SPIRE

SPIRE Photometer Simulator Verification – Scan Map Sensitivity

Prepared by: Tim Waskett
Bruce Sibthorpe

Document number: SPIRE-UCF-NOT-002756

Issue: 1.0 **Date:** 31 May 2005

Contents

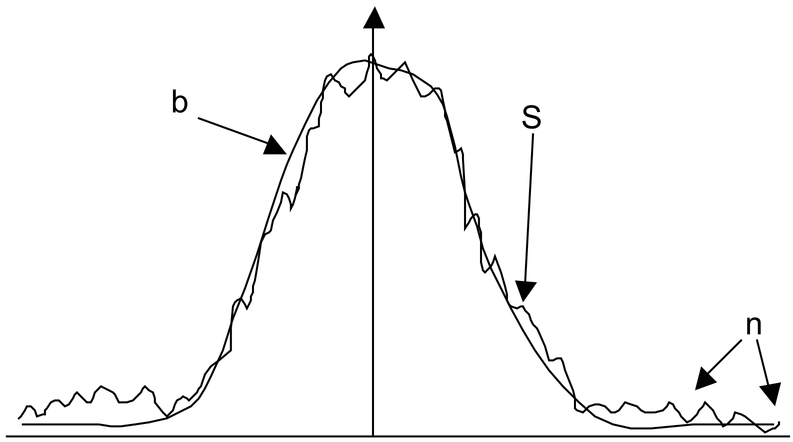
1. Introduction 2
2. Determination of the Optimum Signal-To-Noise of a Point Source........ 2
3. Generalisation to Maps with Non-uniform Noise........ 4
4. Signal to Noise of a Point Source in a Map Compared to Single Point Photometry........ 5
5. Calculation of Effective Integration Time........ 5
6. Measured Values of "ΔS_5σ_1hr_scan" From Simulated Maps........ 7

1. Introduction

This document outlines the basic theory and methods used to verify the performance of the SPIRE photometer simulator while operating in scan map mode (POF 5). We start with the theory behind calculating signal-to-noise for a point source in a map and then describe how this differs from a simple pointed observation. We then describe how the effective integration time of a map is calculated before presenting the results for the sensitivity of SPIRE using a particular scan map example.

2. Determination of the Optimum Signal-To-Noise of a Point Source

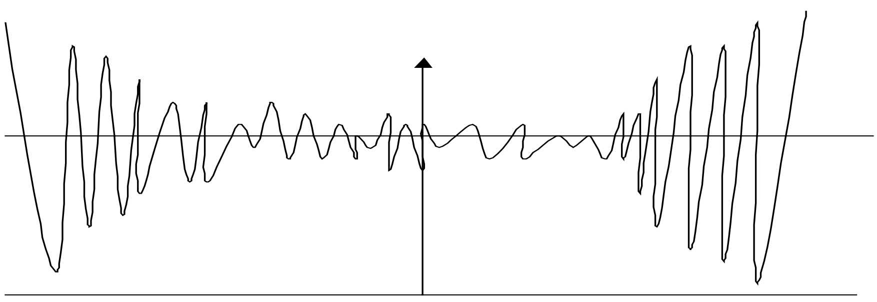
Consider a point source embedded in a noisy map with a signal, in each pixel i, of S_i and with a uniform pixel to pixel noise value of n across the map (measured by the standard deviation of the background pixel values). We would like to know the best possible value for the signal of the source, and also its signal to noise ratio. To do this we must incorporate our knowledge of the telescope beam, or Point Spread Function. We shall call this the "beam function" and, when it is binned using the same pixel size as the map, label it as bi.



Let us assume that the source has been fitted by the beam function so that it has achieved a best fit using some clever method. Now we need to find the signal and the signal to noise ratio of that fitted source.

What we really want is a measure of the peak of the signal, so to use as much information as possible we require every pixel in the map to contribute in some way towards that measurement. However, pixels in the wings of the beam will contain more noise relative to the amount of signal that they contain. Therefore we need to find an appropriate weighting function so that pixels near the centre of the beam contribute more than these noisy pixels.

So, we divide the signal map by the beam function map to get this:



Where the mean value of S_i/b_i is the value of the signal that we are interested in (modulo some normalisation by the peak of the beam function).

Now, the noise in each pixel is not constant anymore but is instead = n/b_i.

The best estimate of S/b is obtained by weighting each measurement appropriately.

$$\frac{S}{b} = \frac{\sum_i^N \frac{S_i}{b_i} \cdot \omega_i}{\sum_i^N \omega_i}$$

Where $\omega_i = \frac{1}{\sigma_i^2}$ and $\sigma_i = \frac{n}{b_i}$, are the weights that optimise this equation giving the smallest final error, and N is the total number of pixels used in the calculation (within some aperture, for example).

So, this leads to:

$$\frac{S}{b} = \frac{\sum_i^N \frac{S_i}{b_i} \cdot \frac{b_i^2}{n^2}}{\sum_i^N \frac{b_i^2}{n^2}} = \frac{\sum_i^N S_i b_i}{\sum_i^N b_i^2} \quad (1)$$

The final error in this measurement is then defined as:

$$\frac{1}{\sigma_{best}^2} = \sum_i^N \frac{1}{\sigma_i^2} = \sum_i^N \omega_i$$

$$\Rightarrow \sigma_{best} = \frac{1}{\left(\sum_i^N \omega_i\right)^{1/2}} = \frac{1}{\left(\frac{1}{n^2}\sum_i^N b_i^2\right)^{1/2}} = \frac{n}{\left(\sum_i^N b_i^2\right)^{1/2}}$$

So, signal/noise is just one divided by the other:

$$SNR = \left[\frac{\sum_i^N S_i b_i}{\sum_i^N b_i^2}\right]\left[\frac{\left(\sum_i^N b_i^2\right)^{1/2}}{n}\right]$$

And finally:

$$\boxed{SNR = \frac{\sum_i^N S_i b_i}{n\left(\sum_i^N b_i^2\right)^{1/2}}}$$

This has several advantages over other methods for calculating signal to noise: it produces an estimate that is independent of both the size of the pixels used to produce the map, and the size of the aperture placed around the source. The first point can be demonstrated by seeing how each term in the above expression varies with the pixel size.

If L is the pixel size then:

$$\sum_i^N S_i b_i \propto \frac{1}{L^2}$$

$$\sum_i^N b_i^2 \propto \frac{1}{L^2}$$

$$n \propto \frac{1}{L}$$

So:

$$SNR \propto \frac{\frac{1}{L^2}}{\frac{1}{L} \cdot \frac{1}{L}} = const$$

The second point can be seen by considering an annulus that is increased in size, beyond the extent of the beam function. As the annulus increases in size the new pixels that are captured by it contain only noise and no signal. Therefore these pixels are weighted such that they contribute nothing to the final measurement, and so have no effect on the derived signal to noise ratio. The best signal to noise is then obtained by selecting an annulus that is big enough to contain all of the significant beam function.

In addition, the beam function need not be normalised so that it matches the amplitude of the signal. As long as the beam function is centred on the position of the source (or the best fit position determined through centroiding, or whatever), the final signal to noise ratio will be independent of the beam normalisation. To obtain the final signal of the source the measurement in (1) must be multiplied by the peak value of *b*.

3. Generalisation to Maps with Non-uniform Noise

When the noise in the map is known to be non-uniform then the above expressions are only an approximation. We need to replace n with n_i in all the calculations and carry it through. This results in expressions for the signal measurement of:

$$\frac{S}{b} = \frac{\sum_i^N \frac{S_i b_i}{n_i^2}}{\sum_i^N \frac{b_i^2}{n_i^2}}$$

And for the error in this measurement of:

$$\sigma = \frac{1}{\left(\sum_i^N \frac{b_i^2}{n_i^2}\right)^{1/2}}$$

Which, when combined, give a signal to noise of:

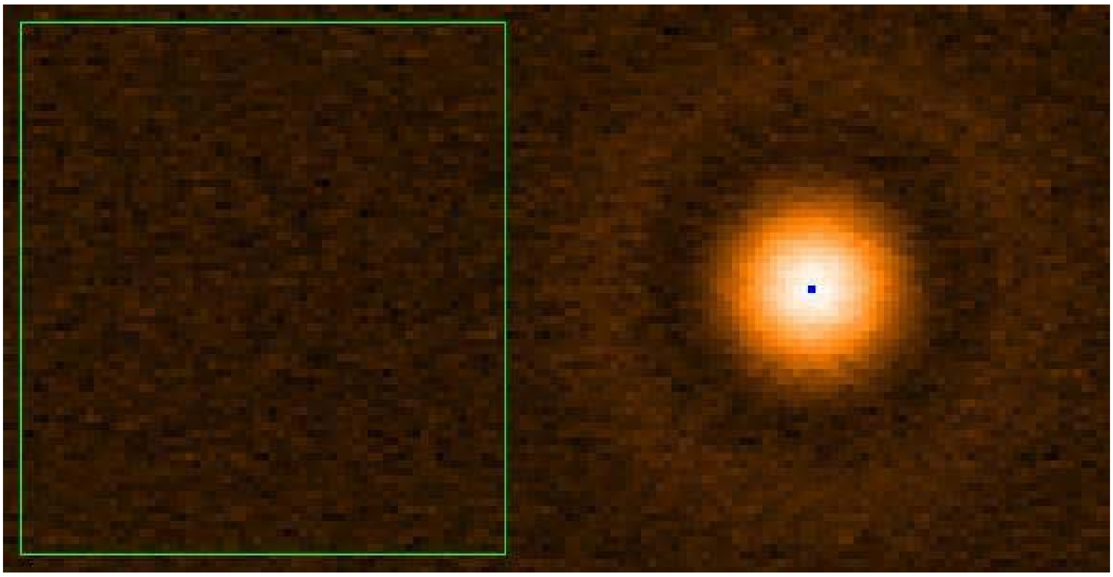
$$\boxed{SNR = \frac{\sum_i^N \frac{S_i b_i}{n_i^2}}{\left(\sum_i^N \frac{b_i^2}{n_i^2}\right)^{1/2}}}$$

The calculation of the noise map n_i is left as an exercise for the interested reader.

4. Signal to Noise of a Point Source in a Map Compared to Single Point Photometry

When calculating the signal to noise of a point source in a map, two factors need to be taken into account when comparing to the theoretical maximum, obtained from simply performing single point photometry: the improvement in SNR by adding up measurements from many map pixels, and the reduction in SNR caused by sharing the total integration time over those same map pixels.

In the example below, showing a bright source in the SPIRE PLW array, the map pixels are 2" square and the total integration time on the source is 90.1s (see below). Each pixel contains data equating to an integration time of 0.106s. The noise in this image is estimated from the 'blank' patch to the left of the source (outlined in green) and has a value of 4.53E-8 (the standard deviation of the pixel values). The value in the central pixel of the bright source (marked in blue) is 1.21E-6 above the mean background level, giving a signal to noise in that pixel of 1.21E-6/4.53E-8 = 26.7.



However, using the method outlined in the previous section, the total signal to noise obtained for this source is actually 342. That is, by summing up the contribution of each pixel to the measurement of the signal, weighted appropriately, the SNR has improved considerably, i.e. by a factor of 342/26.7 = 12.81.

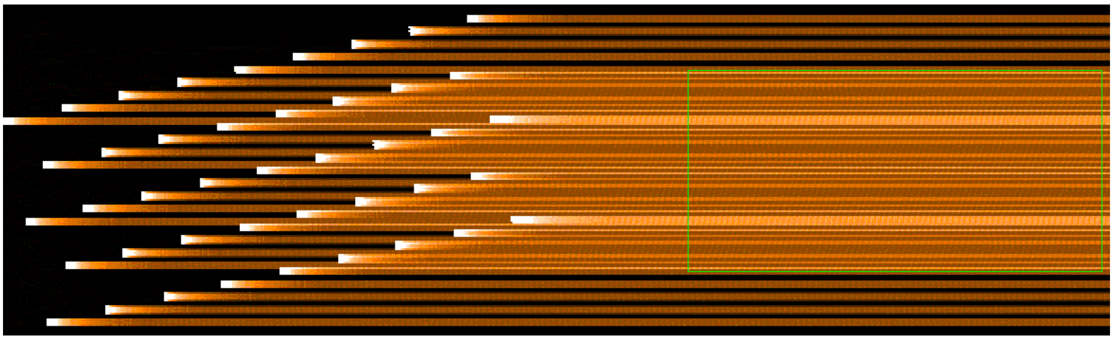
If, on the other hand, we were to simply stare at the centre of the source for the whole 90.1s of integration time, instead of spreading that time out over the rest of the map, then we would actually achieve a SNR of $26.7*(90.1/0.106)^{0.5} = 778$. In other words, the reduction in SNR in any given pixel, from the theoretical maximum, in this case, is $(90.1/0.106)^{0.5} = 29.15$.

Therefore, when making a map, the SNR of a point source can only ever be 12.81/29.15 = 342/778 = 0.44 times that of the same source observed using point source photometry, when integrating for the same length of time. This factor is roughly the same for all three arrays (within the uncertainties of the measurement process) and is robust to changes in the pixel size of the maps and to the source flux. Taking the mean of the factors for the three arrays we get: factor = 0.43.

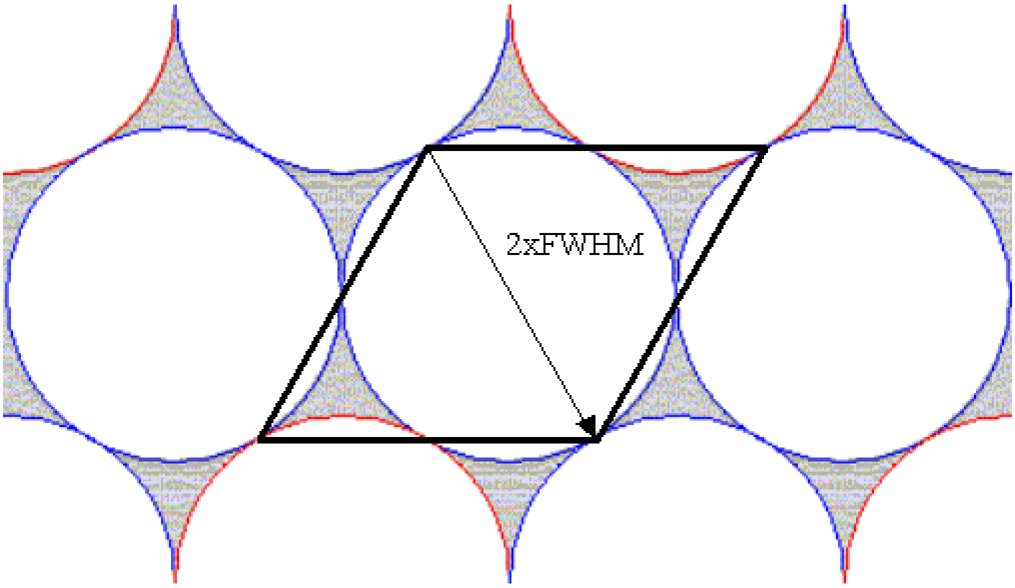
Of course, if the source is fit using some clever method, and then the model source subtracted from the map, then the noise at the exact position of the source can be estimated from the residuals rather than some nearby piece of 'blank' sky.

5. Calculation of Effective Integration Time

Because the sizes of the three SPIRE arrays are not quite the same the effective integration time for any given pixel will vary slightly from map to map (and from pixel to pixel within each map but we'll assume for now that the maps have uniform coverage in themselves). To measure the integration time for a particular scan strategy a 'counts' map can be generated, along with the signal map, that quantifies the number of data points contributing to any particular pixel. The mean number of counts per pixel can be obtained by averaging over an appropriate area, such as the green box in the image below. This example shows the PLW array having been scanned back and forth a total of 4 times with a scan separation of 2" to ensure a nice uniform coverage in the central strip, where we perform these tests.



Now, each detector in the array is responsible for filling in a particular area of the map during the scan, shown as the black rhombus in the schematic below, where the circles represent the feed horns of each detector. Assuming that the map is completely filled with data, as is the case in the central strip above, then adding up the number of counts within this area will give the effective integration time for the map. In other words, if a single detector were moved around inside this area for the effective integration time, then the average number of counts per pixel will match that of the final map. This effective integration time is different to the time it takes for the array to perform its scan.



The area of the rhombus above is simple to calculate if the separation, d, of the detectors is known (this is similar but not exactly equal to twice the FWHM of the beam for each array). To convert this area from arc seconds to pixels the size of the pixels, l, must also be taken into account.

$$A = \frac{\sqrt{3}}{2} \cdot \frac{d^2}{l^2}$$

And finally, to get the effective integration time the mean number of counts per pixel is multiplied by the time each count represents and by the number of pixels covered by each detector.

For example, the PLW array above has a mean 2"x2" pixel count of 8.8, each count represents 12ms of integration time and the area covered by each detector is 853.6 pixels. Therefore the effective integration time is 8.8 x 12e-3 x 853.6 = 90.1s.

The same calculation can be performed for the other two arrays using the appropriate counts maps and detector separations for those arrays.

6. Measured Values of "ΔS_5σ_1hr_scan" From Simulated Maps

A simulation was performed using 4 overlapping scans, separated by 2" and using a scanning speed of 25"/s, to produce final maps that contained at least 1 data point every 2", even for the PLW map. Because of the slight differences in the sizes of the three arrays the effective integration time on any given part of the different maps is not quite the same. The effective integration times for points lying in a strip along the centre of the scans (where it is most uniform) are included in the table below. No sources of 1/f noise were included in this simulation.

Maps were created for all three SPIRE arrays from data that had not had the 5Hz analogue filter applied to them, so that no correlations were introduced to the data.

The SNR of a point source with a flux of 1600mJy was calculated, using the method above, and this value was converted into the ΔS_5σ_1hr_scan values for the three arrays. The measured ΔS_5σ_1hr_scan values are shown in the fourth column of the table below.

The theoretical ΔS_5σ_1hr_scan values in the sensitivity model are effectively obtained by multiplying the basic ΔS_5σ_1hr value by several factors. These factors are as follows:

Background estimation	$2^{0.5}$
S/N loss due to need to make a map	1/0.338
Channel yield.	$1/0.8^{0.5}$

The resulting theoretical values are shown in the final column of the table.

If the channel yield and background estimation factors are ignored, and the S/N loss due to making a map is replaced by the value of 0.43 obtained in the previous section, then the resulting values are those shown in the fifth column of the table.

I.e. the ΔS_5σ_1hr_scan values are obtained from the NEFD_basic values using the following equation:

$$\Delta S\_5\sigma\_1hr\_scan = \frac{NEFD\_basic \cdot 5}{2^{0.5} \cdot 3600^{0.5} \cdot 0.43}$$

The two values in the fifth column represent the predictions obtained by using the NEFD values in the sensitivity model and the simulator, respectively.

			ΔS_5σ_1hr_scan (mJy)		
Array	Effective Integration Time (s)	SNR (1600mJy)	Measured	My Theoretical (Matt / Sim)	Matt's Theoretical
PSW	80.4	416	2.87	3.62 / 2.88	7.3
PMW	85.0	352	3.49	4.22 / 3.34	8.5
PLW	90.1	342	3.70	4.77 / 3.74	9.6

For a direct comparison between what we expect the simulator to produce and the actual measured values, compare the numbers in bold.

	NEFD Matt	NEFD Sim
PSW	26.4	21.03
PMW	30.8	24.37
PLW	34.8	27.31

We find a very good agreement between the predicted sensitivities and those that were actually measure from the output maps. The largest error is for the PMW array at 4.5%, which we deem to be acceptable given uncertainties in the measurement process.

The differences between the sensitivities predicted by the sensitivity model and the simulator can be traced to slight differences in the assumptions made, such as the channel yield and the band-passes that are implemented.

Based on this analysis we conclude that the simulator is working as expected, when operated in scan map mode.