E-mail exchange on the Design and Required Gain Stability in the DCU Band Pass Filtering
SPIRE-RAL-NOT-001140
Issued 4 Feb 2002 as part of DCU design justification for IBDR

0. Summary

Contained in this note are two e-mail exchanges and associated Mathcad models concerning the design of the electronic band pass filter that precedes the LIA in the SPIRE Detector Control Unit (DCU). They are wrapped into a technical note as they are essentially the design justification for the rather broad band of the filter that is implemented in the design.

1. Band Pass Filter Mathcad Models

From: Viktor Hristov [vvh@astro.caltech.edu]
Sent: 19 September 2001 19:41
To: Swinyard, BM (Bruce)
Cc: 'Viktor Hristov'; Delderfield, J (John) ; Jamie Bock (E-mail)
Subject: RE: BPF

Hi Bruce,

Please read my comment, embeded in your message.

Viktor.

On Wed, 19 Sep 2001, Swinyard, BM (Bruce) wrote:

> In the MathCad sheet give give the transfer function for the combined LIA
> and bandpass filter using a "resonance frequency gain" of m/Q where m is the
> order and Q is the (relative) inverse width of the filter =1/deltaW - is
> this correct?

Yes, our goal is to maximize the quality factor Q of the BPF (ratio of the carrier frequency vs the bandwidth of the filter), this way to reduce the contribution of the noises cntered on the odd harmonics of the carrier.

>
> The "trim range" for the bias frequency is set at Fb +- 40 Hz (25*pi/2)
> I confess I don't understand how you have set this - it is not what is in
> the specification which is set at 50-300 Hz. Are you asking that the
> specification be changed?

The trim range should be higher than the signal bandwidth of the spectrometer, so if a microphonics occure on some of the odd harmonics of the carrier, we could move it's position on the output spectra outside of the LIA noise bandwidth (the magic number Pi/2 reflects this for a single pole filter. We use 4-pole so this factor could be relaxed a bit ;). As already defined in the specs, we can shift the carrier frequency around alot. This possibility reflects the FPGA design to synthesize 256-point sine, by dividing a 10 Mhz clock by an integer number. In real life though, if the BPF Q is going to be increased, we have to bite the bulet and set the carrier frequency to some reasonable value (say 150 Hz +

or - 25 Hz to avoid the power lines interference in the lab) and define
the trim range, outside of which the performance of the LIA will be
degradated.

>
> You then use the trim range and the post detection bandwidth to set the
> maximum frequency harmonics against which the BPF has to defend for both the
> spectrometer and photometer cases.

I use the summ of the signal bandwidth & trim range to define the maximuum
Q in the both cases. The trim ranges are assumed to be the same, even
though the signal bandwidth of the photometer is alot smaller. It's
because the photometer and the spectrometer use the same FPGA to produce
the bias. We may define separate trim ranges for the spectrometer and for
the photometer, what will increase the photometer Q, but for that an input
form Frederic will be needed.

Actually I'd be very happy to hear any comments from the CEA.

>You give this as both amplitude and
> intensity? This is why you square the transfer function? Again forgive the
> stupid questions.

I try to quantify how much the odd carrier harmonics are being rejected
(kind of how much the microphonics centered across the odd harmonics of
the carrier and falling inside the signal bandwidth contribute. I think
that's what you wanted?

>
>
>
> > -----Original Message-----
> > From: Viktor Hristov [mailto:vvh@astro.caltech.edu]
> > Sent: 17 September 2001 20:41
> > To: 'Swinyard Bruce'
> > Cc: AUGUERES Jean-Louis DAPNIA; 'Pinsard Frederic'; CARA Christophe
> > SMTP; 'Griffin M.'; 'King Ken'; 'Bock Jamie'; 'Lilienthal Gerald'
> > Subject:
> >
> >
> > Hi Bruce,
> >
> > Attached here is a MCAD document and a ".pdf" print of it,
> > regarding your
> > request of more tight definition of the SPIRE preamplifier Band-Pass
> > Filter. A ratios of the attenuations of the (odd harmonics/first
> > harmonic) signals and powers are listed too.
> >
> > Naturally to implement this stuff, we have to change some
> > component values
> > of the Photometer/Spectrometer LIA preamplifiers. To do so,
> > we will need a
> > request of change, like the one issued on the LIA LPF.
> >

> > Don't forget the ordering of the passive components for the QM1 is
> > stalled, till we receive the new corrected values for
> > whatever from CEA
> > (unleas I get official permission to estimate the new values
> > by myself).
> >
> > Besides studying the CEA schematics/boards I found the
> > component values
> > for the Spectrometer/Photometer gains are ill defined, and there are
> > diodes missing on the inputs of EACH LIA channel.
> >
> > Viktor.
> >
>

2. Bandpass Filter Stability Requirement

Viktor Hristov wrote:

Hi Frederic,

Glad you open this second bias harmonic question. It runs deeper than just the LIA LPF, but also the bolometer temperature fluctuating on the double bias frequency. In your conclusion you assume we will have to keep the bias frequency high, assuming a sinusoidal bias waveform. After the synchronous detector the rectified sine will have the spectral content you described, and naturally the LIA LPF have to be steep enough to attenuate the second harmonic suficiently, else we will get an interference between the ADC sampling frequency and the double bias frequency. Now naturally some could ask, how could we get interference between two synchronous signals (because presumably the sampling is syncronized with the bias). Hence the beat frequency will be located on 0Hz and won't have scientific significance.

Then there is no strict requirement on the shape of the bias. If rectangular, we can bias as low as 50 Hs with no second harmonics problems. Now natirally having a rectangular bias currents through the bolometer is impossible with the current bias scheme due to the parasitic capacitances in the cold front end, so the output of the bolometer will have ripples due to the electrical timeconstant, what will show up at the input of the LIA. Yet the transient contribution at the output of the LPF will be much less than if we had pure sinusoidal bias. Naturally having rectangular bias is not very healty in terms of x-talk, and if some takes in account the electrical timeconstant, then a rectangular bias with sinusoidal edges might be optimal. The frequency of the sinusoidal transition could be choosen high (say >200 Hz) to match the bolometer electrical timeconstant. The frequency change will be equivalent to to keeping the bias at the max/min value of the sine for the clock cycles needed.

Regarding the Band-Pass filter, I allways try to resist making it too narrow-band because it has not been designed to be such. The design has been made, assuming wide range of bias, low power consumption and low input noise. To achieve this initial conditions, the BPF function is embedded into the preamplifier design, and can't be separated. Being part of the preamplifier feedback, it's parameters depend not only on the passive component value and stability, but also on the active component behaviour.

Viktor.

>
> From Frederic Pinsard
>

Dear ….,

I hope that you are well.

I'm writing because I don't think that we are going in the right direction with the BPF.

I've been doing some new simulations, calculations concerning all the analog processing (BPF and LPF.)

First of all, I don't think it's very pertinent that the transfer function of the BPF is so dependent on MAT2 parameters like HFE and their biasing.

Currently in the design that has already been implemented, the biasing of MAT2 is given by 5 volt reference instead of the 6 volt in Viktor's Mathcad files.
To look at this, I did some simulations on Spice with all Viktor's component parameters (resistors, capacitors and the 6V polarization references.) With this simulation I found a resonance frequency Fr =178 Hz instead of the 175Hz that Viktor had calculated. In other words, we don't know which differences we will get with the real components.

I also did a simulation with the same component but with a 5V bias on the MAT02, I found Fr= 197 Hz.

Conclusion:
I'm not sure that we will be able to obtain the same strict characteristics for all the channels with a design that depends on the temperature, component dispersion and bias.

As you know the layouts of the LIA_P and LIA_S boards are finished, so if we have to implement this new filter, we will have to do this job again and it will take at least 4weeks to have the new layouts ready - if it will even fit (that is the other problem that worries me).

The only thing we could do without changing the existing layouts is to increase the 470pF capacitors to 1.5nF capacitors and remove the 10k serial resistors.

I have also done the following calculation:

Square ware Fourier decomposition:

Sq(wt)= 4/pi [sin(wt)+(1/3)sin(3wt)+(1/5).sin(5wt)+.....]

Input signal of the pre-amp:

$$In=(Vb+Nb)\sin(wt)+Nwe$$

Vb= bolometer signal , Nb=bolometer noise , Nwe=noise warm electronic (white noise)

Signal demodulation:

$$= 2/\pi\ (Vb+Nb)\ [1- (2/3)\cos(2wt)- (2/15)\cos(4wt)\ ......] + Nwe.\pi/\sqrt{8}$$

So, for a 19bit resolution system we need an LPF that reduces the ripple from the 2n harmonics to a level under 1LSB. This obliges us to use the following calculation:
(H: filter transfer function)
$H(2wt)/H(0) < 3/1048576$ this means that we have to have about –110dB of attenuation between the main terms and the terms in cos(2wt).

So for the photometer with the 4 poles LPF with a cut off frequency at 5Hz the modulation frequency must be above 120Hz.
And for the spectrometer with the 6 poles LPF with a cut off frequency at 25Hz the modulation frequency must be above 206Hz.

Conclusion
The current frequency range of the bolometer bias is incompatible with the performance requirement for the DCU. The low frequency limit of 100Hz seems more adapted than the current 50Hz.

I look forward to hearing your response to this letter.
Frederic

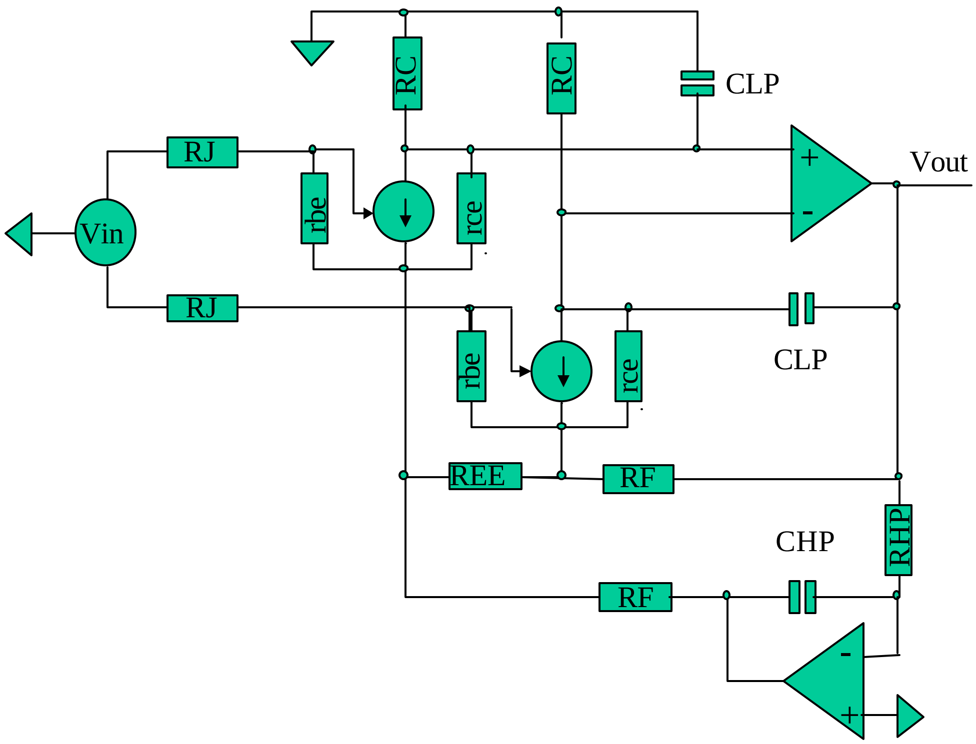
Viktor Hristov wrote:

> Yo Jamie,
>
> assuming a periodic sawtoot wave, with amplitude of (1000/3600)*1 K and
> period of 1000 sec (1 mHz fundamental frequency), for the fourier
> decomposition we got coefficients -> an = (0.3/n*Pi) [K], hence for 100
> mHz, the harmonic will be appr 1 mK. The requrement of 1E-5 gain stability
> per 100 mHz bandwidth needs a stability of the LIA transfer function,
> better than 1E-5/1E-3 = 1%/K. For 10 mHz bandwidth, the tenth harmonic
> will have appr 10 mK amplitude, hence the LIA transfer function stability
> will have to be better than 1E-3/K (1000 ppm/K). Taking in account the 1/3
> factor (assuming the thermal instability to contribute les than one STD
> unit of a gaussian process), for 10 mHz case we will need overal stability
> of the LIA TF better than 330 ppm/K.
>
>
> Viktor.
>
> On Fri, 26 Oct 2001, Jamie Bock wrote:
>
> >
> > Hi Viktor,

> >
> > I calculate that the gain stability has to be < 200 ppm/K in order
> > to be < 1/3 the total noise at 100 mHz for a 1000 s observation.
> > Does that sound right to you? 200 ppm/K was larger than I expected,
> > so you might double check.
> >
> > Your current Q = 0.8 BPF has 500 ppm/K at 170 +/- 50 Hz. Assuming
> > the temperature sensitivity is linear, we meet the specification
> > at 170 +/- 20 Hz. No good. I think we can achieve 150 +/- 50 Hz,
> > 200 ppm/K only if Q = 0.3. So the question is: can we implement
> > a filter in the given board constraints that meets the 200 ppm/K
> > requirement, attenuates the 3rd and 5th harmonic appreciably, and
> > allows tunability from 100 to 200 Hz. If not I think we should
> > forget the BPF.
> >
> > Since I've done a favor from you, can you please answer Christophe's
> > questions on the DRCU specifications? I'll forward it to you again.
> >
> > Jamie
> >

SPIRE PREAMP BPF MODEL

Viktor Hristov



BPF resonance gain, resonance frequency and quality factor @ T0

$Hr = 250$ $\qquad \omega r0 \equiv 2\cdot\pi\cdot 175$ $\qquad Q0 = 1.5$

Component values:

$RF \equiv 15\cdot 10^{3}$ $\qquad RE \equiv 15\cdot 10^{3}$ $\qquad RJ \equiv 5\cdot 10^{3}$

Operational temperatures:

$\delta T \equiv 1$ $\qquad T0 \equiv 300$ $\qquad T1 := T0 + \delta T$

Basic constants:

$qe \equiv 1.60217733\cdot 10^{-19}$ $\qquad kb \equiv 1.380658\cdot 10^{-23}$

$VT(T) := \dfrac{kb\cdot T}{qe}$ $\qquad VT(T0) = 0.026$ $\qquad VT(T1) - VT(T0) = 8.617\times 10^{-5}$

Frontend MAT02 parameters

$IS \equiv 6 \cdot 10^{-13}$ $\qquad$ $\beta \equiv 500$

By the expression for the collector current:

$IC(T, VBE) := IS \cdot \exp\left(\frac{VBE}{VT(T)}\right)$ $\qquad$ $IC(T0, 0.6) = 7.205 \times 10^{-3}$ $\qquad$ $IC(T1, 0.6) = 6.671 \times 10^{-3}$

$LnIC(T, VBE) := \ln(IS) + \frac{VBE}{VT(T)}$ $\qquad$ $LnIC(T0, 0.6) = -4.933$

Find the VBE:

$x := 0.6$ $\qquad$ $Vbe(T) := root(LnIC(T, x) - \ln(0.0001), x)$ $\qquad$ $Vbe(T0) = 0.489$ $\qquad$ $Vbe(T1) = 0.491$

$IC(T0, Vbe(T0)) = 1 \times 10^{-4}$ $\qquad$ $Vbe(T1) - Vbe(T0) = 1.631 \times 10^{-3}$

Find the transistors transconductance, BE and CE impedances

$S(T) := \frac{IC(T, Vbe(T))}{VT(T)}$ $\qquad$ $S(T0) = 3.868 \times 10^{-3}$ $\qquad$ $S(T1) = 3.855 \times 10^{-3}$

$rce(T) := \frac{100}{IC(T, Vbe(T))}$ $\qquad$ $rbe(T) := \frac{\beta}{S(T)}$ $\qquad$ $rce(T0) = 10 \times 10^{5}$ $\qquad$ $rbe(T0) = 1.293 \times 10^{5}$

Find the gain resistor REE by the preamp gain @ T0:

$Gx(REE) := \left(\frac{RF}{rce(T0)} + 1 + \frac{2 \cdot RF}{REE} + \frac{RF}{RE}\right)$ $\qquad$ $Gx(100) \cdot 10^{3} = 3.02 \times 10^{5}$

$x := 2\frac{RF}{Hr}$ $\qquad$ $REE := root(Gx(x) - Hr, x)$ $\qquad$ $REE := floor(REE)$ $\qquad$ $REE = 120$

Define the preamplifier gain thermal dependence:

$G(T) := \left(\frac{RF}{rce(T)} + 1 + \frac{2 \cdot RF}{REE} + \frac{RF}{RE}\right)$ $\qquad$ $G(300) = 252.015$ $\qquad$ $G(301) = 252.015$

Find the BPF timeconstants by the required ωr and Q @ T0

$\tau LP0 := \frac{Q0}{\omega r0}$ $\qquad$ $\tau HP0 := \frac{1}{Q0 \cdot \omega r0}$ $\qquad$ $\tau LP0 = 1.364 \times 10^{-3}$ $\qquad$ $\tau HP0 = 6.063 \times 10^{-4}$

Find the BPF resistance and capacitance values @T0:

$RLP0 := \left[\frac{G(T0)}{\beta} \cdot (rbe(T0) + RJ) + RF \cdot \frac{(\beta + 1)}{\beta}\right]$ $\qquad$ $CLP0 := \frac{\tau LP0}{RLP0}$

$RHP0 := \frac{RLP0}{Q0^2}$ $CHP0 := \frac{\tau HP0}{RHP0}$

Results for the BPF component values @ T0:

$RLP0 = 8.27 \times 10^4$ $CLP0 = 1.65 \times 10^{-8}$ $RHP0 = 3.676 \times 10^4$ $CHP0 = 1.65 \times 10^{-8}$

Define the low-pass capacitor and timeconstant dependence of the temperature:

$CLP(T) := CLP0 \cdot [\, 1 + TCC \cdot (T - T0)\,]$ $CLP(T1) = 1.649 \times 10^{-8}$

$\tau LP(T) := CLP(T) \cdot \left[\frac{G(T)}{\beta} \cdot (rbe(T) + RJ) + RF \cdot \frac{(\beta + 1)}{\beta} \right]$ $\tau LP(T1) = 1.368 \times 10^{-3}$

Define the high-pass capacitor and timeconstant:

$RHP0 = 3.676 \times 10^4$

$CHP(T) := CHP0 \cdot [\, 1 + TCC \cdot (T - T0)\,]$

$\tau HP(dT) := RHP0 \cdot CHP(dT)$

Define the bandpass filter resonance frequency:

$$\omega r(T) := \sqrt{\frac{1}{\tau LP(T) \cdot \tau HP(T)}}$$

Find the relative variation of the resonance frequency for a 1 K temperature variation:

$\frac{\omega r(T0)}{2 \cdot \pi} = 175$ $\frac{\omega r(T1)}{2 \cdot \pi} = 174.776$ $\frac{2 \cdot (\omega r(T0) - \omega r(T1))}{\omega r(T0) + \omega r(T1)} = 1.281 \times 10^{-3}$

Define the BPF badwidth and quality factor temperature dependence:

$BW(T) := \frac{1}{\tau LP(T)}$ $Q(T) := \sqrt{\frac{\tau LP(T)}{\tau HP(T)}}$ $\frac{BW(T0)}{2 \cdot \pi} = 116.667$ $Q(T0) = 1.5$

Define the BPF transfer function:

$H(\omega, T) := G(T) \cdot \frac{j \cdot \omega \cdot BW(T)}{\omega r(T)^2 + j \cdot \omega \cdot BW(T) - \omega^2}$ $j \equiv \sqrt{-1}$ $H(\omega r(T0), T0) = 252.015$

Define the Lock-In transfer function:

$LH(\omega, T) := \cos(\arg(H(\omega, T))) \cdot |H(\omega, T)|$ $LH(\omega r(T0), T0) = 252.015$

Define the relative variation of the BPF transfer function for dT change in temperature

$$RLH(\omega) := \frac{2\cdot(LH(\omega, T1) - LH(\omega, T0))}{LH(\omega, T1) + LH(\omega, T0)} \qquad RLH(\omega r0) = -1.481 \times 10^{-5}$$

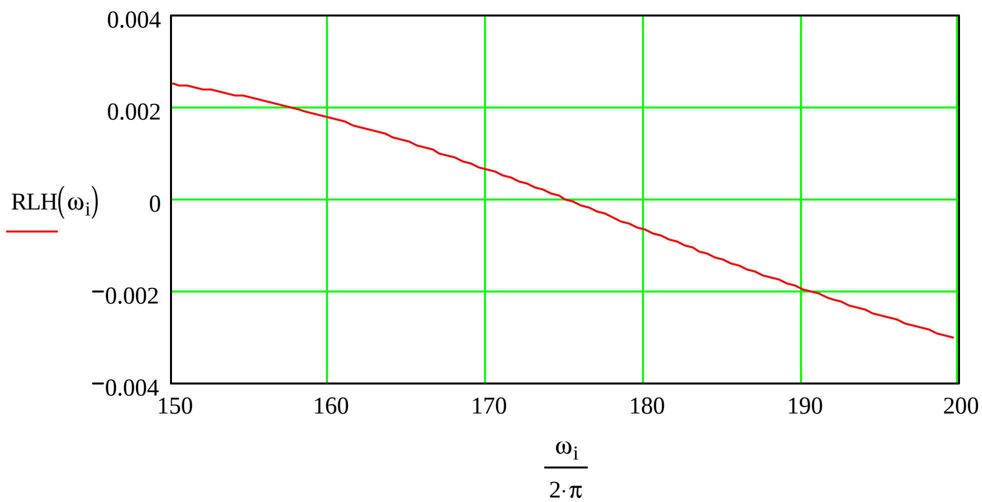
Define the frequency index variable

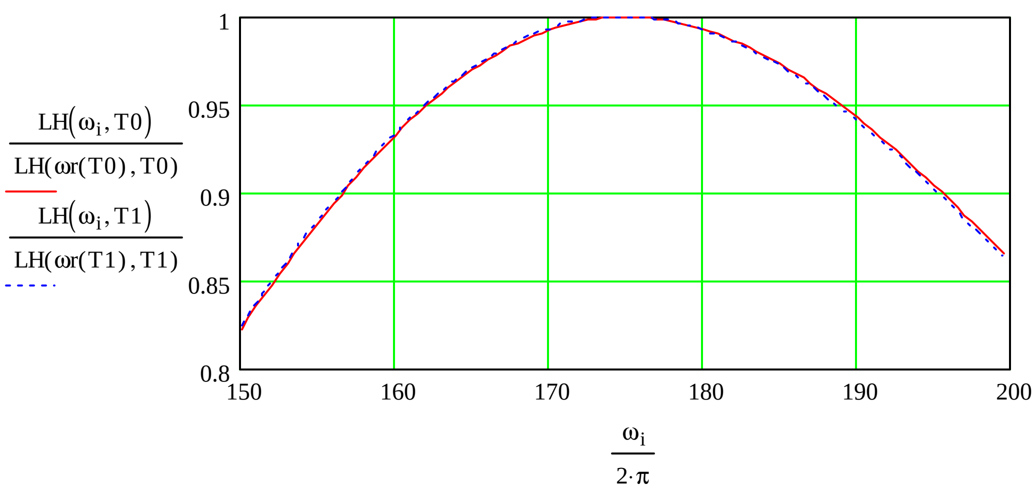
$N := 100 \qquad i := 0 .. N - 1$

$Fmin \equiv 150 \qquad Fmax \equiv 200 \qquad \delta F := \frac{Fmax - Fmin}{N} \qquad \omega_i := (Fmin + i\cdot\delta F)\cdot 2\cdot\pi$

Set the initial values for the capacitance thermal coefficient, Q and gain:

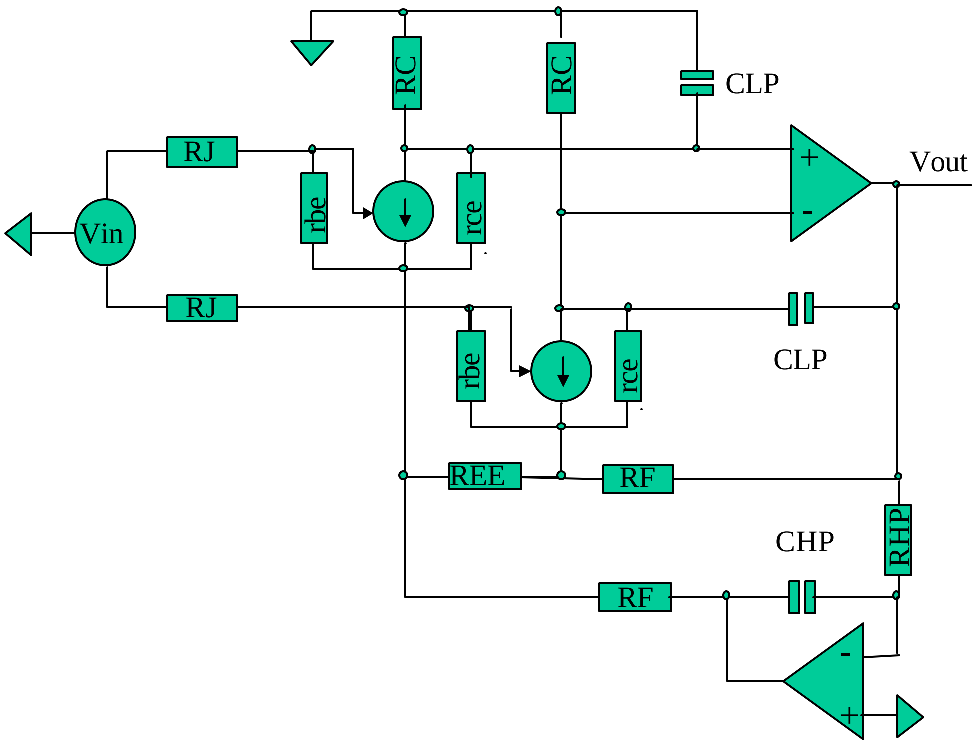
$TCC \equiv -30\cdot 10^{-6} \qquad Q0 \equiv 1.5 \qquad Hr \equiv 250$





SPIRE PREAMP BPF MODEL

Viktor Hristov



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Find the VBE:

$x := 0.6 \qquad Vbe(T) := root(LnIC(T,x) - \ln(0.0001), x) \qquad Vbe(T0) = 0.489 \qquad Vbe(T1) = 0.491$

$IC(T0, Vbe(T0)) = 1 \times 10^{-4} \qquad Vbe(T1) - Vbe(T0) = 1.631 \times 10^{-3}$

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$rce(T) := \frac{100}{IC(T, Vbe(T))} \qquad rbe(T) := \frac{\beta}{S(T)} \qquad rce(T0) = 10 \times 10^{5} \qquad rbe(T0) = 1.293 \times 10^{5}$

Find the gain resistor REE by the preamp gain @ T0:

$Gx(REE) := \left(\frac{RF}{rce(T0)} + 1 + \frac{2 \cdot RF}{REE} + \frac{RF}{RE}\right) \qquad Gx(100) \cdot 10^{3} = 3.02 \times 10^{5}$

$x := 2\frac{RF}{Hr} \qquad REE := root(Gx(x) - Hr, x) \qquad REE := floor(REE) \qquad REE = 306$

Define the preamplifier gain thermal dependence:

$G(T) := \left(\frac{RF}{rce(T)} + 1 + \frac{2 \cdot RF}{REE} + \frac{RF}{RE}\right) \qquad G(300) = 100.054 \qquad G(301) = 100.054$

Find the BPF timeconstants by the required ωr and Q @ T0

$\tau LP0 := \frac{Q0}{\omega r0} \qquad \tau HP0 := \frac{1}{Q0 \cdot \omega r0} \qquad \tau LP0 = 7.276 \times 10^{-4} \qquad \tau HP0 = 1.137 \times 10^{-3}$

Find the BPF resistance and capacitance values @T0:

$RLP0 := \left[\frac{G(T0)}{\beta} \cdot (rbe(T0) + RJ) + RF \cdot \frac{(\beta + 1)}{\beta}\right] \qquad CLP0 := \frac{\tau LP0}{RLP0}$

$$RHP0 := \frac{RLP0}{Q0^2} \qquad CHP0 := \frac{\tau HP0}{RHP0}$$

Results for the BPF component values @ T0:

$$RLP0 = 4.19 \times 10^{4} \qquad CLP0 = 1.737 \times 10^{-8} \qquad RHP0 = 6.546 \times 10^{4} \qquad CHP0 = 1.737 \times 10^{-8}$$

Define the low-pass capacitor and timeconstant dependence of the temperature:

$$CLP(T) := CLP0 \cdot [\,1 + TCC \cdot (T - T0)\,] \qquad CLP(T1) = 1.737 \times 10^{-8}$$

$$\tau LP(T) := CLP(T) \cdot \left[\frac{G(T)}{\beta} \cdot (rbe(T) + RJ) + RF \cdot \frac{(\beta + 1)}{\beta} \right] \qquad \tau LP(T1) = 7.29 \times 10^{-4}$$

Define the high-pass capacitor and timeconstant:

$$RHP0 = 6.546 \times 10^{4}$$

$$CHP(T) := CHP0 \cdot [\,1 + TCC \cdot (T - T0)\,]$$

$$\tau HP(dT) := RHP0 \cdot CHP(dT)$$

Define the bandpass filter resonance frequency:

$$\omega r(T) := \sqrt{\frac{1}{\tau LP(T) \cdot \tau HP(T)}}$$

Find the relative variation of the resonance frequency for a 1 K temperature variation:

$$\frac{\omega r(T0)}{2 \cdot \pi} = 175 \qquad \frac{\omega r(T1)}{2 \cdot \pi} = 174.825 \qquad \frac{2 \cdot (\omega r(T0) - \omega r(T1))}{\omega r(T0) + \omega r(T1)} = 9.979 \times 10^{-4}$$

Define the BPF badwidth and quality factor temperature dependence:

$$BW(T) := \frac{1}{\tau LP(T)} \qquad Q(T) := \sqrt{\frac{\tau LP(T)}{\tau HP(T)}} \qquad \frac{BW(T0)}{2 \cdot \pi} = 218.75 \qquad Q(T0) = 0.8$$

Define the BPF transfer function:

$$H(\omega, T) := G(T) \cdot \frac{j \cdot \omega \cdot BW(T)}{\omega r(T)^2 + j \cdot \omega \cdot BW(T) - \omega^2} \qquad j \equiv \sqrt{-1} \qquad H(\omega r(T0), T0) = 100.054$$

Define the Lock-In transfer function:

$$LH(\omega, T) := \cos(\arg(H(\omega, T))) \cdot |H(\omega, T)| \qquad LH(\omega r(T0), T0) = 100.054$$

Define the relative variation of the BPF transfer function for dT change in temperature

$$\mathrm{RLH}(\omega) := \frac{2\cdot(\mathrm{LH}(\omega, \mathrm{T1}) - \mathrm{LH}(\omega, \mathrm{T0}))}{\mathrm{LH}(\omega, \mathrm{T1}) + \mathrm{LH}(\omega, \mathrm{T0})} \qquad \mathrm{RLH}(\omega r0) = -2.555 \times 10^{-6}$$

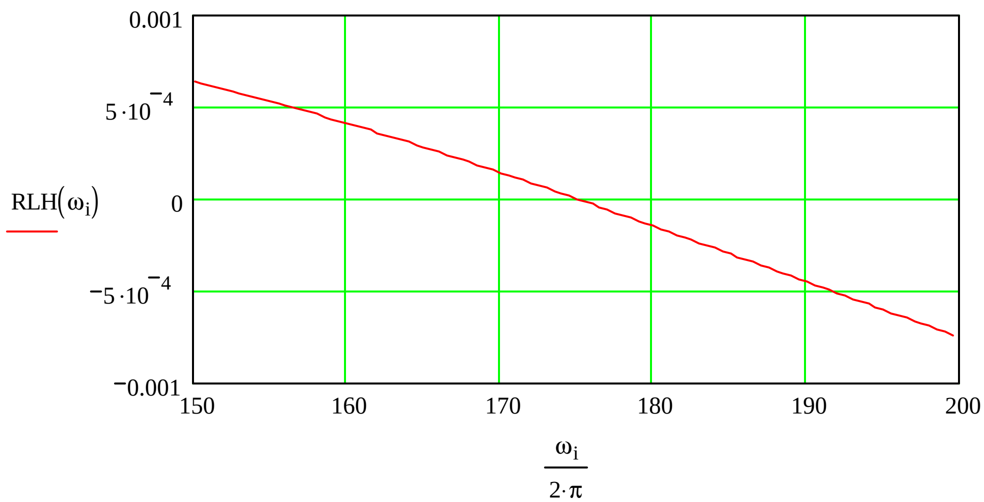
Define the frequency index variable

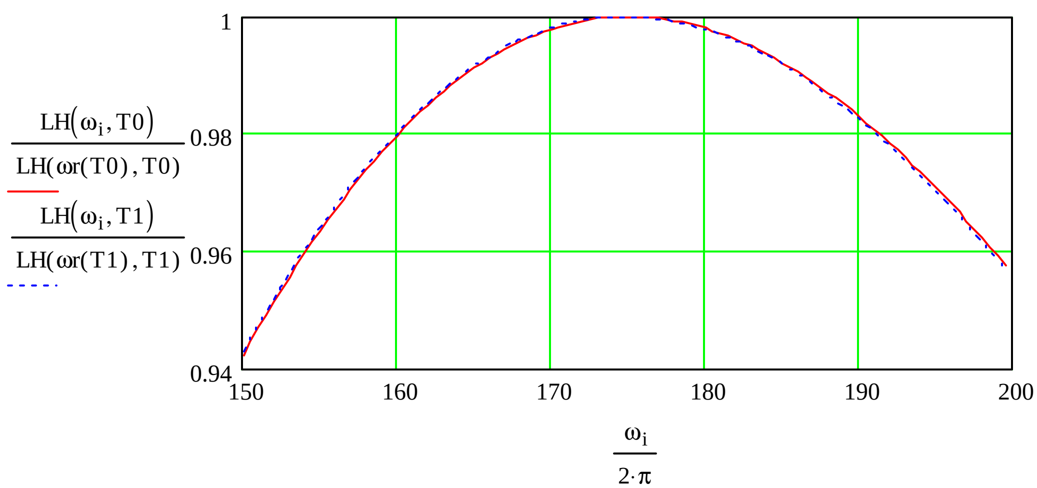
$N := 100 \qquad i := 0 .. N - 1$

$$\mathrm{Fmin} \equiv 150 \qquad \mathrm{Fmax} \equiv 200 \qquad \delta F := \frac{\mathrm{Fmax} - \mathrm{Fmin}}{N} \qquad \omega_i := (\mathrm{Fmin} + i\cdot\delta F)\cdot 2\cdot\pi$$

Set the initial values for the capacitance thermal coefficient, Q and gain:

$\mathrm{TCC} \equiv -30\cdot 10^{-6} \qquad \mathrm{Q0} \equiv 0.8 \qquad \mathrm{Hr} \equiv 100$





SPIRE LOCK-IN BAND PASS FILTER OPTIMIZATION

Viktor Hristov

1. LOCK-IN TRANSFER FUNCTION

$$OUT(t) = \frac{1}{T_{\mathrm{int}}} \int_0^{T_{\mathrm{int}}} X(t) R(t) dt$$

Lock-in output

$$R(t) = \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin((2n+1)\nu_{ref} t)$$

+1, t = 0 ..Tref/2
-1, t = Tref/2 .. Tref

Lock-in reference function

$$X(t) = A_m \sin(m \nu_{ref} t + \varphi_m) \Rightarrow OUT(t) = A_m \frac{2}{m\pi} \cos(\varphi_m) \quad , m = 2n+1$$

2. SECOND ORDER BAND-PASS FILTER TRANSFER FUNCTION

$$HBPF(P) = \frac{HBPF_r \dfrac{P}{Q}}{P^2 + \dfrac{P}{Q} + 1}, \quad HBPF_r \text{ is the resonance frequency gain}$$

$$here: \quad Q = \frac{1}{\Delta\Omega} = \frac{1}{\Omega_{\max} - \Omega_{\min}}, \quad P = j\Omega, \quad \Omega = \frac{\omega}{\omega_r} \quad \text{is the normalized frequency}$$

$$\Omega_{\max/\min} = \sqrt{4 + \Delta\Omega^2} \pm \frac{\Delta\Omega}{2}, \quad -3db \ BPF \ \ cutoff \ \ frequencys$$

$$\left| HBPF \right| = \frac{HBPF_r \dfrac{\Omega}{Q}}{\sqrt{1 + \Omega^2 \left(\dfrac{1}{Q^2} - 2 \right) + \Omega^4}}, \quad \varphi = a\tan\left(Q\left(\frac{1}{\Omega} - \Omega \right) \right)$$

3. Combined BPF and LOCK_IN TRANSFER FUNCTION vs harmonic number m:

$$H_m = \frac{2}{m\pi} \frac{HBPF_r \dfrac{m}{Q}}{\sqrt{1 + m^2 \left(\dfrac{1}{Q^2} - 2 \right) + m^4}} \cos\left(a\tan\left(Q\left(\frac{1}{m} - m \right) \right) \right), \quad m = 2n+1 \, (odd)$$

MATHCAD STUFF

Expression for the combined BPF - LIA response vs the harmonic number m:

$$H(Q,m) := \frac{2}{\pi \cdot m} \cdot \frac{\frac{m}{Q}}{\sqrt{1 + m^2 \cdot \left(\frac{1}{Q^2} - 2\right) + m^4}} \cdot \cos\left[\operatorname{atan}\left[Q \cdot \left(\frac{1}{m} - m\right)\right]\right]$$

Define the expected bias frequency and it's <u>trim range dFr</u>:

$Fr \equiv 150$ $\delta Fr \equiv 25 \cdot \frac{\pi}{2}$ The trim range have to be GEQ the signal bandwidth

Define the sifnal bandwidths for the Photometer and the Spectrometer:

$BW\_P \equiv 5$ $BW\_S \equiv 25$

Define the harmonic variable:

$n := 0 .. 4$ $m_n := 2 \cdot n + 1$

PHOTOMETER:

$Fmax\_P := Fr + \pi \cdot BW\_P + \delta Fr$ $Fmax\_P = 204.978$

$\Omega max\_P := \frac{Fmax\_P}{Fr}$ $\Omega max\_P = 1.367$ $\Omega min\_P := \frac{1}{\Omega max\_P}$ $\Omega min\_P = 0.732$

$\Delta\Omega\_P := \Omega max\_P - \Omega min\_P$ $\Delta\Omega\_P = 0.635$

$Q\_P := \frac{1}{\Delta\Omega\_P}$ $Q\_P = 1.575$

$$HN\_P := \overrightarrow{\frac{H(Q\_P, m)}{H(Q\_P, 1)}} \qquad HN\_P = \begin{pmatrix} 1 \\ 0.018 \\ 3.437 \times 10^{-3} \\ 1.214 \times 10^{-3} \\ 5.637 \times 10^{-4} \end{pmatrix} \qquad HN\_P^2 = \begin{pmatrix} 1 \\ 3.194 \times 10^{-4} \\ 1.181 \times 10^{-5} \\ 1.473 \times 10^{-6} \\ 3.177 \times 10^{-7} \end{pmatrix} \qquad m = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{pmatrix}$$

SPECTROMETER:

$\text{Fmax\_S} := \text{Fr} + \pi\cdot\text{BW\_S} + \delta\text{Fr}$ $\text{Fmax\_S} = 267.81$

$\Omega\text{max\_S} := \dfrac{\text{Fmax\_S}}{\text{Fr}}$ $\Omega\text{max\_S} = 1.785$ $\Omega\text{min\_S} := \dfrac{1}{\Omega\text{max\_S}}$ $\Omega\text{min\_S} = 0.56$

$\Delta\Omega\text{\_S} := \Omega\text{max\_S} - \Omega\text{min\_S}$ $\Delta\Omega\text{\_S} = 1.225$

$\text{Q\_S} := \dfrac{1}{\Delta\Omega\text{\_S}}$ $\text{Q\_S} = 0.816$

$$\text{HN\_S} := \overrightarrow{\frac{H(\text{Q\_S}, m)}{H(\text{Q\_S}, 1)}} \qquad \text{HN\_S} = \begin{pmatrix} 1 \\ 0.058 \\ 0.012 \\ 4.42\times 10^{-3} \\ 2.072\times 10^{-3} \end{pmatrix} \qquad \text{HN\_S}^2 = \begin{pmatrix} 1 \\ 3.377\times 10^{-3} \\ 1.497\times 10^{-4} \\ 1.954\times 10^{-5} \\ 4.293\times 10^{-6} \end{pmatrix} \qquad m = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{pmatrix}$$