SPIRE

SUBJECT:	**Analysis of radiant heating of the SPIRE input filter**		
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Change Record

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Simple calculation of SPIRE input filter heating due to absorption of thermal radiation

Assumptions

- Circular filter (should provide representative results for rectangular geometry)
- Periphery of filter is at fixed temperature T_o
- Radiative cooling is negligible (pessimistic)
- Filter thermal conductivity can be represented by a power law
- Thermal conductivity is dominated by the dielectric component (i.e., metalisation does not increase the effective thermal conductivity)
- Filter is heated by a uniform radiant power density

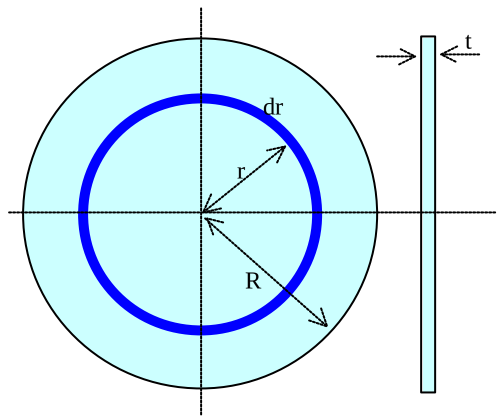
Definitions

Filter radius	R
Filter thickness	t
Filter periphery temperature	T_o
Thermal input to filter (power absorbed per unit area)	r
Thermal conductivity law	$k(T) = k_o T^b$

Consider an annular ring of thickness dr at radius r.
Let the temperatures at the inner and outer edges of the ring be T and T + dT respectively.

The thermal conductance across the ring is

$$G(r) = k_0 T^{\beta} \frac{2\pi rt}{dr}$$



In equilibrium, the power flowing across dr is equal to the total power absorbed within radius r, and is

$$W(r) = \pi r^2 \rho$$

The heat balance equation for the ring is

$$W(r) = G(r) dT$$

Therefore $$\pi r^2 \rho = k_0 T^{\beta} \frac{2\pi rt}{dr} dT$$

So $$rdr = \frac{2k_0 t}{\rho} T^{\beta} dT.$$

Integrating this with the boundary condition $T(R) = T_o$ gives

$$T(r) = \left[\frac{\rho(\beta+1)(R^2 - r^2)}{4k_0 t} + T_0^{\beta+1} \right]^{\frac{1}{\beta+1}}.$$

Calculation of temperature profile as a function of absorbed power density

Fundamental constants: $kb := 1.3806 \cdot 10^{-23}$ $h := 6.626 \cdot 10^{-34}$ $c := 3 \cdot 10^{8}$ $\sigma := 5.67 \cdot 10^{-8}$

Planck function $$B(nu, T) := \frac{2 \cdot h \cdot (nu)^3}{c^2 \cdot \left[e^{\left(\frac{h \cdot nu}{kb \cdot T}\right)} - 1 \right]}$$

Filter radius and thickness (m) $R := 50 \cdot 10^{-3}$ $t := 1 \cdot 10^{-3}$

Filter periphery temperature (K) $To := 5$

Thermal conductivity index $\beta := 0.5$

Thermal conductivity at 1 K (W m-1 K-1) $ko := 0.013$

Representatve date from ALW for CTFE from Reed et al. 1973.

Total power falling on filter (mW) $Ptot(\rho) := 1000 \cdot \rho \cdot \pi \cdot R^2$

Temperature vs. radial distance $$T(r, \rho) := \left[To^{(\beta+1)} + \frac{\rho \cdot (\beta + 1) \cdot (R^2 - r^2)}{4 \cdot ko \cdot t} \right]^{\frac{1}{\beta+1}}$$

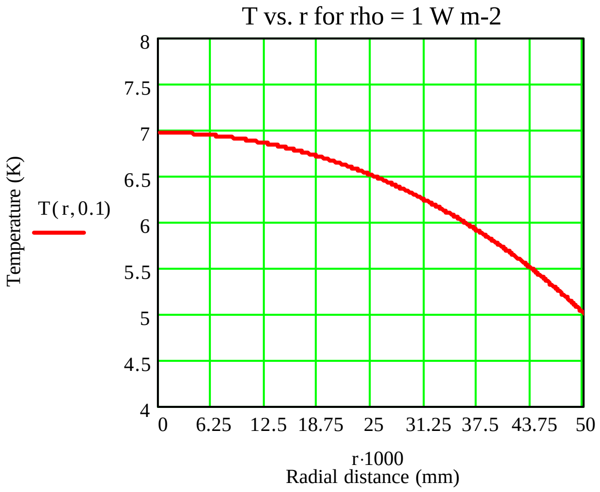
$r := 0, 0.1 \cdot 10^{-3} .. R$

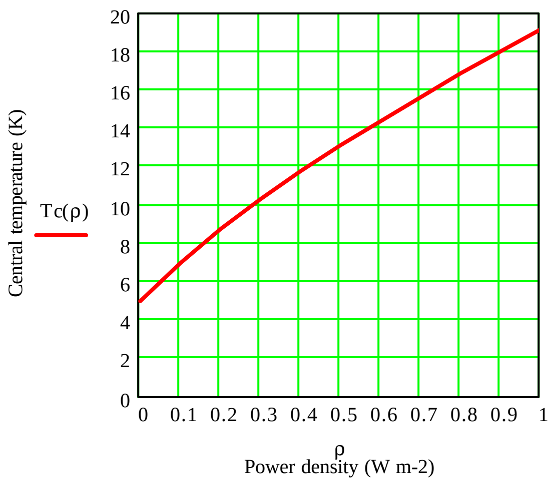
Filter central temperature vs. absorbed power density $$Tc(\rho) := \left[To^{(\beta+1)} + \frac{\rho \cdot (\beta + 1) \cdot R^2}{4 \cdot ko \cdot t} \right]^{\frac{1}{\beta+1}}$$

Average filter temperature (K) vs power density (W m-2) $$Tavg(\rho) := \frac{\int_0^R T(r, \rho) \cdot 2 \cdot \pi \cdot r \, dr}{\pi \cdot R^2}$$ $Tavg(0.1) = 6.0$

T vs. radius for a typical case

Central temp. vs. r $\rho := 0, 0.1 .. 1$





Estimation of heating effect for SPIRE input filter

Filter absorption: assume that the filter reflects 90% of the incident power across the whole spectrum (estimate from Peter Ade)

$\varepsilon filter := 0.1$

1. Power received from the telescope

Telescope: **Focal ratio** $F := 8.68$ **Temperature (K)** $Ttel := 80$

Solid angle subtended by telescope at filter $\Omega := \dfrac{\pi}{4 \cdot F^2}$

AW at filter for telescope background (m^2 sr) $A\Omega := \pi \cdot R^2 \cdot \Omega$ $A\Omega = 8.2 \times 10^{-5}$

Note: We assume here that the flter is well-baffled by the SPIRE FPU structure and sees only the telescope secondary mirror

Define suitable wavelength range (mm) for integration of telescope Planck function $\lambda 1 := 1 \cdot 10^{-6}$ $\lambda 2 := 10000 \cdot 10^{-6}$

Check $\nu 2 := \dfrac{c}{\lambda 1}$ $\nu 1 := \dfrac{c}{\lambda 2}$

$\left(\int_{\nu 1}^{\nu 2} B(nu, 80)\, dnu\right) \cdot \pi = 2.32$ $\sigma \cdot 80^4 = 2.32$ $\nu 2 = 3.0 \times 10^{14}$ $\nu 1 = 3.0 \times 10^{10}$

Telescope emissivity vs. wavelength

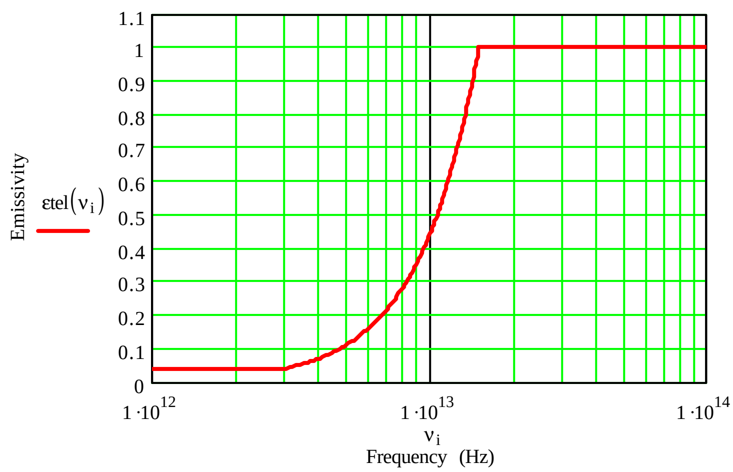
$\varepsilon 1(nu) := 0.04 \cdot \left[\dfrac{nu}{\left(\dfrac{c}{100 \cdot 10^{-6}}\right)}\right]^2$

Assume pessimistically
* **e = 0.04 for all wavelengths above 100 mm**
* **e increases with frequency^2 below 100 mm until it hits unity and = unity for frequencies above that point**

$\varepsilon 2(nu) := if(\varepsilon 1(nu) > 1, 1, \varepsilon 1(nu))$

$\varepsilon tel(nu) := if(\varepsilon 2(nu) < 0.04, 0.04, \varepsilon 2(nu))$

$i := 1, 2 .. 1000$ $\nu_i := 1 \cdot 10^{11} \cdot i$



Power density (W m-2), total power (W) and central temperature (K) of filter when viewing only telescope thermal emission

$\rho tel := \Omega \cdot \varepsilon filter \cdot \int_{\nu 1}^{\nu 2} B(nu, Ttel) \cdot \varepsilon tel(nu)\, dnu$ $\rho tel = 1.8 \times 10^{-4}$

$Ptel := \rho tel \cdot \pi \cdot R^2$ $Ptel = 1.4 \times 10^{-6}$

$Tc(\rho tel) - To = 0.0038$ **(a very small effect)**

2. Stray light power received from the cryostat ambient environment in flight

This is estimated (Tony Richards, private communication) as between 0.1 and 1 mW total power. Here we adopt the latter as a pessimistic estimate.

Corresponding power density (W m-2) $\rho stray := \frac{10^{-3}}{\pi R^2}$ $\rho stray = 0.127$

Corresponding filter central and average temperatures (K) $Tc(\rho stray) = 7.5$ $Tavg(\rho stray) = 6.3$

Values for 1E-4 W total power are 5.3 and 5.1 K respectively.

2. Worst case: instrument in ground calibration facility (with no ND or pre-filtering)

Assume the background is a unit emissivity 300-K black body

Power density (W m-2), total power (W) and temperature (K) of filter

$$\rho room := \Omega \cdot \varepsilon filter \cdot \int_{\nu 1}^{\nu 2} B(nu, 300)\, dnu \qquad \rho room = 1.5 \times 10^{-1}$$

$$Proom := \rho tel \cdot \pi \cdot R^2 \qquad Proom = 1.4 \times 10^{-6} \qquad Tc(\rho room) - To = 2.9$$

$$Tavg(\rho room) = 6.5$$

Comments and conclusions

* **The calculations above are for a 1-mm thick filter 100 mm in diameter. This should give a pessimistic result for the SPIRE input filter (dimensions 120 x 50 mm)**

* **The heating effect is dominated by the stray light environment rather than the telescope background power.**

* **A stray light power of 1 mW on the filter will result in a temperature rise of around 2.5 K at the centre of the filter and an increase in the average temperature to around 6.3 K. This is envisaged to have a negligible effect on the instrument performance. Analysis using the instrument sensitivity model will be carried out to quantify the effect and to derive a temperature stability requirement for the filter.**

* **The above analysis has been carried out using pessimistic assumptions about all of the filter properties. Further refinement of the model is likely to produce lower estimates for the temperature rise.**