefacts. This scenario assumes that the user is mainly interested in unresolved line features.

Applying the apodizing function aNB_15 will significantly reduce various features of the spectrum in order to facilitate data interpretation:

- Reduce the secondary lobes of an unresolved spectral line to 0.674% of the center peak, effectively getting rid of the side-lobes.
- Reduce the SLW channel fringe, which is at the highest OPD and therefore poses the greatest risk of confusing line information, by 82.6 – 86.7%.

The costs for the reduction of the artefacts masquerading the spectral information of the astronomical source are:

- Spectral resolution increases by a factor of 1.5 from 0.048 cm^{-1} to 0.071 cm^{-1}.
- The line amplitude is deprecated by 52.9% of the un-apodized line amplitude.
- The SSW1 channel fringe will be reduced only slightly, by 4.6 – 9.7%, i.e. it will become more prominent in comparison to the line feature.
- The SSW2 channel fringe will be reduced by 46.7 – 55.6%, i.e. about as much as the line feature.
- Extracting line flux information by fitting a Gaussian will lead to an error of 2.1%.
- Extracting line flux information directly from the data will lead to an error of 2.2%.
- Some of the noise is reduced to facilitate line identification.

Apodize to extract weak lines

The premise of this scenario is a user who wants to extract information with minimal effort and good accuracy from a spectrum that is free of instrumental artefacts. This scenario assumes that the user is mainly interested in weak unresolved line features.

Applying the apodizing function aNB_11 will allow for the straightforward retrieval of line flux information:

- Reduce the secondary lobes of an unresolved spectral line to 9.631% of the center peak.
- Reduce the SLW channel fringe, which is at the highest OPD and therefore poses the greatest risk of confusing line information, by 40.4 – 40.7%.
- Some of the noise is reduced to facilitate line identification.

The costs for the reduction of the artefacts masquerading the spectral information of the astronomical source are:

- Spectral resolution increases by a factor of 1.1 from 0.048 cm^{-1} to 0.053 cm^{-1}.
- The line amplitude is deprecated by about 22.4% of the un-apodized line amplitude.
- The SSW1 channel fringe will be reduced only slightly, by 2.5 – 5.3%, i.e. it will become more prominent in comparison to the line feature.
- The SSW2 channel fringe will be reduced by 25.5 – 30.2%, i.e. comparable to the reduction of the line amplitude. There is very little net gain.
- Extracting line flux information by fitting a Gaussian will lead to an error of 3.8%.
- Extracting line flux information directly from the data will lead to an error of 2.5%.

Do not apodize for best results

The premise of this scenario is a user who is willing to perform an often unstable multi-component fit to a sinc profile in order to get data of maximum accuracy.

Not applying an apodizing function has several benefits:

- The spectral resolution is at its optimum with 0.048 cm^{-1}.
- The amplitude of already weak lines is not reduced further.
- Line flux extraction with a sinc fit can lead to precise results.
- Line flux extraction directly from the data and keeping the factor of 1.207 in mind will lead to precise results – in the absence of noise.

The costs for the reduction of the artefacts masquerading the spectral information of the astronomical source are:

- The secondary lobes of an unresolved spectral line are at 21.723% of the center peak.
- The SLW channel fringe, which is at the highest OPD, creates the risk of confusing the channel fringe with line information.
- The SSW1 channel fringe introduces modulation of the continuum which may lead to erroneous results when characterizing the continuum, particularly in the presence of many lines.

- The SSW2 channel fringe, which is at the second highest OPD, creates the risk of confusing the channel fringe with line information.
- Extracting line flux information by fitting a Gaussian will lead to an error of 19.2%.
- Extracting line flux information directly from the data will lead to an error of 20.4%.

7. More work

There is more work to be done:

- What to do about line contamination? Can apodization help to tell instrumental artefacts from source information and cleanly separate the continuum from the line content? The main idea is that the characterization of the continuum should be independent of the applied apodization.
- How is the continuum information affected by the channel fringes?
- How is the line flux information affected by the channel fringes? That will depend on the amplitude of the line and the amplitude of the channel fringe.
- Deriving correction factors from the derived fluxes of the apodized cosine, should further reduce the error in the line flux. These correction factors can be applied when recovering the line flux from a monochromatic source measured during the PFM4 ground-based test campaign. This should be checked for a range of detectors and line shapes. How would I establish a reference of the line amplitude?

APPENDIX: Flux for known line shapes

The flux in a spectral line is equal to the integrated area below the line in a spectrum measured in flux density units, e.g. Jansky, over a wavescale, e.g. frequency. The line flux can be derived analytically if the ILS is a sinc function or a Gaussian.

Sinc function

The line flux for a sinc function is equal to the amplitude of the sinc function times the FWHM divided by 1.207:

$$Flux = \int_{-\infty}^{\infty} Line(\sigma)d\sigma \quad with \quad Line(\sigma) = A\,Sinc(\pi\frac{\sigma-\sigma_0}{\Delta\sigma}) = A\frac{\sin(\pi\frac{\sigma-\sigma_0}{\Delta\sigma})}{(\pi\frac{\sigma-\sigma_0}{\Delta\sigma})}$$

$$Flux = \int_{-\infty}^{\infty} A\frac{\sin(\pi\frac{\sigma-\sigma_0}{\Delta\sigma})}{(\pi\frac{\sigma-\sigma_0}{\Delta\sigma})}d\sigma = A\int_{-\infty}^{\infty}\frac{\sin(\pi\frac{\sigma}{\Delta\sigma})}{(\pi\frac{\sigma}{\Delta\sigma})}d\sigma$$

$$Flux = \int_{-\infty}^{\infty}\frac{\sin(\omega)}{(\omega)}\frac{d\omega}{d\omega}d\sigma \quad with \quad \omega = \pi\frac{\sigma}{\Delta\sigma} \quad and \quad \frac{d\omega}{d\sigma} = \frac{\pi}{\Delta\sigma}$$

$$Flux = A\frac{\Delta\sigma}{\pi}\int_{-\infty}^{\infty}\frac{\sin(\omega)}{(\omega)}d\omega = A\cdot\Delta\sigma$$

$$Flux \approx A\cdot\frac{FWHM}{1.207}$$

Gaussian

The line flux for a Gaussian is equal to the amplitude of the Gaussian times the FWHM times 1.064:

$$Flux = \int_{-\infty}^{\infty} Line(\sigma)d\sigma \quad with \quad Line(\sigma) = A\exp(-\frac{(\sigma-\sigma_0)^2}{2\cdot\Delta\sigma^2})$$

$$Flux = A\int_{-\infty}^{\infty}\exp(-\frac{(\sigma-\sigma_0)^2}{2\cdot\Delta\sigma^2})d\sigma = A\int_{-\infty}^{\infty}\exp(-\frac{\sigma^2}{2\cdot\Delta\sigma^2})d\sigma$$

$$With \quad \int_{-\infty}^{\infty}\exp(-\frac{x^2}{2\cdot\Delta\sigma^2})dx = \Delta\sigma\sqrt{2\pi}: \quad Flux = A\cdot\Delta\sigma\cdot\sqrt{2\pi}$$

$$With \quad FWHM = 2\cdot\Delta\sigma\cdot\sqrt{2\ln 2}: \qquad Flux = A\cdot FWHM\sqrt{\frac{\pi}{4\ln 2}} \approx 1.064\cdot A\cdot FWHM$$