There are 3 spatial sampling choices foreseen for mapping with the Spectrometer: sparse, intermediate and full. Full sampling means Nyquist sampling with a separation of points equal to half the beam size (ie. Fλ/2)

The spectrometer pixel separation is:

Array	Separation	f-number$_{SPIRE}$	Separation	Measured FWHM
SSW	2.25 mm	4.35	32.5"	16"
SLW	3.9 mm	4.85	50.5"	25"

The separation on sky was calculated assuming that the effective telescope diameter is, D_{eff} = 3.287 m. The measured FWHM are from PFM1, using the hot black body source, with an accuracy of a few arcseconds (ie. consistent with 2$F\lambda$ pixel spacing).

Taking the beam size to be exactly half the separation of the pixels gives the following sampling requirements for the three modes:

Mode	Sampling	SSW	SLW
Sparse	2 beam	32.5"	50.5"
Intermediate	1 beam	16.25"	25.25"
Full	½ beam	8.125"	12.625"

A sparse map of the field of view can be achieved with a single pointing of the spectrometer. For intermediate or full sampling, the BSM can be used to move the array and fill in the gaps. This can either be with a jiggle or with a linear scan at an angle.

The size of any map at a single telescope pointing is restricted by the BSM limits. The instrument specifications are ±126" in the spacecraft Y-direction (chop) and ±30" in the Z-direction (jiggle). However, the testing shows that it will probably function up to ±35" in the Z-direction (and possibly further).

It is assumed in the following that the final map coverage at the quoted sampling must be approximately 2.6'×2.6' (the size of the unvignetted field of view). However, it may be more practical to use the area covered up to the outer ring of unvignetted pixels (2.17' for SSW and 1.68' for SLW; see Fig. 1).
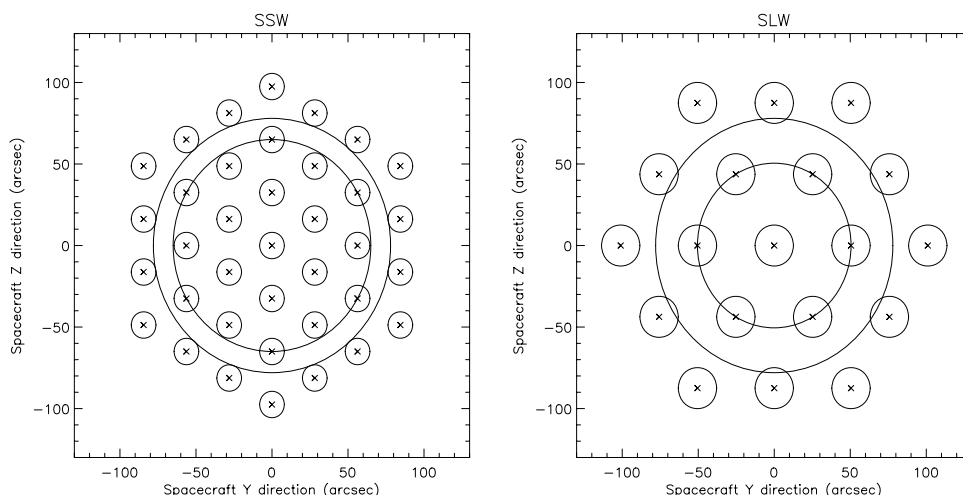

Figure 1: Layout of the arrays with the 2.6' unvignetted field of view shown, as well as the circle going through the centres of the outer unvignetted pixels (65" for SSW; 50.5" for SLW).

1.1 Standard Jiggle Map

The simplest approach to a jiggle would be a regular square grid. However, due to the hexagonal close packed layout, the separation between pixels is not the same in the horizontal and vertical directions. Therefore, a hexagonal grid of points, with alternate rows or columns offset by half the point spacing should be observed, as done for the photometer.

In order to carry out a standard jiggle map to fully sample the spectrometer field of view, 16 jiggle positions are needed for SSW. This involves moving the BSM in a 4×4 raster.

In order to extend this to fill SLW as well, the coverage has to be increased to the SLW pixel separation. In order to get an even sampling and signal to noise across the map in the simplest way possible, the 4×4 raster can be increased to 7×7 positions. This oversamples the SLW map by nearly a factor of 2 and means that the SSW map is covered several times in the central regions. The total number of pointings required is 49. Figure 1 shows the result. The "Operating modes for the SPIRE instrument" document (SPIRE-RAL-DOC-000320) describes this mode in Section 3.2.2 (SOF2).

There is potential for optimisation of the simple jiggle pattern by trying to minimise the oversampling in the SLW array whilst maintaining the sampling necessary for SSW.
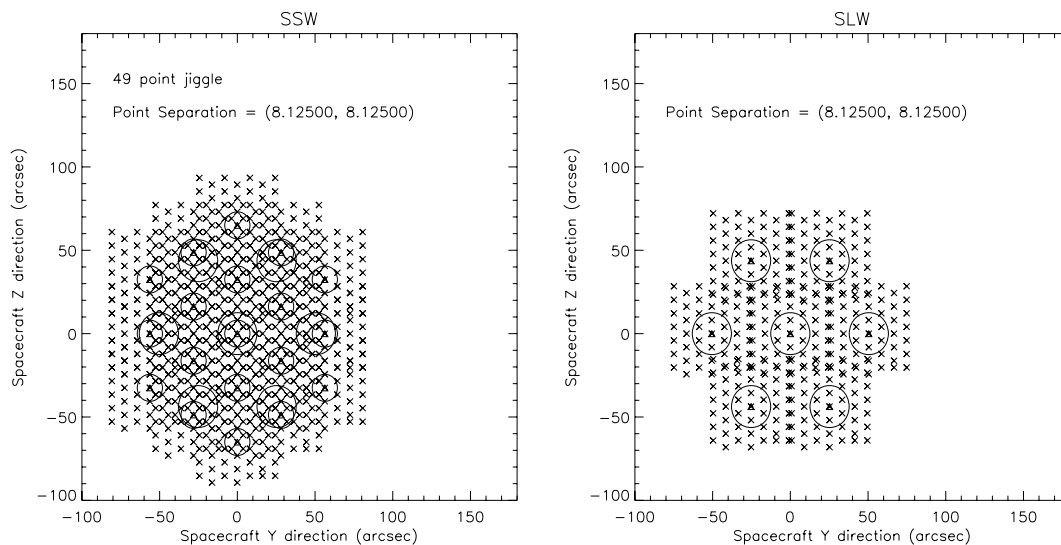

Figure 2: The array positions are shown for SSW and SLW during a 49 point jiggle to sample SLW with point spacing of 8.125".

1.2 Scanning the BSM at an angle

The alternative method is to scan the BSM in a linear way with the array rotated to a 'magic angle' (Bruce's tech note).

As data from the two arrays will be taken simultaneously, the angle chosen should provide the optimum sampling for both. This is complicated by the fact that the hexagonal packing of the pixels is rotated by 90 degrees between SSW and SLW.

There are five possible angles that give *equal* spacing between the rows for SSW and SLW (although for some of these angles there are wider gaps between the rows – particularly for SLW). These are shown in Table 1, with the row separation, the number of rows with overlapping pixels and the number of pointings required to fully sample a distance of 2.6' along the scan direction (taking edge effects into account). These 5 angles have a symmetric partner in the range 30° – 60° (e.g. the row spacing at 19.1° is the same as at (60–19.1)°).

Table 1: The five possible angles for even spacing of rows in a linear scan of SSW and SLW, showing the gap at the start and end of the scan with incomplete sampling, and the total number of pointings required for Nyquist sampling to cover 2.6' in the scan direction. In all cases multiple passes are required. These are shown as $N_{pts} \times N_{rows}$. The separation of the points in each row is 8.125".

Angle	Gap (SSW)	Pointings to cover SLW with SSW sampling
30.0°	0.0"	20×4 = 80
19.1°	116.5"	33×2 = 66
16.1°	85.6"	30×2 = 60
10.9°	55.3"	26×2 = 52
0.0°		7×6 = 42

The main problem with the magic angle method is that even at the angle where SLW rows have equal spacing between them (19.1°), the separation of rows is greater than required for Nyquist sampling. Therefore, SLW will always have to be scanned in at least 2 passes to achieve perfect sampling. Also, Table 1 does not take the BSM limits into account. Assuming BSM limits of ±35" in the jiggle direction would mean scans above 16.8° would not be able to cover the full 2.6'.
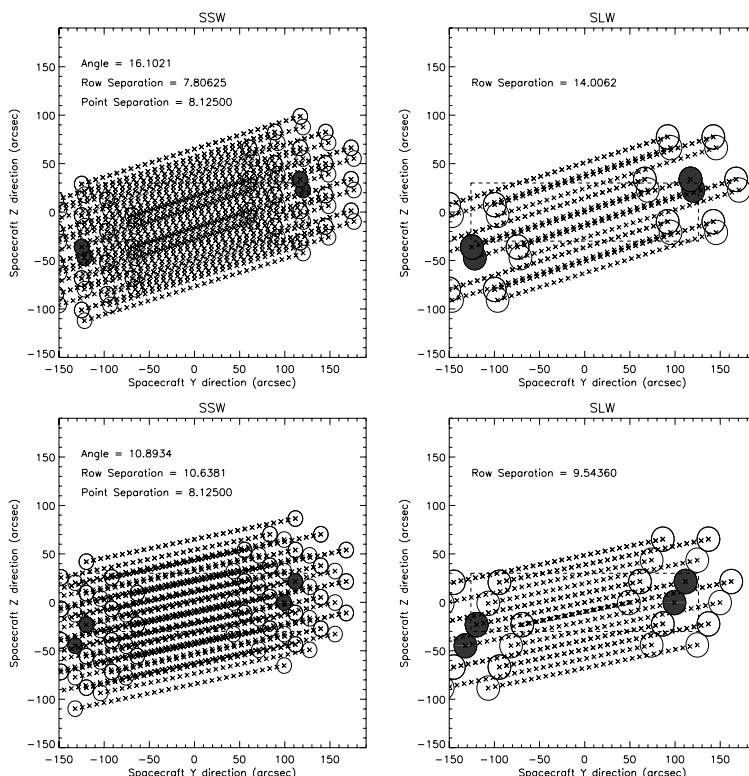

Figure 3: Array positions for SSW and SLW during a scan at the magic angle. Repeated passes to fill in gaps are shown. The dashed box shows the limits of the BSM motion (±126, ±30) on the central pixel (which is shaded grey). Point spacing is set to 8.125".

Technical Note

SPIRE FTS Mapping Modes

E. Polehampton

Ref:	SPIRE-RAL-NOT-002801
Issue:	1.0
Date:	16 January 2007
Page:	4 of 12

1.3 Optimisation of number of points

It is possible to optimise the jiggle and magic angle scans by increasing the point separation and using the repeated passes to fill in the gaps in SSW sampling. Table 2 shows the optimised parameters. After this optimisation, the 7×4 point jiggle shows the least number of points.

Table 2: Revised parameters needed to completely map SLW and SSW with better than Nyquist sampling for each, after optimising the point separation. Columns are the same as Table 3. This does not take into account the restrictions on the number of points in the scan due to the BSM movement.

Mode	N_{pts}	Δ_{pts}	N_{rows}	Δ_{rows}	Total pts	Area SSW	Area SLW
Jiggle	7	12.625"	4	8.125"	28	1.9'×1.9'	1.7'×1.7'
30	12	12.625"	3	9.4"	36	slightly undersampled	
30	12	12.625"	4	7.0"	48	2.5'×2.4'	2.5'×2.0'
19.1	20	10.1"	2	8.25"	40	2 gaps	
16.1	21	10.1"	2	11.5"	42	2.6'×2.3'	2.7'×1.8'
10.9	20	10.1"	2	19"	40	2.6'×1.9'	2.2'×2.1'
0.0	5	10.1"	6	8.125"	30	2.0'×2.6'	1.8'×2.2'

In order to optimise the jiggle even further, both SSW and SLW should be simultaneously sampled without any extra oversampling of the other. This is almost achieved by the simple 16 point jiggle in a hexagonal pattern. If the spacing is set for SSW, Nyquist sampling is nearly achieved for SLW simultaneously. Figure 5 shows the result of this 16-point jiggle. If the smaller area covered by the unvignetted pixels and the slight offset from perfect sampling in SLW is acceptable, then this mode is the one with minimum number of pointings.

The maximum deviation of points from the ideal Nyquist sampling positions in the SLW map is 5". As this is only 20% of the pixel FWHM, it should not significantly affect the map quality.
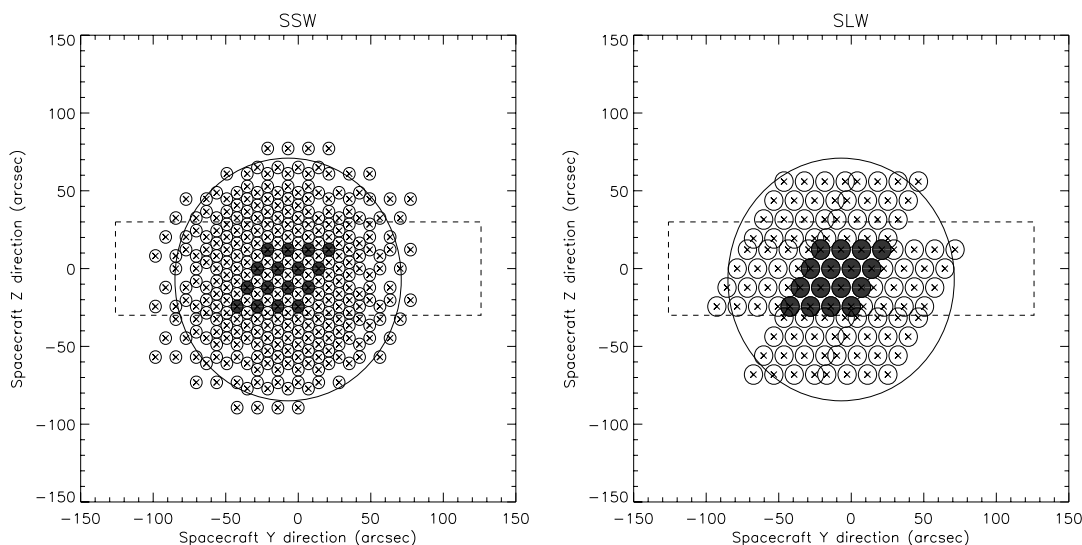

Figure 4: Full Nyquist sampling of both arrays using a 16-point jiggle. Only the unvignetted pixels are shown. The circles are drawn with a diameter equal to half the pixel FWHM. The shaded pixel shows the movement of the centre of the array during the jiggle. The large circle shows the 2.6' unvignetted field of view and the dashed line shows the BSM limits.
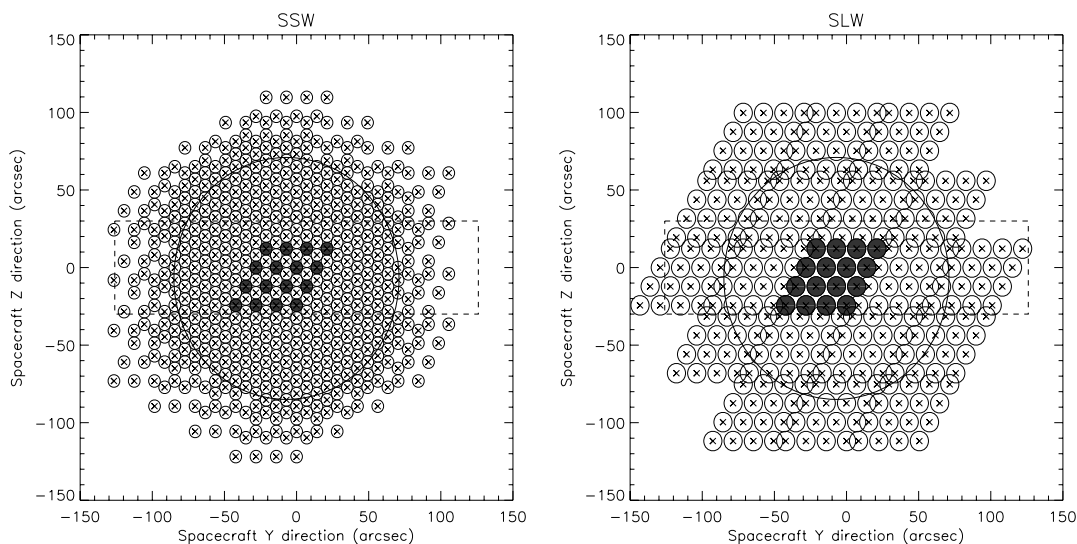
Technical Note

SPIRE FTS Mapping Modes
E. Polehampton

Ref:	SPIRE-RAL-NOT-002801
Issue:	1.0
Date:	16 January 2007
Page:	5 of 12

Figure 5: Full Nyquist sampling of both arrays using a 16-point jiggle. Same as Figure 4 except that all pixels, including those that are partially vignetted, are shown.

1.4 Conclusion

The pattern with the minimum number of pointings to produce a fully sampled map is the 16 point jiggle. This is better than scanning the BSM at a magic angle due to the fact that 2 passes must be made to fully sample SLW and that the scan length must be extended to allow for gaps with incomplete sampling at the edges.

The disadvantages of this method are that it covers less than the 2.6' FOV using the unvignetted pixels, and the sampling is not regular in the SLW map. However, when the partially vignetted pixels are included, the map size will be larger than 2.6'. The maximum deviation from ideal positions in the SLW map is 5", 20% of the beam FWHM.

The other disadvantage is that there is no redundancy of pixels. Therefore, if any pixels die in orbit, there may have to be additional jiggle positions added to the map.

Technical Note

SPIRE FTS Mapping Modes
E. Polehampton

Ref:	SPIRE-RAL-NOT-002801
Issue:	1.0
Date:	16 January 2007
Page:	6 of 12

1.5 Implementation

The above analysis suggests that the most efficient spectrometer mapping modes will be,

Mode	Sampling	Map
Sparse	2 beam	Single pointing
Intermediate	1 beam	4-point jiggle
Full	½ beam	16-point jiggle

The BSM angles needed to carry out these maps are calculated based on the FWHM of the SSW beam, Δ_{SSW}, where Δ_{SSW} = 16.25".

Intermediate Sampling (4 point jiggle)

Separation in Y-direction   = 2 × Δ_{SSW} × cos(30)   **= 28.1458"**
Separation in Z-direction   = 1.5 × Δ_{SSW} **= 24.375"**

Full Sampling (16 point jiggle)

Separation in Y-direction   = Δ_{SSW} × cos(30)   **= 14.0729"**
Separation in Z-direction   = 0.75 × Δ_{SSW} **= 12.1875"**

In order to place the centre of the FOV onto the nearest pixel to the true centre (*due to the even number of jiggle positions, the true centre is between pixels*), the following offsets should be used:

Intermediate sampling 4-pt jiggle:

Position	Y-angle	Z-angle
1	0.0	0.0
2	28.1458	0.0
3	14.0729	-24.375
4	-14.0729	-24.375

Full sampling 16-pt jiggle:

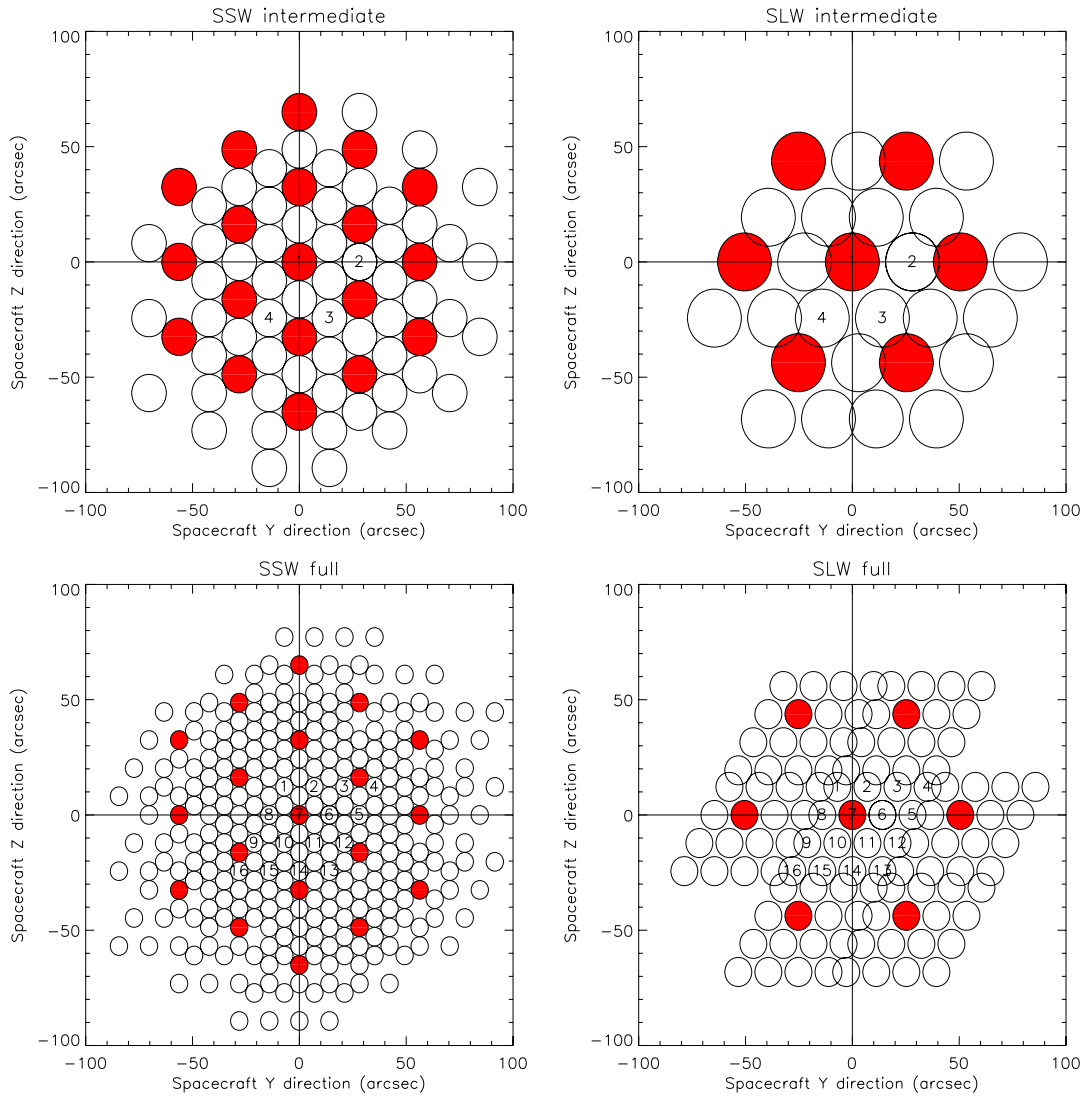| Position	Y-angle	Z-angle
1	-7.03646	12.1875
2	7.03646	12.1875
3	21.1094	12.1875
4	35.1823	12.1875
5	28.1458	0.0
6	14.0729	0.0
7	0.0	0.0
8	-14.0729	0.0
9	-21.1094	-12.1875
10	-7.03646	-12.1875
11	7.03646	-12.1875
12	21.1094	-12.1875
13	14.0729	-24.375
14	0.0	-24.375
15	-14.0729	-24.375
16	-28.1458	-24.375

Figure 6: Diagram showing the final jiggle pattern. The central pixel is labelled with the jiggle position number from the above tables. The central pointing in the FOV is shaded.

Technical Note

SPIRE FTS Mapping Modes

E. Polehampton

Ref:	SPIRE-RAL-NOT-002801
Issue:	1.0
Date:	16 January 2007
Page:	8 of 12

Raster Mapping

When fitting the individual jiggle maps together to form a larger raster, the joins are set by the unvignetted pixels. This gives some overlap with adjacent raster positions using the vignetted pixels. Raster maps are made up by repeating the individual sparse, intermediate or fully sampled jiggle map at multiple positions, using the telescope to offset between them.

All three spectrometer raster modes can be operated with the *same* offsets between the centres of the individual jiggled fields of view. These offsets are best calculated according to the FWHM of the SLW pixel size, Δ_{SLW}.

In the spacecraft Y-Z frame, these are:

ystep	$3.25 \times \Delta_{SLW}$	= 82.06
zstep	$3.75 \times \Delta_{SLW} \times \cos(30)$	= 82.00
yrowstep	$1 \times \Delta_{SLW}$	= 25.25
zrowstep	$6 \times \Delta_{SLW} \times \cos(30)$	= 131.20

where Δ_{SLW} = 25.25"

The *ystep* and *zstep* give the offsets *along* a raster row, and the *yrowstep* and *zrowstep* give the offsets *between* rows.

These offsets ensure that there are no gaps in either the SLW or SSW maps when only un-vignetted pixels are used. However, it would be complicated to create a rectangular map from these offsets – the most straightforward shape is a parallelogram with angle of 98.7°. This is inclined at an angle of 45° to the spacecraft Y-Z axes. These angles are calculated from the offsets along and between rows,

$$\theta_{YZ} = \tan^{-1}\left(\frac{ystep}{zstep}\right) = 45°$$

$$a = \sqrt{(2 zrowstep)^2 + (2 yrowstep)^2}$$

$$b = \sqrt{ystep^2 + zstep^2}$$

$$c = \sqrt{(ystep + 2 yrowstep)^2 + (2 zrowstep - zstep)^2}$$

$$\theta_{par} = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) - 90° = 98.7°$$

The *spacing of raster positions along each row in the frame of the parallelogram, s_a (116.0"), and the spacing between rows, s_b (110.6"), can also be calculated from the step sizes:

$$s_a = \sqrt{(ystep^2 + zstep^2)} = 116.0"$$

$$s_b = \sqrt{(yrowstep^2 + zrowstep^2)} \sin\left(\theta_{YZ} + \tan^{-1}\left(\frac{yrowstep}{zrowstep}\right)\right) = 110.6"$$

The user should input the length (*l*) and height (*h*) of the parallelogram and then the number of raster positions can be calculated from s_a and s_b. In calculating the area covered, the overlap between rows and the extra space at the end of rows is taken into account, with the final parallelogram centred on the users source coordinates.
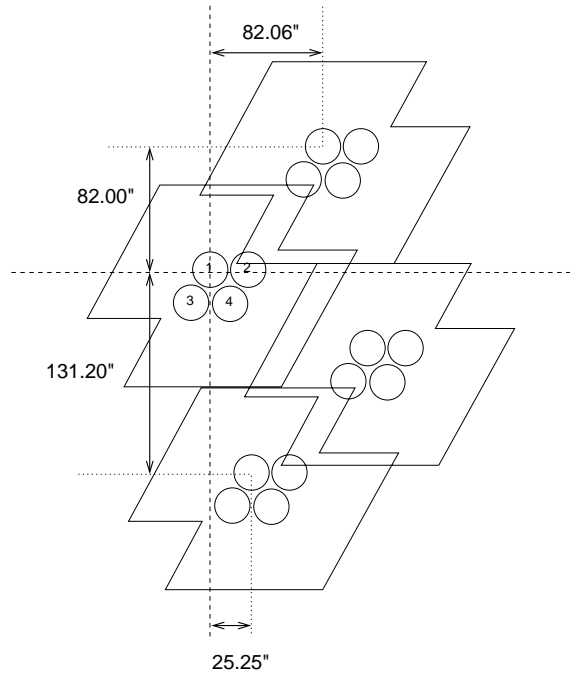
Figure 7: Diagram showing the offsets between individual raster fields of view for SLW. In this case each field of view is a 4-point jiggle and only the centre pixel of the array at each of the 4 positions is shown.
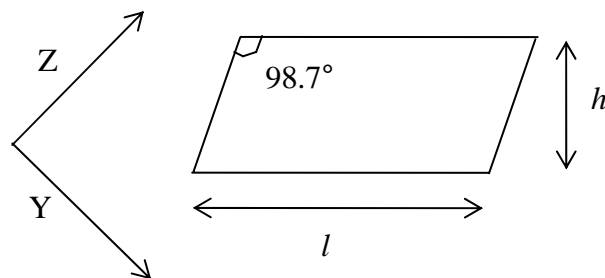

Figure 8: Orientation of the parallelogram covered by a raster map with respect to the spacecraft Y and Z axes, with the definition of raster length (l) and height (h) in the frame of the parallelogram shown.

Along a row, the coverage extends from the centre of the first field in odd rows to the centre of the last field in even rows. Between rows, the coverage extends 1/4 of the row separation above and below the centre of the first and last rows. This is demonstrated in Figure 9. The number of raster points along each direction calculated from,

$$n_l = \text{ceiling}[(l - s_a/2) / s_a] + 1$$
$$n_h = \text{ceiling}[(h - s_b/2) / s_b] + 1$$

In addition, for the case of a single row, the coverage can be extended by $s_a/2$ at the beginning of the row, ie. when $n_h = 1$,

$$n_l = \text{ceiling}[(l - s_a) / s_a] + 1$$

Technical Note

SPIRE FTS Mapping Modes

E. Polehampton

Ref:	SPIRE-RAL-NOT-002801
Issue:	1.0
Date:	16 January 2007
Page:	10 of 12

The actual area that is covered is always greater than the requested length and height because the raster can only be increased in size by integer multiples of s_a and s_b. Example sizes are shown below. Note the extra length (increased by $s_a/2$) when n_h=1.

Raster points	length	height
1×1	s_a	$1/2\ s_b$
1×2	$1/2\ s_a$	$3/2\ s_b$
2×1	$2\ s_a$	$1/2\ s_b$
2×2	$3/2\ s_a$	$3/2\ s_b$
3×3	$5/2\ s_a$	$5/2\ s_b$
4×4	$7/2\ s_a$	$7/2\ s_b$

Once the number of raster positions is known, their coordinates in spacecraft Y-Z coordinates can be calculated from the centre of the parallelogram and the step sizes. In order to obtain the parallelogram with angle of 98.7°, an extra offset of (*ystep, zstep*) must be applied every 2 rows, ie,

$$Y\_offset = start\_y + (l \times ystep) + (h \times yrowstep) + \text{int}((h + 1)/2) \times ystep$$
$$Z\_offset = start\_z + (l \times zstep) - (h \times zrowstep) + \text{int}((h + 1)/2) \times zstep$$

where (*start_y, start_z*) are the coordinates of the first field in the raster (top left corner of the parallelogram), *l* is the number of fields along a row (starting at 0), and *h* is the row number (starting at 0 and counting from top to bottom). The value of $\text{int}((h + 1)/2)$ is 0 for the first row (*h*=0), 1 for the second and third rows (*h*=1 & 2), 2 for the forth and fifth rows (*h*=3 & 4), etc.

One important point when joining the raster maps is that the spacecraft roll angle will be set by the time of the observation. Therefore it is better when constructing the final rectangular raster if the orientation of the individual jiggle maps on the sky is not critical to the pattern. Due to the length of time it takes for a high resolution SMEC scan, large maps with the highest spectral resolution will have to be split into several observations and the roll angle may have changed between them. However, the overlap between fields that has been allowed should mean that even if the roll angle changes there will not be gaps in the map.

Technical Note

SPIRE FTS Mapping Modes

E. Polehampton

Ref:	SPIRE-RAL-NOT-002801
Issue:	1.0
Date:	16 January 2007
Page:	11 of 12

The following plots show the layout of different rasters in detail.
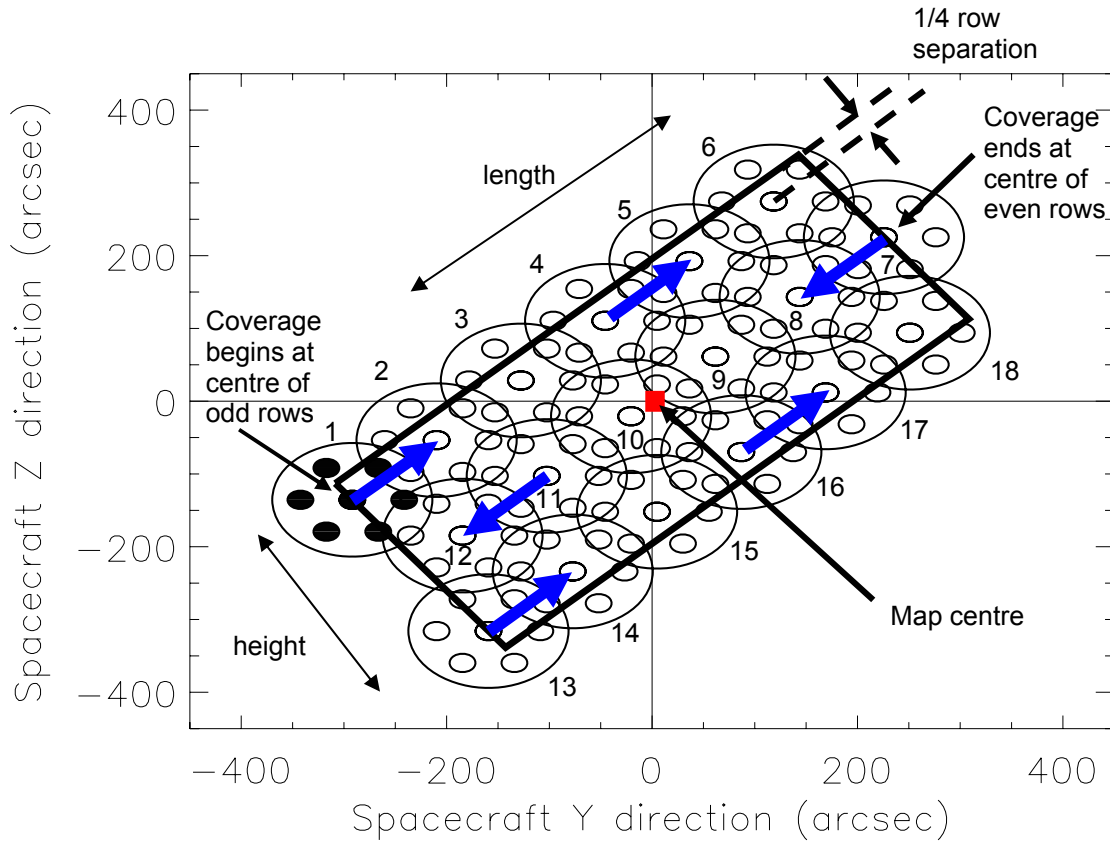

Figure 9: Plot showing the layout of a 6×3 raster. The 2.6' unvignetted field of view is shown for each raster position, as well as the positions (for a sparse map) of the unvignetted SLW pixels. The parallelogram defining the guaranteed coverage is plotted as a thick line – the source position was set to be at (0, 0). The direction of telescope movement is shown by the blue arrows – the raster begins at the top left corner of the parallelogram.
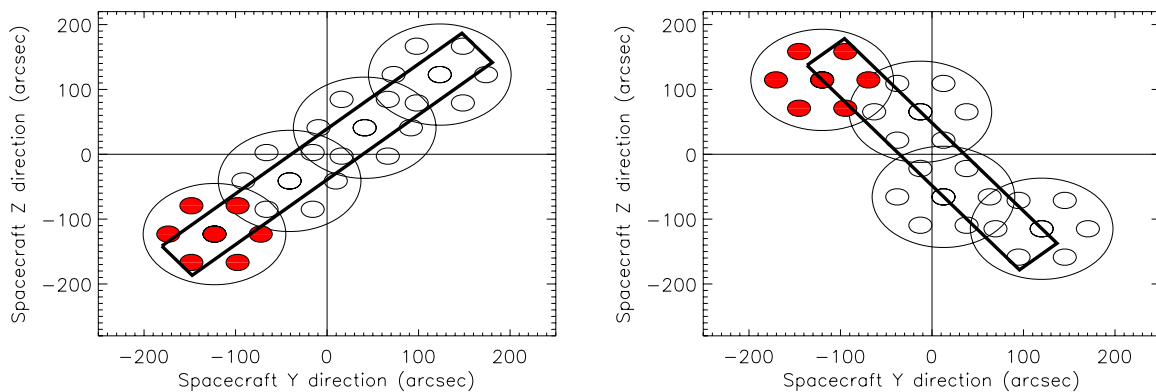

Figure 10: Plot showing the layout of a raster with a single row (user input height < ½ s_b), and with only one field in each row (user input length < ½ s_a). In both cases the source position was set to be at (0, 0). The first field to be observed is coloured red.
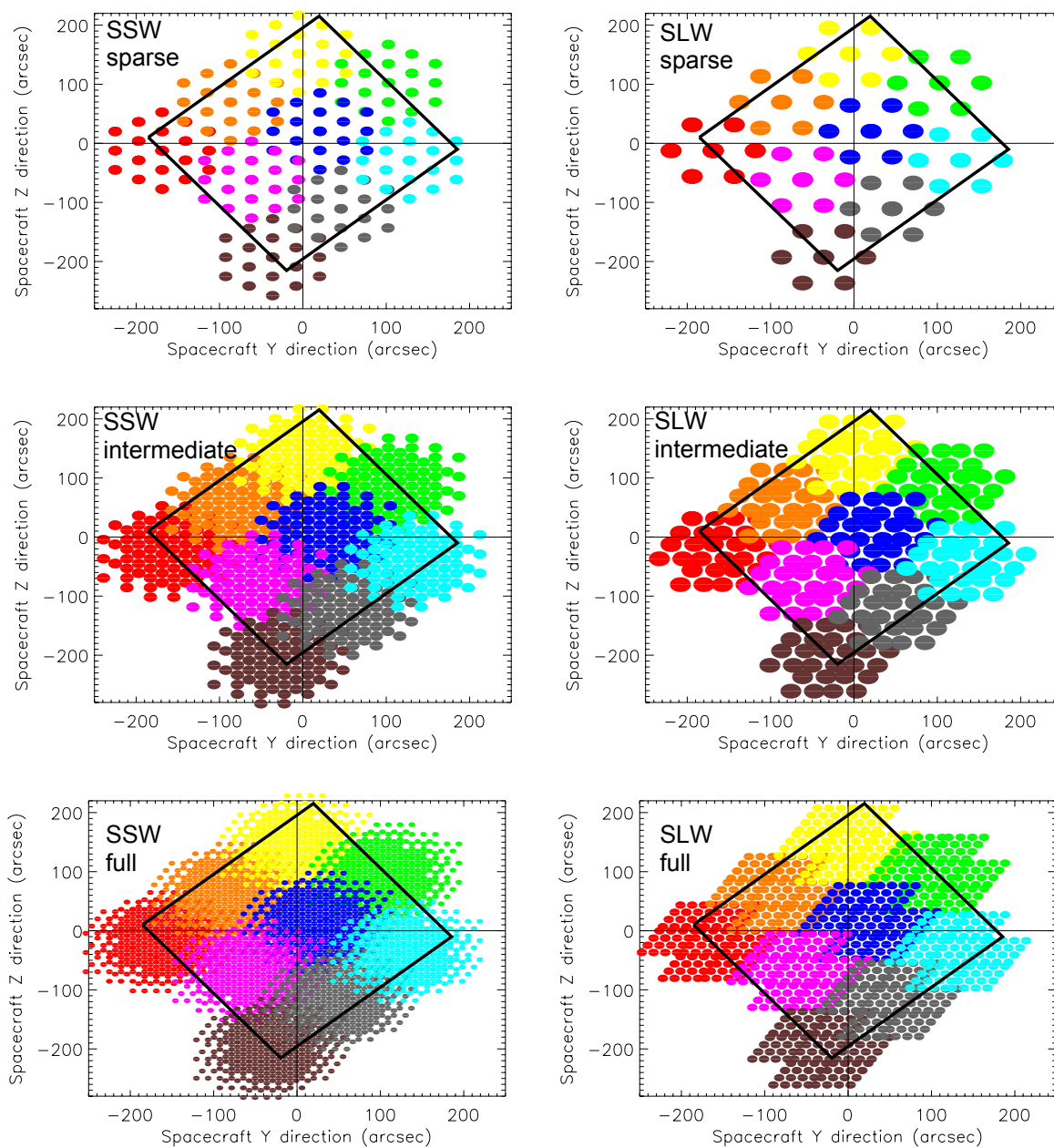
Figure 11: Plot showing the layout of individual pixels in a 3×3 raster for sparse, intermediate and full sampling. The plots on the left are for SSW and on the right for SLW. The pixels are drawn with a diameter equal to the FWHM for sparse and intermediate sampling, and 1/2 FWHM for full sampling.