SPIRE Sensitivity Models

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DOCUMENT No.: SPIRE-QMW-NOT-000642

Issue 3.0

Date: 6 June 2003

Contents

1. Introduction

This document is an update to the note issued for the SPIRE IBDR (22 Jan. 2002).

The attached MathCad models describe the computed sensitivities of the SPIRE photometer and spectrometer for point source and mapping observations.

Note: The spectrometer model has not yet been updated - this work is in progress and is expected to be completed by the time of the IHDR meeting. The relative changes in sensitivity are expected to be comparable to the photometer case.

2. Changes to previous photometer model

The main changes to the previous photometer model are as follows:

- A physical model of the bolometers has been explicitly included to allow the response of the bolometers to changes in other parameters to be taken into account. The bolometer DQE is now a derived parameter rather than an input parameter.
- The loss due to telescope obscuration has been updated from 3% to 13%, taking into account the larger hole in the primary and the shadowing of the secondary support structure
- The bolometer/electronics noise model has been revised to take into account an additional noise contribution that was not included in the previous versions of the model: Johnson noise from the bolometer and its load resistor, and amplifier noise, are increased by a factor of $2^{1/2}$ by the LIA de-modulation.
- The bolometer resistance parameter R_o has been given a nominal value of 100 Ω rather than the previously assumed 180 Ω, taking into account poorer than expected values measured on the CQM detectors.
- The nominal filter transmission efficiency has been increased from 30% to 40% to take into account envisaged filter transmission performance that is better than the pessimistic assumptions formerly adopted.
- The feed efficiencies for the PMW and PSW bands have increased slightly on the basis of measured performance of prototype horns.
- The nominal bolometer temperature has been increased to 320 mK, seen as a more realistic number due to the difficulties in optimising SPIRE in its thermal environment.
- Nominal, best, and worst case values have been defined for all key parameters, and the relative influence of individual parameters on the overall performance is analysed.
- These changes have so far only been implemented for the photometer - similar updates will be done for the spectrometer model in the near future.

3. Assumptions

The main assumptions made in estimating the scientific performance of the instrument are listed in the table below. Additional assumptions are given in the attached worksheets.

Telescope			
Telescope temperature (K)	Best 60 Nominal 80 Worst 90		
Telescope emissivity	Best 0.02 Nominal 0.04 Worst 0.08		
Telescope used diameter (m) (1)	3.29		
Obscuration loss factor	0.87		
No. of observable hours per 24-hr period	21		
Photometer			
Bands	PSW	PMW	PLW
Centre wavelength (μm)	250	363	517
Numbers of detectors	139	88	43
Beam FWHM (arcsec.)	17	24	35
Filter widths ($\lambda/\Delta\lambda$)	3.0	3.2	3.0
Feed-horn/cavity efficiency (2)	Best 0.8 Nominal 0.7 Worst 0.6	Best 0.8 Nominal 0.7 Worst 0.6	Best 0.8 Nominal 0.7 Worst 0.6
Feedhorn point source coupling efficiency	0.7		
Throughput	λ^2		
Field of view (arcmin.) Scan mapping Field mapping	4 x 8 4 x 4		
Chopping efficiency factor	0.45		
Reduction in telescope background by cold stop (3)	0.8		
Bolometer resistance parameter R_o (Ω)	Best 180 Nominal 100 Worst 70		
Bolometer temperature (K)	Best 0.300 Nominal 0.320 Worst 0.340		
JFET voltage noise (nV Hz$^{-1/2}$)	Best 7 Nominal 10 Worst 15		
Bolometer yield	Best 0.9 Nominal 0.8 Worst 0.75		
Overall instrument transmission	Best 0.48 Nominal 0.40 Worst 0.32		
Observing efficiency (slewing, setting up, etc.)	Best 0.95 Nominal 0.85 Worst 0.75		
FTS spectrometer (not yet updated)			
Bands (μm)	200-315	315-670	
Numbers of detectors	37	19	
Bolometer DQE	0.70	0.65	
Feed-horn/cavity efficiency	0.70	0.65	
Usable field of view diameter (arcmin.)	2.6		
Max. spectral resolution (cm^{-1})	0.04		
Overall instrument transmission	0.15		
Signal modulation efficiency	0.5		
Observing efficiency	0.8		
Electrical filter efficiency	0.8		

Notes:

1. The telescope secondary mirror is the pupil stop for the system, so that the outer edges of the primary mirror are not seen by the detectors. This is important to make sure that radiation from highly emissive elements beyond the primary reflector does not contribute stray light.

2. This is the overall absorption efficiency of the combination of feed-horn, cavity and bolometer element.

3. A fraction of the feedhorn throughput falls outside the solid angle defined by the photometer 2-K cold stop and is thus terminated on a cold (non-emitting) surface rather than on the 4% emissive 80-K telescope. This reduces the background power on the detector.

4. Results

4.1 Background power, photon noise levels, and detector quantum efficiency

The background power levels, NEPs and DQEs for the nominal case are summarised in Table 1 below. The previous values are given in brackets.

		Photometer band (mm)			**FTS band (mm)**	
Nominal case		**PSW**	**PMW**	**PLW**	**200-315**	**315-670**
Background power/detector (previous values in brackets)	PW	5.7 (3.8)	4.1 (3.0)	3.4 (2.6)	TBD (11)	TBD (11)
Background-limited NEP	W Hz$^{-1/2}$ x 10^{-17}	9.7 (7.9)	6.9 (5.9)	5.3 (4.6)	TBD (11)	TBD (14)
Overall NEP (inc. detector)	W Hz$^{-1/2}$ x 10^{-17}	13.6 (9.7)	10.7 (7.1)	9.1 (5.9)	TBD (13)	TBD (16)
Bolometer DQE		0.51 (0.73)	0.42 (0.68)	0.34 (0.61)		

Table 1 - Background power and photon noise-limited NEPs for SPIRE.

Comments:

- The nominal-case background power estimates have increased mainly due to the combination of higher filter and higher feed efficiencies transmission now assumed.
- DQE is now a derived parameter rather than an assumption.

4.2 Photometer sensitivity

The photometer sensitivity results are summarised in Table 2 below. Previous values are given in brackets.

Photometry				
Band		PSW	PMW	PLW
ΔS(5-σ; 1-hr) mJy	Point source (7-point mode)	2.7 (2.4)	3.5 (2.8)	4.2 (3.1)
	4’ x 4’ jiggle map	9.5 (8.5)	11.5 (9.3)	13.2 (9.7)
	4’ x 8’ scan map	7.6 (6.8)	9.2 (7.4)	10.5 (7.7)
Time (days) to map 1 deg.^{2} to 3 mJy 1-σ	Nominal case IBDR values	**2.06** (1.7)	**3.01** (2.0)	**3.92** (2.1)
Best case parameters as indicated with other parameters at their nominal values	ε_{tel} = best	1.10	1.66	2.14
	T_{tel} = best	1.37	2.12	2.81
	$t_{filters}$ = best	1.73	2.52	3.31
	R_o = best	1.74	2.47	3.11
	en_{FET} = best	1.79	2.55	3.22
	η_{feed} = best	1.81	2.64	3.46
	T_o = best	1.82	2.58	3.30
	Yield = best	1.83	2.67	3.48
	η_{obs} = best	1.84	2.69	3.51
Nominal telescope; instrument best		1.02	1.38	1.67
Nominal instrument; telescope best		0.80	1.29	1.70
Worst case parameters as indicated with other parameters at their nominal values	ε_{tel} = worst	3.22	4.66	6.19
	en_{FET} = worst	2.72	4.14	5.62
	$t_{filters}$ = worst	2.58	3.81	4.92
	T_{tel} = worst	2.45	3.50	4.54
	η_{feed} = worst	2.40	3.53	4.57
	T_o = worst	2.37	3.58	4.73
	R_o = worst	2.36	3.52	4.69
	η_{obs} = worst	2.33	3.41	4.44
	Yield = worst	2.20	3.21	4.18
Nominal telescope, instrument worst		5.69	9.15	12.71
Nominal instrument; telescope worst		3.93	5.58	7.39

Table 2 - Photometer sensitivity model results

Comments:

- The nominal case mapping speed estimates are worse than the IBDR values by factors of (1.2, 1.5, 1.9) for (PSW, PMW, PLW). These correspond to factors of (1.1, 1.2, 1.4) in NEFD, since observing speed depends on the square of NEFD.

- In the nominal case, the following instrument parameters are the same or better than before:

  ε_{tel}, T_{tel} $t_{filters}$, η_{feed}, Yield and the following are worse than before:

R_o, en_{FET}, T_o, η_{obs}.

- The impact of the additional noise component now included (due to LIA de-modulation) is such that the nominal-case mapping speeds would be (1.6, 2.2, 2.7) vs. the above values of (2.1, 3.0, 3.9) days.

- In the tabulation of best and worst case parameters, the parameters are listed in decreasing order of their influence over the sensitivity. The most influential parameter is the telescope emissivity, since this parameter has the largest uncertainty and dictates the background power on the detectors. The instrument parameters which are most influential on the sensitivity are R_o, en_{FET}, T_o, and $t_{filters}$.

- With all instrument parameters at their worst case values and the nominal telescope, mapping speed is degraded by about a factor of 3 compared to the nominal case.

4.3 Spectrometer sensitivity (not yet updated)

The photometer sensitivity results are summarised in Table 3 below.

Line spectroscopy Ds **= 0.04 cm^{-1}**					
λ	μm		200 - 315	315 - 500	500-670
ΔS (5-σ; 1-hr)	W m^{-2} x 10^{-17}	Point source	TBD (4.7)	TBD (4.0)	TBD (4.0 - 5.6)
		2.6’ map	TBD (13)	TBD (11)	TBD (11 - 15)

Low-resolution spectrophotometry Ds **= 1 cm^{-1}**					
λ	μm		200 - 315	315 - 500	500-670
ΔS (5-σ; 1-hr)	mJy	Point source	TBD (160)	TBD (140)	TBD (140 - 190)
		2.6’ map	TBD (430)	TBD (360)	TBD (360-500)

Table 3 - Spectrometer sensitivity model results

5. List of annexes

A	Photometer sensitivity model	SPIRE_Phot_6.mcd	June 6 2003
B	FTS sensitivity model	SPIRE_FTS_3_IBDR.mcd	22 January 2002

A. Photometer sensitivity model SPIRE_Phot_6.MCD June 6 2003

SPIRE_Phot_6_IHDR.MCD June 6 2003
* Version prepared for SPIRE IHDR
* Updated telescope obscuration
* Detailed bolometer model based on Jamie's EXCEL speadsheet and envisaged operating temperature and JFET noise.
* Previously unaccounted-for noise contribution due to LIA demodulation now included.
* Feed efficiencies updated to take Rownd et al. results into account

SPIRE_Phot_5_IBDR.MCD 22 Jan 2002
* Prepared for SPIRE IBDR
* Includes estimation of sensitivity to temperatures of telescope and SPIRE instrument stages
* Filter bands updated to reflect filter DDR and horn specification definition (now 250, 363, 517 μm)
* Feedhorn efficiencies and DPdetE values updated according to the BDA SSSD
SPIRE-JPL-PRJ-000456
Working Version, 7 November 2001
* Revised analysis of PCAL requirements
* Best and worst case analysis included (not in version submitted for IBDR)

Previous versions:

SPIRE_Phot_1.MCD 21 November 2000
* Version prepared for Systems Design Review and Toledo Meeting

SPIRE_Phot_2_IIDR.MCD April 2001
* Version prepared for IIDR

SPIRE_Phot_3_SCM.MCD April 2001
* Version prepared for SPIRE Consortium meeting July 2001
* Proposed 0.56-m central obscuration included

Constants $h \equiv 6.626\cdot 10^{-34}$ $c \equiv 3\cdot 10^{8}$ $kb \equiv 1.38\cdot 10^{-23}$

$i \equiv 1, 2..3$ $origin \equiv 1$

Planck function

$$B(nu,T) := \frac{2\cdot h\cdot (nu)^3}{c^2\cdot\left[e^{\left(\frac{h\cdot nu}{kb\cdot T}\right)} - 1\right]}$$

Key Input parameters

Case to be computed: place chosen option last in the list

$\text{case} \equiv \text{"best"}$ $\quad \text{case} \equiv \text{"worst"}$ $\quad \text{case} \equiv \text{"special"}$

$\text{case} \equiv \text{"nominal"}$

	Best case	**Nominal case**	**Worst case**	**Special case**
Telescope temp. (K)	$\text{Ttel\_goal} := 60$	$\text{Ttel\_nom} := 80$	$\text{Ttel\_min} := 90$	$\text{Ttel\_special} := \text{Ttel\_goal}$
Telescope emissivity	$\varepsilon\text{tel\_goal} := 0.02$	$\varepsilon\text{tel\_nom} := 0.04$	$\varepsilon\text{tel\_min} := 0.06$	$\varepsilon\text{tel\_special} := \varepsilon\text{tel\_goal}$
Bolometer yield	$\text{y\_goal} \equiv 0.9$	$\text{y\_nom} \equiv 0.8$	$\text{y\_min} \equiv 0.75$	$\text{y\_special} := \text{y\_nom}$
Observing efficiency	$\eta\text{obs\_goal} := 0.95$	$\eta\text{obs\_nom} := 0.85$	$\eta\text{obs\_min} := 0.75$	$\eta\text{obs\_special} := \eta\text{obs\_nom}$
He-3 temp. (K)	$\text{To\_goal} := 0.300$	$\text{To\_nom} := 0.320$	$\text{To\_min} := 0.340$	$\text{To\_special} := \text{To\_nom}$
Bolometer Res. param (W)	$\text{RS\_goal} := 180$	$\text{RS\_nom} := 100$	$\text{RS\_min} := 70$	$\text{RS\_special} := \text{RS\_nom}$
JFET noise (V Hz-1/2)	$\text{en\_J\_goal} := 7 \cdot 10^{-9}$	$\text{en\_J\_nom} := 10 \cdot 10^{-9}$	$\text{en\_J\_min} := 15 \cdot 10^{-9}$	$\text{en\_J\_special} := \text{en\_J\_nom}$
Feed efficiency	$\eta\text{feed\_goal}_i := \begin{array}{\|c\|}\hline 0.8 \\ \hline 0.8 \\ \hline 0.8 \\ \hline \end{array}$	$\eta\text{feed\_nom}_i := \begin{array}{\|c\|}\hline 0.7 \\ \hline 0.7 \\ \hline 0.7 \\ \hline \end{array}$	$\eta\text{feed\_min}_i := \begin{array}{\|c\|}\hline 0.6 \\ \hline 0.6 \\ \hline 0.6 \\ \hline \end{array}$	$\eta\text{feed\_special}_i := \eta\text{feed\_nom}_i$

Filter transmission (see below) Nominal value = 40%, best and worst 20% different $\quad \text{trans\_fac\_special} \equiv 1.0$

Summary of parameters

Telescope Temperature

$\text{Ttel} := \text{if}(\text{case} = \text{"best"}, \text{Ttel\_goal}, \text{Ttel\_nom})$

$\text{Ttel} := \text{if}(\text{case} = \text{"worst"}, \text{Ttel\_min}, \text{Ttel})$

$\text{Ttel} := \text{if}(\text{case} = \text{"special"}, \text{Ttel\_special}, \text{Ttel})$ $\qquad \text{Ttel} = 80$

Emissivity

$\varepsilon\text{tel} := \text{if}(\text{case} = \text{"best"}, \varepsilon\text{tel\_goal}, \varepsilon\text{tel\_nom})$

$\varepsilon\text{tel} := \text{if}(\text{case} = \text{"worst"}, \varepsilon\text{tel\_min}, \varepsilon\text{tel})$

$\varepsilon\text{tel} := \text{if}(\text{case} = \text{"special"}, \varepsilon\text{tel\_special}, \varepsilon\text{tel})$ $\qquad \varepsilon\text{tel} = 0.04$

Diameter $\text{Dtel} \equiv 3.285$ **Focal ratio** $\text{Ftel} \equiv 8.68$

Obscuration factor $\text{Obs\_factor} \equiv 0.872$

Detector system

Filter central wavelengths

$\lambda_i \equiv$	$R_i \equiv$
250	3.00
363	3.18
517	3.00

Final optics focal ratio $Ffin \equiv 5$

Feedhorn point source coupling efficiency $\eta tel \equiv 0.7$

Feed efficiency

$\eta feed_i := \text{if}\left(case = \text{"best"}, \eta feed\_goal_i, \eta feed\_nom_i\right)$

$\eta feed_i := \text{if}\left(case = \text{"worst"}, \eta feed\_min_i, \eta feed_i\right)$

$\eta feed_i := \text{if}\left(case = \text{"special"}, \eta feed\_special_i, \eta feed_i\right)$

$\eta feed_i =$
0.700
0.700
0.700

Cold stop attenuation of telescope background $\eta cs \equiv 0.8$

Warm amplifier noise voltage (V Hz-1/2) $en\_warm := 7 \cdot 10^{-9}$

JEET noise voltage (V Hz-1/2)

$en\_J := \text{if}(case = \text{"best"}, en\_J\_goal, en\_J\_nom)$

$en\_J := \text{if}(case = \text{"worst"}, en\_J\_min, en\_J)$

$en\_J := \text{if}(case = \text{"special"}, en\_J\_special, en\_J)$ $en\_J = 1.000 \times 10^{-8}$

Total amplifier noise $enamp := \left(en\_J^2 + en\_warm^2\right)^{0.5}$ $enamp = 1.221 \times 10^{-8}$

Bolometer yield

$yield := \text{if}(case = \text{"best"}, y\_goal, y\_nom)$

$yield := \text{if}(case = \text{"worst"}, y\_min, yield)$

$yield := \text{if}(case = \text{"special"}, y\_special, yield)$ $yield = 0.800$

Chopping efficiency $\eta ch \equiv 0.45$

Observing efficicency

$\eta obs := \text{if}\left(case = \text{"best"}, \eta obs\_goal, \eta obs\_nom\right)$

$\eta obs := \text{if}\left(case = \text{"worst"}, \eta obs\_min, \eta obs\right)$

$\eta obs := \text{if}\left(case = \text{"special"}, \eta obs\_special, \eta obs\right)$ $\eta obs = 0.850$

Thermal system

He-3 stage temp. (K)

$To := \text{if}(case = \text{"best"}, To\_goal, To\_nom)$

$To := \text{if}(case = \text{"worst"}, To\_min, To)$

$To := \text{if}(case = \text{"special"}, To\_special, To)$ $To = 0.320$

Level-0 temp. (K) $T2 \equiv 2.0$ **Level-1 temp. (K)** $T4 \equiv 5.0$

Detector Numbers

$$Ndets_i \equiv \begin{pmatrix} 15\cdot 5 + 16\cdot 4 \\ 13\cdot 4 + 12\cdot 3 \\ 9\cdot 3 + 8\cdot 2 \end{pmatrix} \qquad Ndets_i = \begin{pmatrix} 139 \\ 88 \\ 43 \end{pmatrix}$$

Array dimension cente-centre (pixels)

$$Nmax_i \equiv \begin{pmatrix} 15 \\ 12 \\ 8 \end{pmatrix} \qquad Nmin_i \equiv \begin{pmatrix} 8 \\ 6 \\ 4 \end{pmatrix}$$

Optical system

$$j \equiv 0,1..11 \qquad k \equiv 0,1..12$$

Transmission, emissivity and temp. of optical elements

0 = Telescope
1 = input filter
2 = M3
3 = M4
4 = M5
5 = 4-K filter
6 = M6
7 = 2-K filter
8 = M7
9 = Dichroic
10 = M8
11 = Bandpass filter
12 = Blocker

$k =$	$t_k \equiv$	$\varepsilon_k :=$	$T_k :=$	$td_j =$
0	0.960	εtel	Ttel	0.388
1	0.900	0.100	T4	0.431
2	0.995	0.005	T4	0.433
3	0.995	0.005	T4	0.435
4	0.995	0.005	T4	0.438
5	0.900	0.100	T4	0.486
6	0.995	0.005	T4	0.489
7	0.900	0.300	T2	0.543
8	0.995	0.005	T2	0.546
9	0.900	0.100	T2	0.606
10	0.995	0.005	T2	0.609
11	0.677	0.100	To	0.900
12	0.900	0.100	To	

$$trans\_fac \equiv if(case = \text{"best"}, 1.2, 1.0)$$

$$trans\_fac \equiv if(case = \text{"worst"}, 0.8, trans\_fac)$$

$$trans\_fac \equiv if(case = \text{"special"}, trans\_fac\_special, trans\_fac)$$

$$trans\_fac = 1.0$$

Note:
The 2-K filter is located at the 2-K pupil stop. An emissivity of 0.3 is ascribed to this component:0.1 for the filter itself plus 0.2 for the spillover onto the cold-stop

Transmission from element to detector

$$td_j \equiv trans\_fac \cdot \prod_{k=j+1}^{12} t_k$$

Derived parameters

Effective telescope area (m^2)

$$\mathrm{Atel} := \frac{\pi \cdot \mathrm{Dtel}^2}{4} \cdot \mathrm{Obs\_factor} \qquad \mathrm{Atel} = 7.39$$

Plate scale at telescope focus (arcsec/mm):

$$\mathrm{PS} := \frac{1}{\mathrm{Dtel} \cdot \mathrm{Ftel}} \cdot \frac{360}{2 \cdot \pi} \cdot 3.6 \qquad \mathrm{PS} = 7.23$$

Plate scale at arrays (arcsec/mm):

$$\mathrm{PSA} := \mathrm{PS} \cdot \frac{8.68}{5} \qquad \mathrm{PSA} = 12.6$$

Beamwidths (arcsec.):

$$\mathrm{FWHM}_i := \frac{1.11 \cdot \lambda_i \cdot 10^{-6}}{\mathrm{Dtel}} \cdot \frac{360}{2 \cdot \pi} \cdot 3600 \qquad \mathrm{FWHM}_i = \begin{array}{|c|}\hline 17.4 \\ \hline 25.3 \\ \hline 36.0 \\ \hline \end{array}$$

Horn aperture outside dia. (mm)

$$\mathrm{Dhorn}_i := \frac{2 \cdot \mathrm{Ffin} \cdot \lambda_i}{1000} \qquad \mathrm{Dhorn}_i = \begin{array}{|c|}\hline 2.5 \\ \hline 3.6 \\ \hline 5.2 \\ \hline \end{array}$$

Horn size projected onto telescope focus (mm):

$$\mathrm{Dpix}_i := \left(\mathrm{Dhorn}_i\right) \cdot \frac{\mathrm{Ftel}}{\mathrm{Ffin}} \qquad \mathrm{Dpix}_i = \begin{array}{|c|}\hline 4.3 \\ \hline 6.3 \\ \hline 9.0 \\ \hline \end{array}$$

Array dimensions at telescope focus centre-centre (mm):

$$\mathrm{Lmm}_i := \mathrm{Nmax}_i \cdot \mathrm{Dpix}_i \qquad \mathrm{Wmm}_i := \mathrm{Nmin}_i \cdot \mathrm{Dpix}_i$$

Field size (arcmin):

$$\mathrm{Larcmin}_i := \frac{\mathrm{Lmm}_i \cdot \mathrm{PS}}{60} \qquad \mathrm{Warcmin}_i := \frac{\mathrm{Wmm}_i \cdot \mathrm{PS}}{60}$$

$\lambda_i =$	$\mathrm{Lmm}_i =$	$\mathrm{Wmm}_i =$	$\mathrm{Larcmin}_i =$	$\mathrm{Warcmin}_i =$
250	65	35	7.8	4.2
363	76	38	9.1	4.6
517	72	36	8.7	4.3

Bands: defined by central wavelengths (in µm) and resolution of the filters

$$\nu_i := \frac{c}{\lambda_i \cdot 10^{-6}} \qquad \lambda L_i := \lambda_i - \frac{\lambda_i}{2 \cdot R_i} \qquad \lambda U_i := \lambda_i + \frac{\lambda_i}{2 \cdot R_i}$$

$$\nu L_i := \frac{c}{\lambda U_i \cdot 10^{-6}} \quad \nu U_i := \frac{c}{\lambda L_i \cdot 10^{-6}} \qquad \Delta\lambda_i := \frac{\lambda_i}{R_i} \qquad \Delta\nu_i := \frac{\nu_i}{R_i}$$

$i =$	$\lambda_i =$	$R_i =$	$\lambda L_i =$	$\lambda U_i =$	$\Delta\lambda_i =$	$\nu_i \cdot 10^{-9} =$	$\nu L_i \cdot 10^{-9} =$	$\nu U_i \cdot 10^{-9} =$	$\Delta\nu_i \cdot 10^{-9} =$
1	250	3.00	208.3	291.7	83	1200	1029	1440	400
2	363	3.18	305.9	420.1	114	826	714	981	260
3	517	3.00	430.8	603.2	172	580	497	696	193

Waveguide diameters (µm):

$$D\_wg_i := \frac{\lambda U_i \cdot 1.841}{\pi}$$

$D\_wg_i =$
170.9
246.2
353.5

Background power levels on the detectors

Throughput:

$$A\Omega_i := \eta cs \cdot \left(\lambda_i \cdot 10^{-6}\right)^2$$

Power contribution absorbed by detector from any element (pW)

$$Power_{i,j} := td_j \cdot \varepsilon_j \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_j) \cdot A\Omega_i \, d\nu$$

Total power **absorbed** by detector (pW)

$$Pdet_i := \sum_{n=0}^{9} Power_{i,n}$$

$Power_{1,j} =$	$Power_{2,j} =$	$Power_{3,j} =$
5.65	4.14	3.38
0.00	0.00	0.01
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.02
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00

Note that the background power on the detectors is totally dominated by the telescope

$Pdet_i =$
5.65
4.15
3.41

Photon noise limited NEP (full expression)

$$NEPph_i := \left[\frac{4\cdot A\Omega_i\cdot h^2}{c^2}\cdot\left[\int_{\nu L_i}^{\nu U_i}\frac{\varepsilon tel\cdot td_0\cdot \eta feed_i\cdot nu^4}{e^{\left(\frac{h\cdot nu}{kb\cdot T_0}\right)}-1}\cdot\left[1+\frac{\varepsilon tel\cdot td_0\cdot \eta feed_i}{e^{\left(\frac{h\cdot nu}{kb\cdot T_0}\right)}-1}\right]dnu\right]\right]^{0.5}\cdot 10^{17}$$

$NEPph_i =$

9.73
6.92
5.28

Bolometer model

Material band-gap temperature (K)	$TG \equiv 41.8$
Load resistance (MΩ)	$RL \equiv 20$
Heat capacity at 300 mK (pJ K-1)	$Co := 1$
Thermal conductivity index	$\beta \equiv 1.5$
Heat capacity index	$\rho := 1$
R-T power law index	$n \equiv 0.5$

Resistance parameter (Ω)

$RS := \text{if}(case = \text{"best"}, RS\_goal, RS\_nom)$

$RS := \text{if}(case = \text{"worst"}, RS\_min, RS)$

$RS := \text{if}(case = \text{"special"}, RS\_special, RS)$

$RS = 100$

Material parameter

$$\delta := \frac{TG}{To}$$

Static thermal conductance at 300 mK (pW K-1)

$G_i :=$

66
53
40

Static thermal conductance at bath temperature (PW K-1)

$$GS0_i := G_i\cdot 10^{-12}\cdot\left(\frac{To}{0.300}\right)^{\beta}$$

Loading parameter

$$\gamma_i := \frac{Pdet_i\cdot 10^{-12}}{To\cdot GS0_i}$$

Bias parameter

$b := 0.1..200$

$$\phi_b := 1+\frac{b}{200}$$

Resistance (MΩ)

$$R_b := RS\cdot \exp\left[\left[\left(\frac{\delta}{\phi_b}\right)^{n}\right]\right]\cdot 10^{-6}$$

Electrical power (if P < 0, set P = 0)

$$PP_{b,i} := To\cdot GS0_i\cdot\left[\frac{(\phi_b)^{\beta+1}-1}{\beta+1}-\gamma_i\right]$$

$$P_{b,i} := \text{if}\left(PP_{b,i} < 0, 0, PP_{b,i}\right)$$

Temp. coeff of resistance

$$\alpha_b := \frac{-n\cdot(\delta)^n}{(\phi_b)^{n+1}\cdot To}$$

V (mV) and I (nA)

$$V_{b,i} := \left(P_{b,i}\cdot R_b\right)^{0.5}\cdot 10^{6}$$

$$I_{b,i} := \left(\frac{P_{b,i}}{R_b\cdot 10^{6}}\right)^{0.5}\cdot 10^{9}$$

Gd and Ge (pW K-1)

$$Gd_{b,i} := GS0_i\cdot\left[(\phi_b)^{\beta}\right]$$

$$Ge_{b,i} := Gd_{b,i}-\alpha_b\cdot P_{b,i}\cdot\left(\frac{RL-R_b}{RL+R_b}\right)$$

Dynamic impedance (MW)

$$Z_{b,i} := \frac{Gd_{b,i}+\alpha_b\cdot P_{b,i}}{Gd_{b,i}-\alpha_b\cdot P_{b,i}}\cdot R_b$$

Heat capacity (J K-1)

$$C_b := Co\cdot 10^{-12}\cdot\left(\frac{To\cdot\phi_b}{0.300}\right)^{\rho}$$

Responsivity (V W-1):

$$S_{b,i} := \text{if}\left[I_{b,i} = 0, 1, \frac{\left(R_b - Z_{b,i}\right)\cdot 10^{6}}{2\cdot V_{b,i}\cdot 10^{-3}}\cdot\frac{RL}{Z_{b,i}+RL}\right]$$

Phonon NEP:

$$\mathrm{NEPp}_{b,i} := \mathrm{if}\left[I_{b,i} = 0, 1, \left[4\cdot kb\cdot(To)^2\cdot GS0_i\cdot\frac{\beta+1}{2\cdot\beta+3}\cdot\frac{(\phi_b)^{2\cdot\beta+3}-1}{(\phi_b)^{\beta+1}-1}\right]^{0.5}\right]$$

Johnson NEP:

$$\mathrm{NEPj}_{b,i} := \left[4\cdot kb\cdot(To)^2\cdot GS0_i\cdot\frac{(\phi_b)^{2\cdot\beta+2\cdot n+3}}{n^2\cdot(\delta)^{2\cdot n}\cdot\left[\frac{(\phi_b)^{\beta+1}-1}{\beta+1}-\gamma_i\right]}\right]^{0.5}$$

Load resistor NEP:

$$\mathrm{NEPload}_{b,i} := \left(\frac{4\cdot kb\cdot To}{RL\cdot 10^6}\right)^{0.5}\cdot\left|\frac{Z_{b,i}\cdot RL\cdot 10^6}{Z_{b,i}+RL}\right|\cdot\frac{1}{S_{b,i}}$$

Amplifier NEP:

$$\mathrm{NEPamp}_{b,i} := \frac{enamp}{S_{b,i}}$$

Detector NEP

$$\mathrm{NEPdet}_{b,i} := \left[\left(\mathrm{NEPp}_{b,i}\right)^2 + \left(\mathrm{NEPj}_{b,i}\right)^2 + \left(\mathrm{NEPload}_{b,i}\right)^2 + \left(\mathrm{NEPamp}_{b,i}\right)^2\right]^{0.5}$$

Total NEP at JFET output (W Hz-1/2):

$$\mathrm{NEPtot}_{b,i} := \left[\left(\mathrm{NEPdet}_{b,i}\right)^2 + \left(\mathrm{NEPph}_i\cdot 10^{-17}\right)^2\right]^{0.5}$$

Total noise at JFET output (V Hz-1/2)

$$\mathrm{entot}_{b,i} := \mathrm{NEPtot}_{b,i}\cdot S_{b,i}$$

Total NEP at LIA output (W Hz-1/2):

$$\mathrm{NEPlia}_{b,i} := \left[\left(\mathrm{NEPph}_i\cdot 10^{-17}\right)^2 + \left(\mathrm{NEPp}_{b,i}\right)^2 + 2\cdot\left[\left(\mathrm{NEPj}_{b,i}\right)^2 + \left(\mathrm{NEPamp}_{b,i}\right)^2 + \left(\mathrm{NEPload}_{b,i}\right)^2\right]\right]^{0.5}$$

DQE:

$$\mathrm{DQE}_{b,i} := \left(\frac{\mathrm{NEPph}_i\cdot 10^{-17}}{\mathrm{NEPlia}_{b,i}}\right)^2$$

$\mathrm{DQE\_PSW}_b := \mathrm{DQE}_{b,1}$ $\mathrm{DQEop}_1 := \max(\mathrm{DQE\_PSW})$ $\mathrm{DQEop}_1 = 0.512$

$\mathrm{DQE\_PMW}_b := \mathrm{DQE}_{b,2}$ $\mathrm{DQEop}_2 := \max(\mathrm{DQE\_PMW})$ $\mathrm{DQEop}_2 = 0.420$

$\mathrm{DQE\_PLW}_b := \mathrm{DQE}_{b,3}$ $\mathrm{DQEop}_3 := \max(\mathrm{DQE\_PLW})$ $\mathrm{DQEop}_3 = 0.339$

Optimum bias points:

$\mathrm{index\_PSW}_b := \mathrm{if}\left(\mathrm{DQE\_PSW}_b = \max(\mathrm{DQE\_PSW}), b, 0\right)$ $\mathrm{p\_SW} := \max(\mathrm{index\_PSW})$ $\mathrm{p\_SW} = 61$

$\mathrm{index\_PMW}_b := \mathrm{if}\left(\mathrm{DQE\_PMW}_b = \max(\mathrm{DQE\_PMW}), b, 0\right)$ $\mathrm{p\_MW} := \max(\mathrm{index\_PMW})$ $\mathrm{p\_MW} = 58$

$\mathrm{index\_PLW}_b := \mathrm{if}\left(\mathrm{DQE\_PLW}_b = \max(\mathrm{DQE\_PLW}), b, 0\right)$ $\mathrm{p\_LW} := \max(\mathrm{index\_PLW})$ $\mathrm{p\_LW} = 61$

$$p_i := \begin{array}{|l|}\hline \mathrm{p\_SW} \\ \hline \mathrm{p\_MW} \\ \hline \mathrm{p\_LW} \\ \hline \end{array}$$

Responsivity (V W-1), time constants (ms), and 3-dB freq. at optimum bias

Effective time constant

Physical time constant

3-dB freq.

NEPtot

$Sop_i := S_{p_i,i}$

$Sop_i =$

$2.05\cdot10^{8}$
$2.41\cdot10^{8}$
$2.64\cdot10^{8}$

$$\tau e_i := \frac{C_i}{Ge_{p_i,i}}$$

$\tau e_i \cdot 1000 =$

7.73
9.82
12.87

$$\tau phys_i := \frac{Ge_{p_i,i}}{Gd_{p_i,i}}\cdot\tau e_i$$

$\tau phys_i \cdot 1000 =$

9.89
12.59
16.48

$$\nu o_i := \frac{1}{2\cdot\pi\cdot\tau e_i}$$

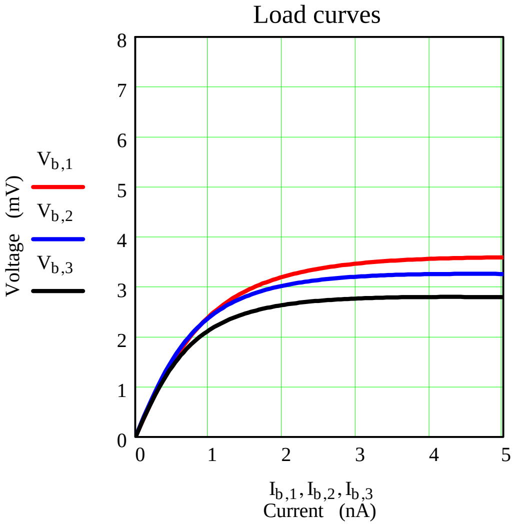
$\nu o_i =$

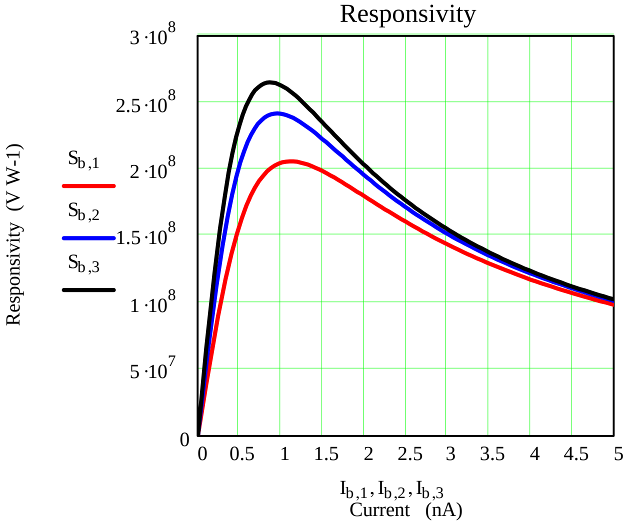
20.58
16.2
12.37

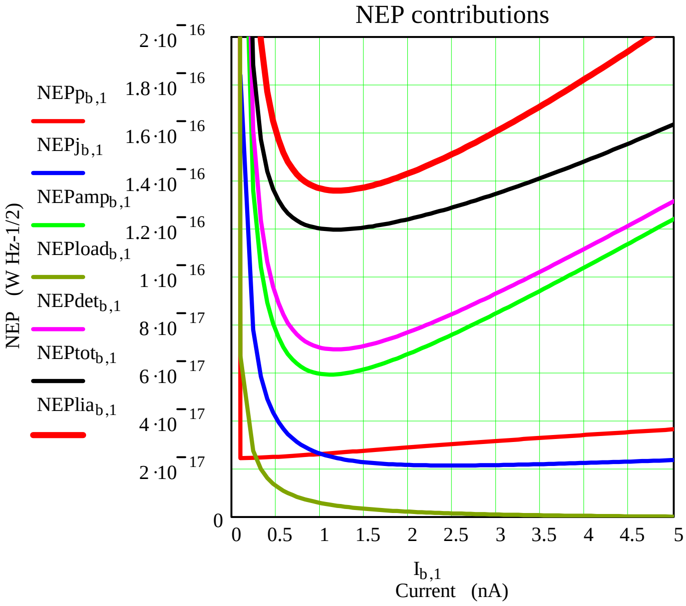
$NEPtotop_i := NEPlia_{p_i,i}$

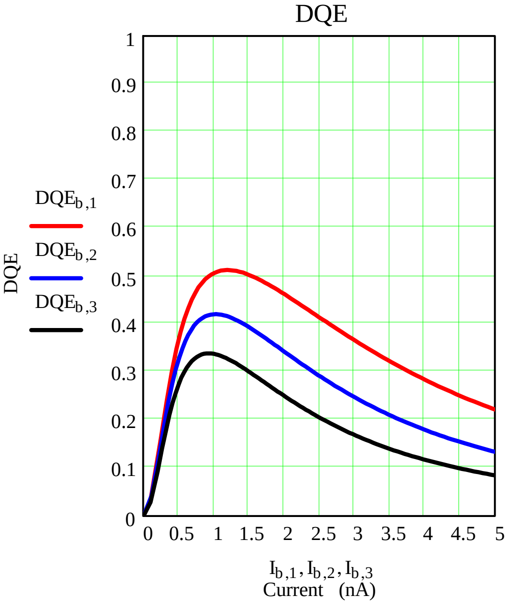
$NEPtotop_i =$

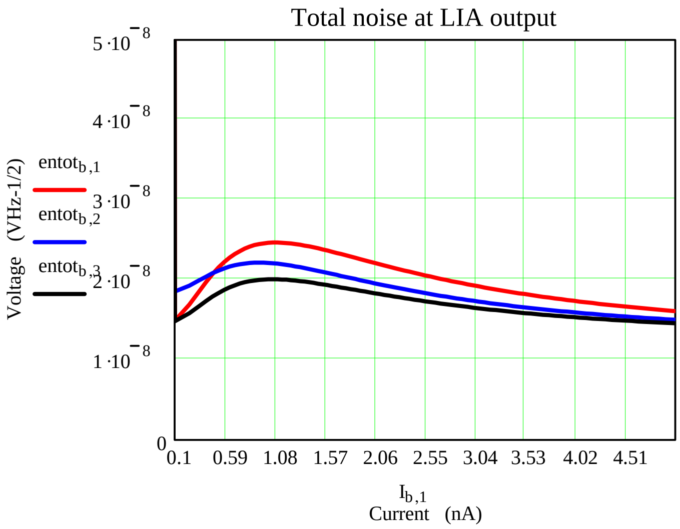
$1.36\cdot10^{-16}$
$1.07\cdot10^{-16}$
$9.07\cdot10^{-17}$











Summary of bolometer performance parameters

$DQEop_i =$	$Sop_i =$	$\tau phys_i \cdot 1000 =$
0.512	$2.05 \cdot 10^8$	9.89
0.420	$2.41 \cdot 10^8$	12.59
0.339	$2.64 \cdot 10^8$	16.48

Photon noise levels and single-detector NEFD

Photon noise limited NEP (full expression)

$$NEPph_i := \left[\frac{4 \cdot A\Omega_i \cdot h^2}{c^2} \cdot \left[\int_{vL_i}^{vU_i} \frac{\varepsilon tel \cdot td_0 \cdot \eta feed_i \cdot nu^4}{e^{\left(\frac{h \cdot nu}{kb \cdot T_0}\right)} - 1} \cdot \left[1 + \frac{\varepsilon tel \cdot td_0 \cdot \eta feed_i}{e^{\left(\frac{h \cdot nu}{kb \cdot T_0}\right)} - 1} \right] dnu \right] \right]^{0.5} \cdot 10^{17}$$

Overall NEP (W Hz-1/2 x 10-17)

$$NEPtotop_i := \frac{NEPph_i}{\left(DQEop_i\right)^{0.5}}$$

referred to the power absorbed by the detector

NEFD (mJy Hz-1/2) for point source chopped observations

$$NEFDp_i := \frac{NEPtotop_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta ch \cdot \eta tel \cdot 2^{0.5} \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed_i}$$

Factor of SQRT(2) from pixel-pixel chopping

NEFD (mJy Hz-1/2) for field mapping (jiggle mode)

$$NEFDf_i := \frac{NEPtotop_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta ch \cdot \eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed_i}$$

No factor of SQRT(2) in the denominator as we are not pixel-pixel chopping

NEFD (mJy Hz-1/2) for scan map observations without chopping

$$NEFDs_i := \frac{NEPtotop_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed_i} \cdot 2^{0.5}$$

Factor of SQRT(2) assumes need for background subtraction (probably pessimistic as background can be estimated by averaging a number of scan points)

1-s ; 1 sec. limiting flux densities (mJy):

$$S\_1\sigma\_1s\_point_i := \frac{NEFDp_i}{2^{0.5}} \qquad S\_1\sigma\_1s\_field_i := \frac{NEFDf_i}{2^{0.5}} \qquad S\_1\sigma\_1s\_scan_i := \frac{NEFDs_i}{2^{0.5}}$$

1-s ; 1 hr. limiting flux densities (mJy):

$$Slim\_point\_1hr_i := \frac{S\_1\sigma\_1s\_point_i}{(3600 \cdot \eta obs)^{0.5}} \qquad Slim\_field\_1hr_i := \frac{S\_1\sigma\_1s\_field_i}{(3600 \cdot \eta obs)^{0.5}} \qquad Slim\_scan\_1hr_i := \frac{S\_1\sigma\_1s\_scan_i}{(3600 \cdot \eta obs)^{0.5}}$$

Point source photometry in 7-point mode:

Loss in S/N for point source due to need to make a 7-point map:

$$\text{loss}_i := \begin{array}{|c|}\hline 0.06 \\ \hline 0.13 \\ \hline 0.20 \\ \hline \end{array}$$

$$\text{Slim\_point\_1hr}_i = \begin{array}{|c|}\hline 0.506 \\ \hline 0.612 \\ \hline 0.698 \\ \hline \end{array}$$

5-σ 1 hr flux density limit (mJy) in 7-point mode:

$$\text{Slim\_7\_pt\_5\_}\sigma\text{\_1hr}_i := 5\cdot\text{Slim\_point\_1hr}_i\cdot\left(1+\text{loss}_i\right)$$

$$\text{Slim\_7\_pt\_5\_}\sigma\text{\_1hr}_i = \begin{array}{|c|}\hline 2.7 \\ \hline 3.5 \\ \hline 4.2 \\ \hline \end{array}$$

Deep mapping of one field for 1 hour in jiggle-map mode:

Loss in S/N for point source due to need to make a map:

S/N improvement through pixel co-addition

$$\text{SN\_imp} := 1.5$$

S/N reduction through decrease in integration time/point by factor of 16

$$\text{SN\_red} := 4$$

Overall reduction in S/N

$$\text{factor} := \frac{\text{SN\_imp}}{\text{SN\_red}} \qquad \text{factor} = 0.375$$

1 σ; 1-hr limiting flux density for field map (mJy)

$$\Delta\text{S\_field\_1hr}_i := \frac{\text{Slim\_field\_1hr}_i}{\text{factor}}$$

$$\Delta\text{S\_field\_1hr}_i = \begin{array}{|c|}\hline 1.9 \\ \hline 2.3 \\ \hline 2.6 \\ \hline \end{array}$$

Deep mapping of one field for 1 hour in scan-map mode:

Note: this would not be done in practice as the telescope turn-around overhead would be unacceptable. But the calculation allows the sensitivity for large-scale maps to be estimated.

1 σ; 1 hr limiting flux density for scan map (mJy)

$$\Delta\text{S\_scan\_1hr}_i := \frac{\text{Slim\_scan\_1hr}_i}{\text{factor}}$$

$$\Delta\text{S\_scan\_1hr}_i = \begin{array}{|c|}\hline 1.2 \\ \hline 1.5 \\ \hline 1.7 \\ \hline \end{array}$$

5-σ flux density limit (mJy) for 4 x 8 arcminute field (allow 25% margin)

$$\text{margin} := 1.25$$

$$\Delta\text{S\_scan\_5\_}\sigma\text{\_1hr}_i := \Delta\text{S\_scan\_1hr}_i\cdot 5\cdot\text{margin}$$

$$\Delta\text{S\_scan\_5\_}\sigma\text{\_1hr}_i = \begin{array}{|c|}\hline 7.6 \\ \hline 9.2 \\ \hline 10.5 \\ \hline \end{array}$$

Time to map 1 sq.deg. to the confusion limit in scan-map mode

Confusion limit for 1 source per 40 beams (mJy) using source count models of Rowan-Robinson (2000)

$$\Delta Sconf\_MRR_i := \begin{array}{|c|}\hline 19\\\hline 20\\\hline 15\\\hline\end{array}$$

Take 15 mJy as required 5-σ limit for all three bands

$$\Delta Sconf_i := \begin{array}{|c|}\hline 15\\\hline 15\\\hline 15\\\hline\end{array}$$

Time to reach confusion limit for one field at 5-σ (minutes)

$$T\_1\_field_i := \left(\frac{\Delta S\_scan\_5\_\sigma\_1hr_i}{\Delta Sconf_i}\right)^2 \cdot 60 \qquad T\_1\_field_i = \begin{array}{|c|}\hline 15.4\\\hline 22.5\\\hline 29.3\\\hline\end{array}$$

Required overlap between fields:

$$overlap := 1.2$$

Number of fields to be mapped for 1 sq. deg.

$$Nfields := \frac{60^2}{4\cdot 8}\cdot\frac{overlap}{yield}$$

Time needed (days)

$$T\_1\_sq\_deg_i := Nfields\cdot T\_1\_field_i\cdot\frac{1}{60\cdot 21} \qquad T\_1\_sq\_deg_i = \begin{array}{|c|}\hline 2.059\\\hline 3.009\\\hline 3.918\\\hline\end{array}$$

Note: It is assumed (pessimistically) that the overlap between fields does not lead to any S/N enhancement

Large area deep survey (nominally 100 sq. deg; 180 days):

Area of one field (sq. arcmin) taking bolometer yield into account

$$Afield := 4\cdot 8\cdot yield \qquad Afield = 25.6$$

Area to be surveyed (sq. deg.)

$$Asurv := 100$$

Number of fields to be observed:

$$Nfields := \frac{Asurv\cdot 60^2}{Afield}\cdot overlap \qquad Nfields = 16875$$

Time for survey:

$$Tdays := 180 \qquad Tmonths := Tdays\cdot\frac{12}{365} \qquad Tmonths = 5.9$$

$$Thrs := Tdays\cdot 21 \qquad Thrs = 3780$$

Time for each field (hrs):

$$TField := \frac{Thrs}{Nfields} \qquad TField = 0.224$$

1-σ; 1-hr limiting flux density for scan map (mJy)

$$\Delta S\_scan\_1hr_i := \frac{Slim\_scan\_1hr_i\cdot margin}{factor}$$

Large survey 5-σ flux density limit (mJy):

$$\Delta S\_surv\_5\sigma_i := \Delta S\_scan\_1hr_i\cdot\left(\frac{1}{TField}\right)^{0.5}\cdot 5 \qquad \Delta S\_surv\_5\sigma_i = \begin{array}{|c|}\hline 16.0\\\hline 19.4\\\hline 22.1\\\hline\end{array}$$

Summary of power loading and sensitivity calculations

Key parameters case = "nominal"

$\eta feed_i =$

0.700
0.700
0.700

$yield = 0.800$

$To = 0.320$

$enamp = 1.221 \times 10^{-8}$

$\eta obs = 0.850$

$\varepsilon tel = 0.040$

$RS = 100$

$DQEop_i =$

0.512
0.420
0.339

$\lambda_i =$	**Pdet absorbed (pW)** $Pdet_i =$	**NEPs (W Hz-1/2 E-17)** $NEPph_i =$	$NEPtotop_i =$	**NEFDs (mJy Hz-1/2)** $NEFDp_i =$	$NEFDf_i =$	$NEFDs_i =$
250	5.6	9.7	13.6	40	56	36
363	4.1	6.9	10.7	48	68	43
517	3.4	5.3	9.1	55	77	49

$\lambda_i =$	**Point source 7-point (mJy) 5 s 1 hr** $Slim\_7\_pt\_5\_\sigma\_1hr_i =$	**Field Map (mJy 5 s 1 hr)** $\Delta S\_field\_1hr_i \cdot 5 =$	**Scan Map (mJy 5 s 1 hr)** $\Delta S\_scan\_5\_\sigma\_1hr_i =$	**1 sq.deg. (15 mJy 5s) Days** $T\_1\_sq\_deg_i =$	**100 sq.deg. 180 day survey (mJy 5s)** $\Delta S\_surv\_5\sigma_i =$
250	2.7	9.5	7.6	2.06	16.0
363	3.5	11.5	9.2	3.01	19.4
517	4.2	13.2	10.5	3.92	22.1

$p := 1, 2 .. 3$

Plot sensitivity results

$\lambda SW_p :=$

λL_1
λ_1
λU_1

$\lambda MW_p :=$

λL_2
λ_2
λU_2

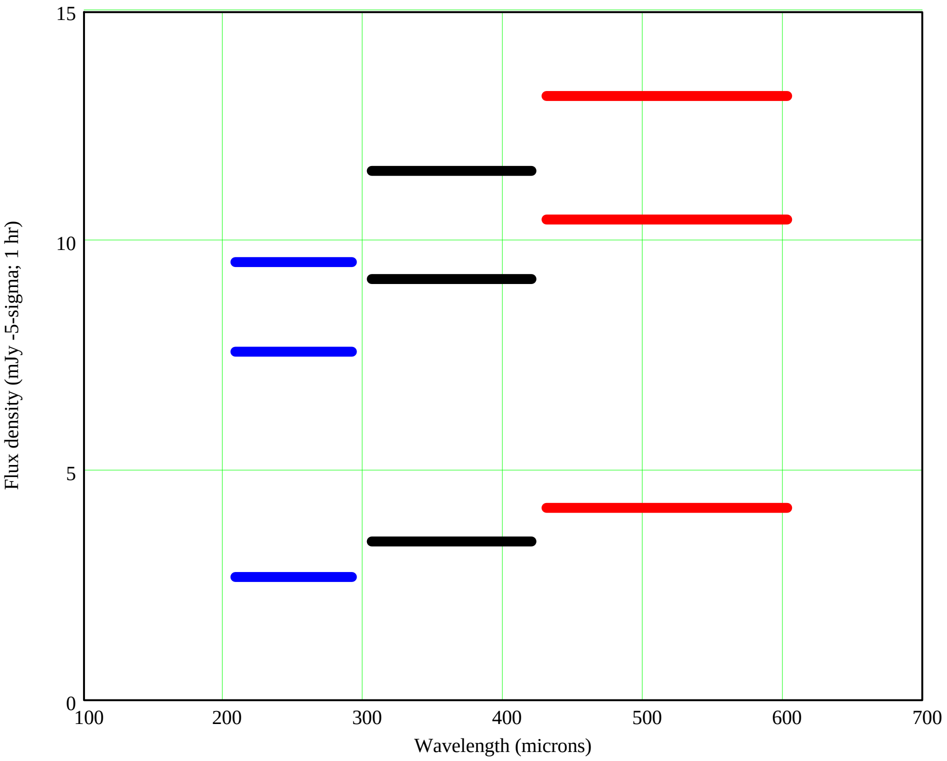
$\lambda LW_p :=$

λL_3
λ_3
λU_3

$SevptSW_p := Slim\_7\_pt\_5\_\sigma\_1hr_1$ $FieldSW_p := \Delta S\_field\_1hr_1 \cdot 5$ $ScanSW_p := \Delta S\_scan\_5\_\sigma\_1hr_1$

$SevptMW_p := Slim\_7\_pt\_5\_\sigma\_1hr_2$ $FieldMW_p := \Delta S\_field\_1hr_2 \cdot 5$ $ScanMW_p := \Delta S\_scan\_5\_\sigma\_1hr_2$

$SevptLW_p := Slim\_7\_pt\_5\_\sigma\_1hr_3$ $FieldLW_p := \Delta S\_field\_1hr_3 \cdot 5$ $ScanLW_p := \Delta S\_scan\_5\_\sigma\_1hr_3$



Temperature stability analysis

1. Telescope

Power from telescope absorbed by detector for nominal telescope temp.

$$Ptel\_nom_i := td_0 \cdot \varepsilon_0 \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_0) \cdot A\Omega_i \, d\nu$$

Power from telescope absorbed by detector for telescope temp. 1 K higher

$$Ptel\_plus_i := td_0 \cdot \varepsilon_0 \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_0 + 1) \cdot A\Omega_i \, d\nu$$

Rate of change of Ptel with Ttel (pW K-1)

$$dPteldT_i := Ptel\_plus_i - Ptel\_nom_i$$

Maximum telescope temperature noise (mK Hz-1/2) based on criterion: ΔPtel < NEP/3

$$Tn\_tel\_max_i := \frac{\frac{1}{3} \cdot NEPtotop_i \cdot 10^{-17}}{dPteldT_i \cdot 10^{-12}} \cdot 1000$$

Summary

$Ptel\_nom_i =$	$Ptel\_plus_i =$	$dPteldT_i =$	$Tn\_tel\_max_i =$
5.65	5.75	0.101	0.45
4.14	4.21	0.066	0.54
3.38	3.43	0.050	0.60
pW	pW	pW K-1	mK Hz-1/2

2. 4-K and 2-K stages

Elements 1 - 6 are at the "4-K" stage

$o\_min := 1$ $o\_max := 6$ $o := o\_min, o\_min + 1 .. o\_max$

Elements 7 - 10 are at the "2-K" stage

$p\_min := 7$ $p\_max := 10$ $p := p\_min, p\_min + 1 .. p\_max$

$k =$	$T_k =$
0	80.0
1	5.0
2	5.0
3	5.0
4	5.0
5	5.0
6	5.0
7	2.0
8	2.0
9	2.0
10	2.0
11	0.3
12	0.3

Power contributions (pW) from each 4-K and 2 K element at nominal temp.

$$P4K\_nom_{i,o} := td_o \cdot \varepsilon_o \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_o) \cdot A\Omega_i \, d\nu$$

$$P2K\_nom_{i,p} := td_p \cdot \varepsilon_p \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_p) \cdot A\Omega_i \, d\nu$$

Total power contributions (pW) from 4-K and 2-K at nominal temp.

$$Ptot4K\_nom_i := \sum_{o = o\_min}^{o\_max} P4K\_nom_{i,o}$$

$$Ptot2K\_nom_i := \sum_{p = p\_min}^{p\_max} P2K\_nom_{i,p}$$

Power contributions (pW) absorbed by detector from each 4-K and 2-K element at nominal temps. + 0.1 K

$$P4K\_plus_{i,o} := td_o \cdot \varepsilon_o \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_o + 0.1) \cdot A\Omega_i \, d\nu$$

$$P2K\_plus_{i,p} := td_p \cdot \varepsilon_p \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_p + 0.1) \cdot A\Omega_i \, d\nu$$

Total power contributions (pW) from 4-K and 2-K stages at nominal temps. + 0.1 K

$$\mathrm{Ptot4K\_plus}_i := \sum_{o\,=\,\mathrm{o\_min}}^{\mathrm{o\_max}} \mathrm{P4K\_plus}_{i,o}$$

$$\mathrm{Ptot2K\_plus}_i := \sum_{p\,=\,\mathrm{p\_min}}^{\mathrm{p\_max}} \mathrm{P2K\_plus}_{i,p}$$

Rate of change of P4K and P2K with T4K and T2K (pW K-1)

$$\mathrm{dP4KdT}_i := 10\cdot\left(\mathrm{Ptot4K\_plus}_i - \mathrm{Ptot4K\_nom}_i\right)$$

$$\mathrm{dP2KdT}_i := 10\cdot\left(\mathrm{Ptot2K\_plus}_i - \mathrm{Ptot2K\_nom}_i\right)$$

Maximum Level-1 temperature noise (mK Hz-1/2) based on criterion:
ΔP4K < NEP/3

$$\mathrm{Tn\_4K\_max}_i := \frac{\frac{1}{3}\cdot \mathrm{NEPtotop}_i\cdot 10^{-17}}{\mathrm{dP4KdT}_i\cdot 10^{-12}}\cdot 1000$$

Maximum Level-0 temperature noise (mK Hz-1/2) based on criterion:
ΔP2K < NEP/3

$$\mathrm{Tn\_2K\_max}_i := \frac{\frac{1}{3}\cdot \mathrm{NEPtotop}_i\cdot 10^{-17}}{\mathrm{dP2KdT}_i\cdot 10^{-12}}\cdot 1000$$

Summary

$\mathrm{Ptot4K\_nom}_i =$	$\mathrm{Ptot4K\_plus}_i =$	$\mathrm{dP4KdT}_i =$	$\mathrm{dP2KdT}_i =$	$\mathrm{Tn\_4K\_max}_i =$	$\mathrm{Tn\_2K\_max}_i =$
$3.940\cdot 10^{-4}$	$4.890\cdot 10^{-4}$	$9.500\cdot 10^{-4}$	$2.773\cdot 10^{-9}$	47.7	$2\cdot 10^{7}$
$5.692\cdot 10^{-3}$	$6.635\cdot 10^{-3}$	$9.432\cdot 10^{-3}$	$2.248\cdot 10^{-6}$	3.8	$2\cdot 10^{4}$
0.031	0.035	0.036	$1.862\cdot 10^{-4}$	0.8	162
pW	pW	pW K-1	pW K-1	mK Hz-1/2	mK Hz-1/2

Chopped observations:
Assume chopping at 2 Hz, so one cycle = 0.5 sec.
Assume drift must correspond to less than NEPtot/3 over this timescale of 0.5 sec

Allowed telescope temperature drift rate (mK s-1)

Allowed 4-K temperature drift rate (mK s-1)

Allowed 2-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_tel\_max}_i := \frac{\mathrm{Tn\_tel\_max}_i}{0.5}$$

$$\mathrm{Drift\_4K\_max}_i := \frac{\mathrm{Tn\_4K\_max}_i}{0.5}$$

$$\mathrm{Drift\_2K\_max}_i := \frac{\mathrm{Tn\_2K\_max}_i}{0.5}$$

$\mathrm{Drift\_tel\_max}_i =$	$\mathrm{Drift\_4K\_max}_i =$	$\mathrm{Drift\_2K\_max}_i =$
0.90	95.39	$3.3\cdot 10^{7}$
1.07	7.55	$3.2\cdot 10^{4}$
1.20	1.67	324.7

Scan-map observations: point source extraction:
Assume timescale corresponds to ~ 100 mHz (cf mapping speed note) or 10 seconds
Assume drift must correspond to less than NEPtot/3 over this timescale of 10 sec.

Note: this is pessimistic in that temperature fluctuations of the telescope will produce a correlated signal across the arrays which can be used to subtract the drift signal from those detectors involved in the point source detection

Allowed telescope temperature drift rate (mK s-1)

$$\mathrm{Drift\_tel\_max}_i := \frac{\mathrm{Tn\_tel\_max}_i}{10}$$

$$\mathrm{Drift\_tel\_max}_i = \begin{array}{|c|}\hline 0.045 \\ \hline 0.054 \\ \hline 0.060 \\ \hline \end{array}$$

Allowed 4-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_4K\_max}_i := \frac{\mathrm{Tn\_4K\_max}_i}{10}$$

$$\mathrm{Drift\_4K\_max}_i = \begin{array}{|c|}\hline 4.770 \\ \hline 0.377 \\ \hline 0.083 \\ \hline \end{array}$$

Allowed 2-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_2K\_max}_i := \frac{\mathrm{Tn\_2K\_max}_i}{10}$$

$$\mathrm{Drift\_2K\_max}_i = \begin{array}{|r|}\hline 1.6\cdot10^{6} \\ \hline 1.6\cdot10^{3} \\ \hline 16.2 \\ \hline \end{array}$$

Conclusions:

The strongest requirement is set by point source extraction from scan map observations.
Required temperature stabilities (for 5-K Level-1 temperature) are:
Telescope: Better than 50 μK s-1 (180 mK hr-1)
4-K: Better than 80 μK s-1 (290 mK hr-1)
(decreases to 42 μK s-1 if the Level-1 stage rises to 6 K)
2-K: Better than 16 mK s-1 - should be no problem

Comparison with Jamie Bock's calculations (SPIRE-JPL-NOT-000623 "Temperature Stability Requirements for SPIRE"):

Jamie's results 4 K: 260 mK hr-1
Telescope: 50 mK hr-1 *

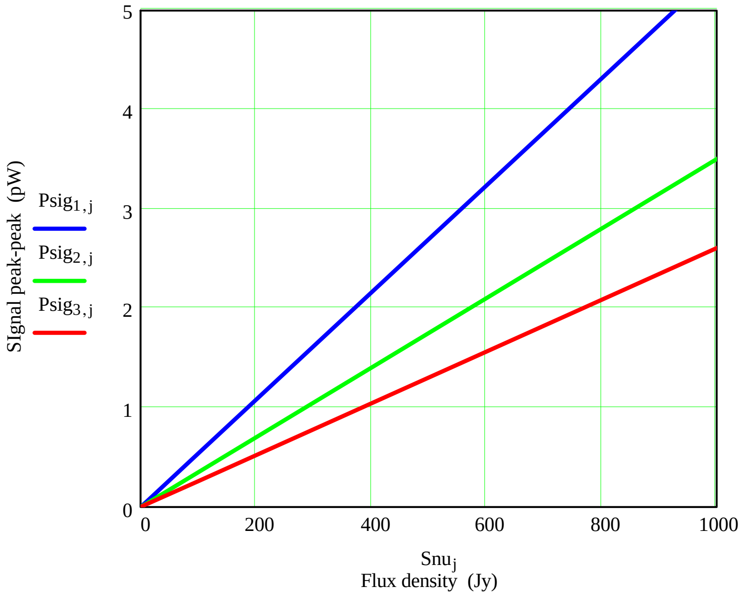
* Jamie assumed $\Delta\nu = \nu$, leading to factor of three greater background so a factor of three greater sensitivity to telescope temperature fluctuations

Large astronomical signals

$j := 1, 2 .. 11 \quad Snu_j := 10^{j-7}$

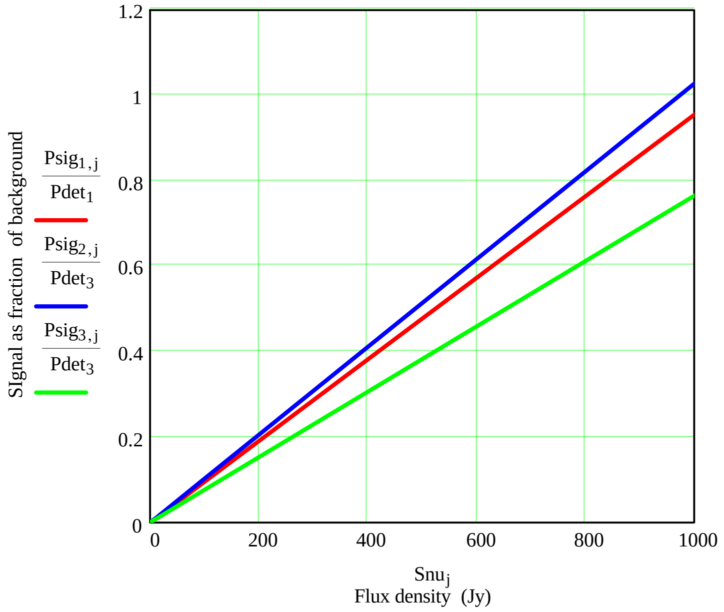
Signal power absorbed by detector

$Psig_{i,j} := Snu_j \cdot 10^{-26} \cdot 10^{12} \cdot \eta tel \cdot \eta feed_i \cdot Atel \cdot td_0 \cdot t_0 \cdot \Delta\nu_i$ pW



Typical 350-μm flux densities (factor of 2 higher or lower for 250 and 500 for planets):

OMC1	**1500**
W3(OH)	**680**
K3-50	**320**
W75N	**650**
Neptune	**100**
Uranus	**250**
Saturn	**7300**
Jupiter	**24000**



$\lambda_i =$	$Pdet_i =$
250	5.6
363	4.1
517	3.4

Comments and conclusions:

- We define a signal power equivalent to 20% of the background to correspond to a high level of loading (where one would expect corrections for detector non-linearity to become problematic).
- This corresponds to a flux density level of aroud 200 Jy for all bands. This means that Uranus will be rather too bright to use as a primary calibrator, but Neptune shold be usable.
- Sources in the 1E5 Jy range will result in signal powers more than 20 times the telescope background. This would completely swamp the detectors such that observations would be impossible to carry out or to calibrate.
- A reasonable limit to adopt at present to define observability is 1000 Jy. Sources near this limit will be difficult to calibrate (achievable accuracy will need to be assessed in ground calibration and PV phase).

Requirements for internal calibrator (PCAL)

Assumptions: Calibrator = black body located at centre of M4

M4 diameter and area (mm; mm^2) $DM4 := 26 \quad AM4 := \frac{\pi \cdot DM4^2}{4}$

PCAL size and area (mm; mm^2) $DCal := 2.8 \quad ACal := \frac{\pi \cdot DCal^2}{4}$

Required instantaneous S/N $SN\_Req := 500$

Overall NEPs (W Hz-1/2 E-17) for the three bands, referred to power absorbed by detector)

$\lambda_i =$	$NEPtotop_i =$
250	13.6
363	10.7
517	9.1

Required calibrator power absobed by detector (pW)

$$PCal\_Req_i := NEPtotop_i \cdot SN\_Req \cdot 10^{-17} \cdot 10^{12}$$

$PCal\_Req_i =$
0.068
0.053
0.045

Calibrator power at detector as a percentage of telescope power at detector

$$\frac{PCal\_Req_i}{Pdet_i} \cdot 100 =$$

1.20
1.29
1.33

Effect of 8-dB Gaussian illumination profile on the pupil

The pupil is illuminated by the photometer feedhorns with a nominal edge taper of 8 dB. The telescope power received from the pupil is therefore weighted according to this illumination profile. PCAL is at the centre of the pupil, and so is viewed more efficiently than the pupil as a whole. Let PCAL be ascribed an efficiency of unity. The relative efficiency of the whole pupil illumination is calculated as follows.

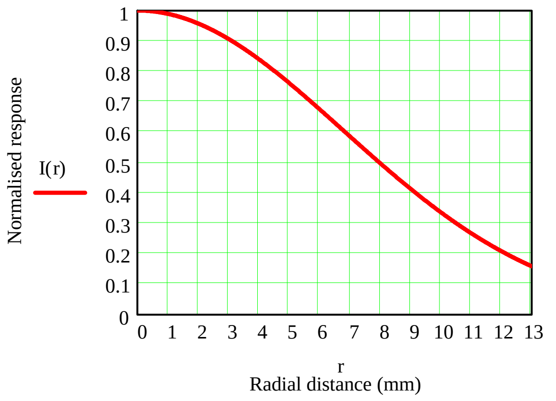
Pupil diameter and radius (mm) $D := 26 \quad R := 0.5 \cdot D$

Pupil edge taper (dB and linear) $taper\_dB := 8 \quad taper\_lin := 10^{\frac{taper\_dB}{10}} \quad taper\_lin = 6.3$

Define Gaussian illumination pattern

$$ro := \frac{R}{\left(-\ln\left(\frac{1}{taper\_lin}\right)\right)^{0.5}} \qquad ro = 9.58 \qquad I(r) := \exp\left[-\left(\frac{r}{ro}\right)^2\right]$$

$$dB(r) := 10\log(I(r))$$



Relative efficiency factor for illumination of telescope pupil

$$\eta\_taper := \frac{\int_0^R I(x) \cdot 2 \cdot \pi \cdot x \, dx}{\pi \cdot R^2}$$

$$\eta\_taper = 0.457$$

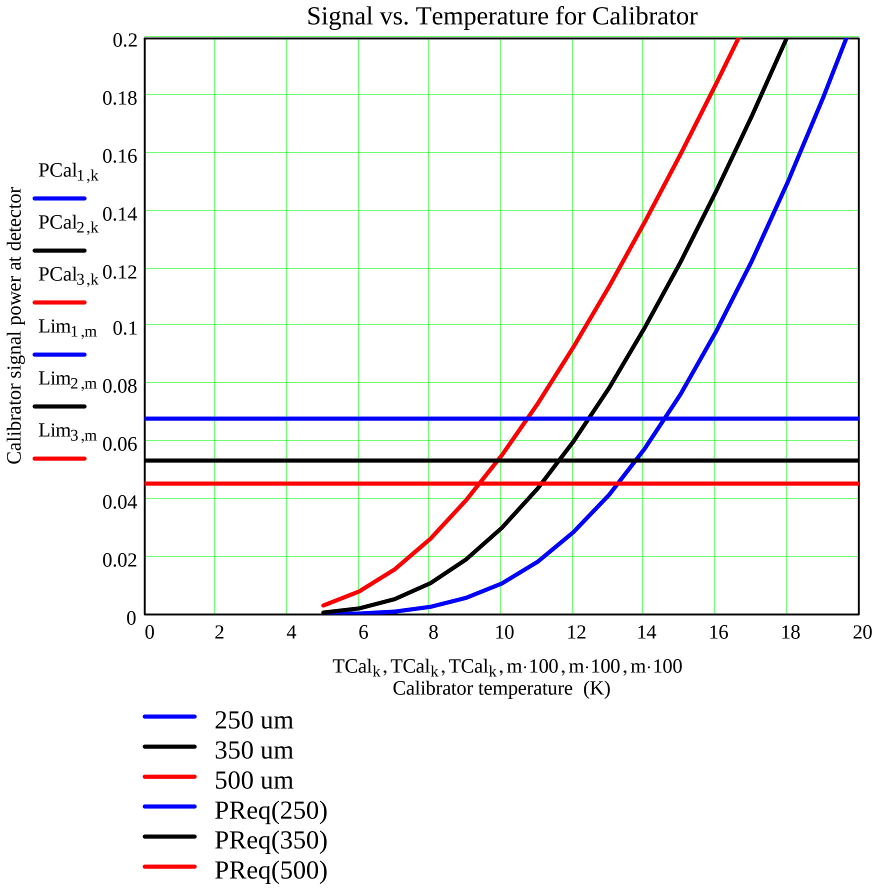
Brightness temperature range for calibrator (K) $k := 5, 6 .. 100$ $TCal_k := k$

Ratio at detector of calibrator power to telescope power

$$Ratio_{i,k} := \frac{ACal \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, TCal_k)\, d\nu}{AM4 \cdot \varepsilon tel \cdot \eta\_taper \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, Ttel)\, d\nu}$$

Calibrator power at detector (pW) $PCal_{i,k} := Ratio_{i,k} \cdot Pdet_i$

Plot calibrator power vs. calibrator brightness temperature: $m := 0, 1 .. 1$ $Lim_{i,m} := PCal\_Req_i$



Horizontal lines show the required powers for S/N = 500 in the three bands

A brightness temperature of 15 K provides S/N > 500 in all three bands.

B. SPIRE FTS sensitivity model SPIRE_FTS_3_IBDR.mcd 22 January 2002

This version:

* Prepared for SPIR IBDR
* Filter bands updated to reflect filter DDR and finalised horn specifications as in ECR on BDA SSSD 11 October 2002
* Feed and bolomneter parameters from *SPIRE-JPL-PRJ-000456 Working Version,* 7 November 2001

Previous versions:

SPIRE_FTS_1.MCD: 21 November 2000:
Version for Toledo meeting and System Review
* Multi-moding of both SW and LW bands now taken into account
* NEP contributions from each mode calculated separately and added in quadrature
* NEP now referred to the power absorbed by the detector
* Calculations done for the minimum and goal parameters of the detectors and feedhorns

SPIRE_FTS_2_IIDR.MCD: 6 April 2001:
* Updated for IIDR

Constants:

$h \equiv 6.626\cdot 10^{-34}$ $kb \equiv 1.3806\cdot 10^{-23}$

$origin := 1$

$c \equiv 2.998\cdot 10^{8}$

$b := 1, 2 .. 2$

Planck function:

$$B(\nu, T) := \frac{2\cdot h\cdot \nu^{3}}{c^{2}\cdot\left[\exp\left(\left(\frac{h\cdot\nu}{kb\cdot T}\right)\right) - 1\right]}$$

Assumptions

Telescope	Temp.	Emissivity	Diameter	Diameter of central obscuration	Focal ratio
	$Ttel \equiv 80$	$\varepsilon tel \equiv 0.04$	$Dtel \equiv 3.285$	$Dobs \equiv 0.56$	$Ftel := 8.68$

Area $Atel := 0.25 \cdot \pi \cdot \left(Dtel^2 - Dobs^2\right)$ $Atel = 8.2$

Percentage obscuration $Obs\_percent := \left(1 - \frac{8.229}{8.475}\right) \cdot 100$

$Obs\_percent = 2.9$

Observing efficiency $\eta obs\_m \equiv 0.8$ (jiggle map)

Cold stop attenuation of telescope background: $\eta cs := 0.8$

FTS efficiency	Observing efficiency	Elec. filter efficiency	Cos^2 modn efficiency
	$\eta obs := 0.8$	$\eta elec \equiv 0.8$	$\eta cosq \equiv 0.5$

Bolometer and feedhorn properties (see BDA Subsystem Spec. Doc. SPIRE-JPL-PRJ-000456):

Overall optical efficiency of horn + bolometer combination $\eta feed\_min := 0.45$ $\eta feed\_goal := 0.75$ $\eta feed\_nom_b :=$

$$\begin{array}{ll} LW & \boxed{0.65} \\ SW & \boxed{0.70} \end{array}$$

DQE of bolometer (wrt absorbed power)

$$\begin{array}{llll} & DQE\_min_b := & DQE\_goal_b := & DQE\_nom_b := \\ LW & 0.60 & 0.71 & 0.65 \\ SW & 0.67 & 0.80 & 0.70 \end{array}$$

$\eta feed_b := \eta feed\_nom_b$

$DQE_b := DQE\_nom_b$

Beam divider reflection transmission, emissivity

$tbd \equiv 0.487$ $rbd \equiv 0.487$ $\eta bd1 \equiv 2 \cdot tbd \cdot rbd$ $\eta bd2 \equiv tbd^2 + rbd^2$

$\eta bd1 = 0.47$ $\eta bd2 = 0.47$ $\varepsilon bd \equiv 1 - (tbd + rbd)$ $\varepsilon bd = 0.03$

Temperatures of filters $T4 \equiv 5$

Diffraction loss at each mirror $diffraction \equiv 0.97$

Emissivity of each mirror $\varepsilon\_mirr \equiv 1 - 0.995$

Effective transmission of each mirror $t\_mirr \equiv 0.995$ $t\_mirr = 0.99$

Overall diffraction loss $diff\_loss := diffraction^{11}$

$diff\_loss = 0.72$

This is applied only to the signal, not to the background (see below)

Transmission, emissivity and temperature of optical elements

$j \equiv 0, 1 .. 16 \quad k \equiv 0, 1 .. 17$

0 = Telescope
1 = CFI1 (4 K)
2 = CFIL2 (4 K)
3 = CIPM (M3)
4 = CBSM (M4)
5 = CRIM (M5)
6 = SPOM (M6)
7 = SIFM
8 = SIRM
9 = SBD_overall
10 = SCOM
11 = SRTM
12 = SDCM
13 = SBD2
14 = SCAM
15 = SFIL3 (2 K)
16 = Bandpass (0.3 K)
17 = Blocker (2 K)

$k =$	$t_k \equiv$	$T_k \equiv$	$\varepsilon_k \equiv$	$td_j =$
0.0	0.96	80	0.04	0.206
1.0	0.90	T4	0.1	0.229
2.0	0.9	T4	ε_mirr	0.255
3.0	t_mirr	T4	ε_mirr	0.256
4.0	t_mirr	T4	ε_mirr	0.257
5.0	t_mirr	T4	ε_mirr	0.258
6.0	t_mirr	T4	ε_mirr	0.260
7.0	t_mirr	T4	ε_mirr	0.261
8.0	t_mirr	T4	εbd	0.262
9.0	ηbd1	T4	ε_mirr	0.553
10.0	t_mirr	T4	2·ε_mirr	0.556
11.0	t_mirr^2	T4	ε_mirr	0.561
12.0	t_mirr	T4	εbd	0.564
13.0	1	T4	ε_mirr	0.564
14.0	t_mirr	T4	0.1	0.567
15.0	0.9	2	0.4	0.630
	0.7	0.3	0.1	
	0.9	0.3		

Transmission from element to detector

$$td_j \equiv \prod_{k=j+1}^{17} t_k$$

td is the transmission efficiency for the telescope background power. The diffraction loss factor is also applied below for the source signal.

Array parameters

Horn external diameter (μm) (internal diameters are 100 μm smaller)

$d\_horn_b :=$

LW	390
SW	225

The horns are thus 2Fλ at 225 and 390 μm.

Array side (centre-centre)

$$Warray_1 := 4 \cdot 2 \cdot \frac{d\_horn_1 \cdot 10^{-6}}{Dtel} \cdot \frac{360}{2 \cdot \pi} \cdot 60 \qquad Warray_2 := 6 \cdot 2 \cdot \frac{d\_horn_2 \cdot 10^{-6}}{Dtel} \cdot \frac{360}{2 \cdot \pi} \cdot 60$$

$Warray_b =$

LW	3.3
SW	2.8

arcmin

Bands

1. Old SPIRE bands

OldBands :=

Array	Design	Horn	Waveguide	Horn	Waveguide	Defocus	No. of	lL	lU	lo	Res.
	l	Length	Length	Dia.	Diameter		horns				
	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)					
S/SW	275	**TBD**	550	2.15	185	0	37	200	315	258	2.24
S/LW	450	45.36	900	3.78	393	0	19	305	670	488	1.34

2. New bands

NewBands :=

Array	Design	Horn	Waveguide	Horn	Waveguide	Defocus	No. of	lL	lU	lo	Res.
	l	Length	Length	Dia.	Diameter		horns				
	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)					
S/SW	275	23.68	575	2.15	190	0	37	190	325	258	1.91
S/LW	450	46.36	925	3.80	393	0	19	300	670	485	1.31

Note: Waveguide diameter is given by λL*(1.841/π)

Select new or old bands for calculation (Index = 1 for new and 0 for old) $\text{Index} := 1$

Extract band limits from the tables above

$$\lambda L_2 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{3,8}, \text{OldBands}_{3,8}\right) \qquad \lambda L_2 = 190$$

$$\lambda U_2 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{3,9}, \text{OldBands}_{3,9}\right) \qquad \lambda U_2 = 325$$

$$\lambda L_1 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{4,8}, \text{OldBands}_{4,8}\right) \qquad \lambda L_1 = 300$$

$$\lambda U_1 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{4,9}, \text{OldBands}_{4,9}\right) \qquad \lambda U_1 = 670$$

Crossover wavelength (μm)

$$\lambda\_\text{cross} := 0.5\cdot\left(\lambda L_1 + \lambda U_2\right) \qquad \lambda\_\text{cross} = 313$$

Waveguide radii (μm) $roS := \frac{\lambda U_2 \cdot 1.841}{2 \cdot \pi}$ $roL := \frac{\lambda U_1 \cdot 1.841}{2 \cdot \pi}$

Waveguide diameter (μm) $doS := 2 \cdot roS$ $doS = 190$ $doL := 2 \cdot roL$ $doL = 393$

Band limits (cm-1) $\sigma L_1 := \frac{10000}{\lambda U_1}$ $\sigma U_1 := \frac{10000}{\lambda L_1}$ $\sigma L_2 := \frac{10000}{\lambda U_2}$ $\sigma U_2 := \frac{10000}{\lambda L_2}$

Band limits (mm and Hz) $\nu L_b := c \cdot \sigma L_b \cdot 100$ $\nu U_b := c \cdot \sigma U_b \cdot 100$

Band centre (mm and Hz) $\nu 0_b := \frac{\nu L_b + \nu U_b}{2}$ $\lambda 0_b := \frac{c \cdot 10^6}{\nu 0_b}$

Band λ/Δλ $R_b := \frac{\sigma U_b + \sigma L_b}{2 \cdot (\sigma U_b - \sigma L_b)}$

Band limits (mm and THz)

	$\lambda L_b =$	$\lambda 0_b =$	$\lambda U_b =$	$\sigma L_b =$	$\sigma U_b =$	$\nu L_b \cdot 10^{-12} =$	$\nu 0_b \cdot 10^{-12} =$	$\nu U_b \cdot 10^{-12} =$	$R_b =$
LW	300	414	670	14.9	33.3	0.45	0.72	1.00	1.31
SW	190	240	325	30.8	52.6	0.92	1.25	1.58	1.91

Aperture and cavity efficiencies

Jason Glenn's caluclations (as summarised in his note of July 2 2001):

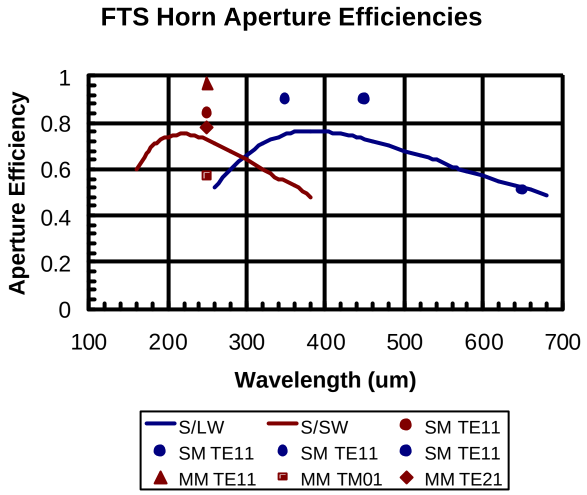
Efficiencies :=

Worksheet

$o := 0, 1 .. 11$ $p := 0, 1 .. 21$

$\lambda\_SW := Efficiencies^{\langle 0 \rangle}$ $\eta app\_SW := Efficiencies^{\langle 1 \rangle}$

$\lambda\_LW := Efficiencies^{\langle 2 \rangle}$ $\eta app\_LW := Efficiencies^{\langle 3 \rangle}$


FTS Horn Aperture Efficiencies

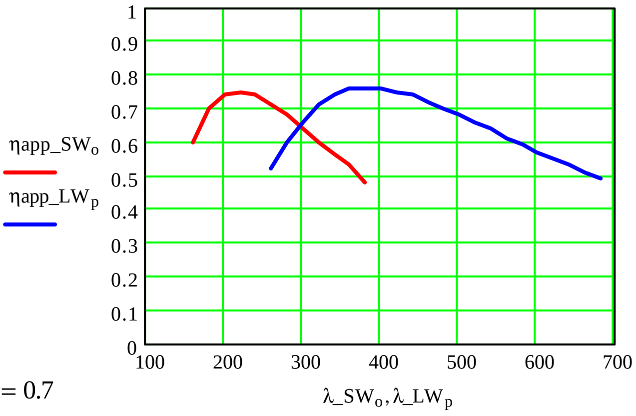
Telescope coupling efficiency (point source)

Assume this is the same as the horn aperture efficiency

Simplified wavelength dependence:

* Constant at 0.7
from 200 - 500 μm

* Declines linearly from 0.7
to 0.5 from 500 to 670 μm
(factor of 0.7/0.5 = 1.4 assumed
below for degradation in sensitivity)



$\eta tel := 0.7$

Background power levels on the detectors and photon noise limited NEPs

Assumptions:

1. All modes carry equal background power (per unit bandwidth) from the telescope
2. Calibrator contributes same amount of power as the telescope
3. Efficiencies of higher order modes are as in Jason's note (independent of crossover wavelength)

Throughput per mode $A\Omega(\nu) := \left(\frac{c}{\nu}\right)^2 \cdot \eta cs$

SW horn (multimoded)

$\eta\_SW\_TE11(\nu) := 0.84$

$\eta\_SW\_TM01(\nu) := 0.57$

$\eta\_SW\_TE21(\nu) := 0.78$

These values are used in calculating the background, but a pessimistic value of ηfeed_nom is assumed for TE11 in calculating the NEFD

$$\eta feed\_nom = \begin{pmatrix} 0.0 \\ 0.7 \\ 0.7 \end{pmatrix}$$

LW horn (multi-moded)

Fundamental mode efficiency vs. frequency

$q := 0, 1 .. 2$

$$\lambda LW_q := \begin{array}{|l|}\hline 650 \\ \hline 450 \\ \hline 350 \\ \hline \end{array} \qquad \nu LW_q := \frac{c \cdot 10^6}{\lambda LW_q} \qquad \eta LW_q := \begin{array}{|c|}\hline 0.51 \\ \hline \eta feed\_nom_1 \\ \hline \eta feed\_nom_1 \\ \hline \end{array} \qquad \eta LW_q = \begin{array}{|c|}\hline 0.51 \\ \hline 0.65 \\ \hline 0.65 \\ \hline \end{array}$$

$\eta feed\_nom_1 = 0.65$

$\eta\_LW\_TE11(nu) := \mathrm{linterp}(\nu LW, \eta LW, nu)$ $\qquad \eta\_LW\_TE11\left(\frac{c \cdot 10^6}{650}\right) = 0.51$

LW horn higher order modes:
(not modelled yet so assume 50%)

$\eta\_LW\_higher := 0.5$

SW band (b = 2)

Cut-off wavelength for TE11 mode

$\lambda cSW := \lambda U_2$ $\qquad \lambda cSW = 325$ $\qquad \nu o\_TE11_2 := 0.5 \cdot (\nu L_2 + \nu U_2)$

Waveguide radius (µm)

$roS = 95$

Cut-off wavelengths of higher modes

$$\lambda c\_TM01 := \frac{2 \cdot \pi \cdot roS}{2.405} \qquad \lambda c\_TM01 = 249 \qquad \nu c\_TM01 := \frac{c \cdot 10^6}{\lambda c\_TM01} \qquad \nu o\_TM01_2 := \frac{\nu c\_TM01 + \nu U_2}{2}$$

$$\lambda c\_TE21 := \frac{2 \cdot \pi \cdot roS}{3.054} \qquad \lambda c\_TE21 = 196 \qquad \nu c\_TE21 := \frac{c \cdot 10^6}{\lambda c\_TE21} \qquad \nu o\_TE21_2 := \frac{\nu c\_TE21 + \nu U_2}{2}$$

Power absorbed by detector from each element and for each mode (pw)

Note: 1. Factor of 2 accounts for same background from calib. source in 2nd port
2. Power is set to zero if the mode is not propagated

TE11 $$P\_TE11_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_2 \cdot 10^{12} \cdot \int_{\nu L_2}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_SW\_TE11(\nu)\, d\nu$$

$$P\_TE11_{2,j} := \mathrm{if}(P\_TE11_{2,j} < 0, 0, P\_TE11_{2,j}) \qquad Power\_TE11_2 := \sum_{n=0}^{9} P\_TE11_{2,n}$$

TM01 $$P\_TM01_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_2 \cdot 10^{12} \cdot \int_{\nu c\_TM01}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_SW\_TM01(\nu)\, d\nu$$

$$P\_TM01_{2,j} := \mathrm{if}(P\_TM01_{2,j} < 0, 0, P\_TM01_{2,j}) \qquad Power\_TM01_2 := \sum_{n=0}^{9} P\_TM01_{2,n}$$

TE21 $$P\_TE21_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_2 \cdot 10^{12} \cdot \int_{\nu c\_TE21}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_SW\_TE21(\nu)\, d\nu$$

$$P\_TE21_{2,j} := \mathrm{if}(P\_TE21_{2,j} < 0, 0, P\_TE21_{2,j}) \qquad Power\_TE21_2 := \sum_{n=0}^{9} P\_TE21_{2,n}$$

NEPph contributions from each mode

TE11 $$NEPph\_TE11_2 := \left(2 \cdot Power\_TE11_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TE11_2\right)^{0.5} \cdot 10^{17}$$

TM01 $$NEPph\_TM01_2 := \left(2 \cdot Power\_TM01_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TM01_2\right)^{0.5} \cdot 10^{17}$$

TE21 $$NEPph\_TE21_2 := \left(2 \cdot Power\_TE21_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TE21_2\right)^{0.5} \cdot 10^{17}$$

Summary of power (pW)
and NEPph contributions
(W Hz-1/2 E-17) in SW band

$$Power_{2,j} := P\_TE11_{2,j} + P\_TM01_{2,j} + P\_TE21_{2,j}$$

$$NEPph_2 := \left[\left(NEPph\_TE11_2\right)^2 + \left(NEPph\_TM01_2\right)^2 + \left(NEPph\_TE21_2\right)^2\right]^{0.5}$$

$$Ptot_2 := Power\_TE11_2 + Power\_TM01_2 + Power\_TE21_2$$

LW band (b = 1)

Cut-off wavelength for TE11 mode (μm) $\lambda cLW := \lambda U_1$ $\lambda cLW = 670$

Waveguide radius (mm) $roL = 196$ $\nu o\_TE11_1 := 0.5 \cdot (\nu L_1 + \nu U_1)$

Cut-off wavelengths of higher modes

$\lambda c\_TM01 := \frac{2 \cdot \pi \cdot roL}{2.405}$ $\lambda c\_TM01 = 513$ $\nu c\_TM01 := \frac{c \cdot 10^6}{\lambda c\_TM01}$ $\nu o\_TM01_1 := \frac{\nu c\_TM01 + \nu U_1}{2}$

$\lambda c\_TE21 := \frac{2 \cdot \pi \cdot roL}{3.054}$ $\lambda c\_TE21 = 404$ $\nu c\_TE21 := \frac{c \cdot 10^6}{\lambda c\_TE21}$ $\nu o\_TE21_1 := \frac{\nu c\_TE21 + \nu U_1}{2}$

$\lambda c\_TE01 := \frac{2 \cdot \pi \cdot roL}{3.832}$ $\lambda c\_TE01 = 322$ $\nu c\_TE01 := \frac{c \cdot 10^6}{\lambda c\_TE01}$ $\nu o\_TE01_1 := \frac{\nu c\_TE01 + \nu U_1}{2}$

$\lambda c\_TE31 := \frac{2 \cdot \pi \cdot roL}{4.201}$ $\lambda c\_TE31 = 294$ $\nu c\_TE31 := \frac{c \cdot 10^6}{\lambda c\_TE31}$ $\nu o\_TE31_1 := \frac{\nu c\_TE31 + \nu U_1}{2}$

TE11 $P\_TE11_{1,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu L_1}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_LW\_TE11(\nu)\, d\nu$

$P\_TE11_{1,j} := if(P\_TE11_{1,j} < 0, 0, P\_TE11_{1,j})$ $Power\_TE11_1 := \sum_{n=0}^{9} P\_TE11_{1,n}$

TM01 $P\_TM01_{1,j} := \eta\_LW\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu c\_TM01}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$

$P\_TM01_{1,j} := if(P\_TM01_{1,j} < 0, 0, P\_TM01_{1,j})$ $Power\_TM01_1 := \sum_{n=0}^{9} P\_TM01_{1,n}$

TE21 $P\_TE21_{1,j} := \eta\_LW\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu c\_TE21}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$

$P\_TE21_{1,j} := if(P\_TE21_{1,j} < 0, 0, P\_TE21_{1,j})$ $Power\_TE21_1 := \sum_{n=0}^{9} P\_TE21_{1,n}$

TE01 $P\_TE01_{1,j} := \eta\_LW\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu c\_TE01}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$

$P\_TE01_{1,j} := if(P\_TE01_{1,j} < 0, 0, P\_TE01_{1,j})$ $Power\_TE01_1 := \sum_{n=0}^{9} P\_TE01_{1,n}$

NEPph contributions from each mode

TE11 $NEPph\_TE11_1 := \left(2 \cdot Power\_TE11_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE11_1\right)^{0.5} \cdot 10^{17}$

TM01 $NEPph\_TM01_1 := \left(2 \cdot Power\_TM01_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TM01_1\right)^{0.5} \cdot 10^{17}$

TE21 $NEPph\_TE21_1 := \left(2 \cdot Power\_TE21_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE21_1\right)^{0.5} \cdot 10^{17}$

TE01 $NEPph\_TE01_1 := \left(2 \cdot Power\_TE01_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE01_1\right)^{0.5} \cdot 10^{17}$

Summary of power (pW) and NEPph contributions (W Hz-1/2 E-17) in LW band

$$\mathrm{Power}_{1,j} := \mathrm{P\_TE11}_{1,j} + \mathrm{P\_TM01}_{1,j} + \mathrm{P\_TE21}_{1,j} + \mathrm{P\_TE01}_{1,j}$$

$$\mathrm{NEPph}_1 := \left[\left(\mathrm{NEPph\_TE11}_1\right)^2 + \left(\mathrm{NEPph\_TM01}_1\right)^2 + \left(\mathrm{NEPph\_TE21}_1\right)^2 + \left(\mathrm{NEPph\_TE01}_1\right)^2\right]^{0.5}$$

$$\mathrm{Ptot}_1 := \mathrm{Power\_TE11}_1 + \mathrm{Power\_TM01}_1 + \mathrm{Power\_TE21}_1 + \mathrm{Power\_TE01}_1$$

Summary

SW band

$\mathrm{Power\_TE11}_2 = 7.6$ $\mathrm{NEPph\_TE11}_2 = 11.2$

$\mathrm{Power\_TM01}_2 = 2.8$ $\mathrm{NEPph\_TM01}_2 = 7.2$

$\mathrm{Power\_TE21}_2 = 0.5$ $\mathrm{NEPph\_TE21}_2 = 3.1$

$\mathrm{Ptot}_2 = 10.8$ $\mathrm{NEPph}_2 = 13.6$

LW band

$\mathrm{Power\_TE11}_1 = 5.2$ $\mathrm{NEPph\_TE11}_1 = 7.1$

$\mathrm{Power\_TM01}_1 = 3.1$ $\mathrm{NEPph\_TM01}_1 = 5.7$

$\mathrm{Power\_TE21}_1 = 1.9$ $\mathrm{NEPph\_TE21}_1 = 4.6$

$\mathrm{Power\_TE01}_1 = 0.5$ $\mathrm{NEPph\_TE01}_1 = 2.5$

$\mathrm{Ptot}_1 = 10.6$ $\mathrm{NEPph}_1 = 10.5$

Note that total power is dominated by the telescope contribution

SW band $\mathrm{Power}_{2,j} =$	LW band $\mathrm{Power}_{1,j} =$
10.8	10.6
$4.6\cdot10^{-4}$	0
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$1.4\cdot10^{-4}$	$5.7\cdot10^{-3}$
$5.5\cdot10^{-5}$	$2.3\cdot10^{-3}$
$1.1\cdot10^{-4}$	$4.7\cdot10^{-3}$
$5.6\cdot10^{-5}$	$2.3\cdot10^{-3}$
$2.9\cdot10^{-4}$	0
$5.7\cdot10^{-5}$	$2.4\cdot10^{-3}$
$1.1\cdot10^{-3}$	0
$3.1\cdot10^{-9}$	$8.5\cdot10^{-5}$

Photon noise levels and single-detector NEFD

Overall NEP
(W Hz-1/2 x 10-17)

$$\mathrm{NEPtot}_b := \frac{\mathrm{NEPph}_b}{(\mathrm{DQE}_b)^{0.5}}$$

referred to the power absorbed by the detector

Detector NEP
(W Hz-1/2 x 10-17)

$$\mathrm{NEPdet}_b := \left[(\mathrm{NEPtot}_b)^2 - (\mathrm{NEPph}_b)^2\right]^{0.5}$$

NEFD (Jy Hz-1/2)

$$\mathrm{NEFD}_b := \frac{\mathrm{NEPtot}_b \cdot 10^{-17} \cdot 10^{26}}{\eta\mathrm{elec} \cdot \eta\mathrm{cosq} \cdot \eta\mathrm{tel} \cdot \mathrm{Atel} \cdot \mathrm{td}_0 \cdot \Delta\nu \cdot t_0 \cdot \eta\mathrm{feed}_b \cdot \mathrm{diff\_loss}}$$

$$\mathrm{NEFD}_b = \begin{array}{|c|}\hline 2.04 \\ \hline 2.38 \\ \hline\end{array}$$

Diffraction loss is taken into account here.

Note: this is pessimistic in that the higher order modes are assumed to couple to the telescope background but not to the source - ηtel is taken to be the same value as for single-mode coupling

Point source observation

Spectral resolution (cm-1 and Hz) $\Delta\sigma \equiv 1$ $\Delta\nu \equiv c \cdot \Delta\sigma \cdot 100$

Limiting flux density
(mJy 5-σ 1-hr)

$$\mathrm{Slim}_b := \frac{1000 \cdot \mathrm{NEFD}_b \cdot 5}{(2 \cdot 3600 \cdot \eta\mathrm{obs})^{0.5}}$$

Limiting line strength
(mJy 5-σ 1-hr)

$$\mathrm{Flim}_b := \left(\frac{\mathrm{Slim}_b \cdot 10^{-26}}{1000} \cdot \Delta\nu\right)$$

Deep mapping of one field for 1 hour:

Loss in S/N for point source due to need to make a map:

S/N improvement through co-addition of pixels $\mathrm{SN\_imp} := 1.5$

S/N reduction through decrease in integration time per point by factor of 16 $\mathrm{SN\_red} := 4$

Overall reduction in S/N

$$\mathrm{factor} := \frac{\mathrm{SN\_imp}}{\mathrm{SN\_red}} \qquad \mathrm{factor} = 0.375$$

Limiting flux density (mJy)

$$\Delta\mathrm{S\_1hr}_b := \frac{\mathrm{Slim}_b}{\mathrm{factor}} \qquad \Delta\mathrm{F\_1hr}_b := \frac{\mathrm{Flim}_b}{\mathrm{factor}}$$

Band centre and edges: wavlengths and resolving powers

$$\mathrm{ResL}_b := \frac{\nu\mathrm{U}_b}{\Delta\nu} \qquad \mathrm{Res0}_b := \frac{\nu 0_b}{\Delta\nu} \qquad \mathrm{ResU}_b := \frac{\nu\mathrm{L}_b}{\Delta\nu}$$

	$\lambda\mathrm{L}_b =$	$\mathrm{ResL}_b =$	$\lambda 0_b =$	$\mathrm{Res0}_b =$	$\lambda\mathrm{U}_b =$	$\mathrm{ResU}_b =$
LW	300.0	33.3	414.4	24.1	670.0	14.9
SW	190.0	52.6	239.8	41.7	325.0	30.8

Summary:

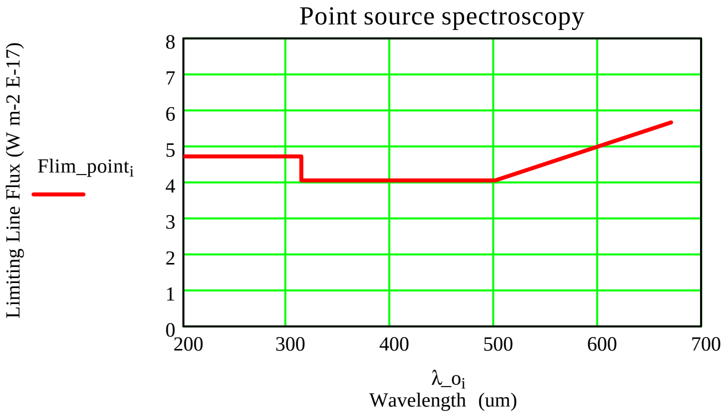
Crossover wavelength $\lambda\_cross = 313$ $\Delta\sigma = 1.0$

	Pdet absorbed (pW)	NEPs (W Hz-1/2 E-17)		Spectrophotometry (mJy 5-s;1-hr) Point source	Map	Spectroscopy (W m-2 5-s;1-hr) Point source	Map
	$Ptot_b =$	$NEPph_b =$	$NEPtot_b =$	$Slim_b =$	$\Delta S\_1hr_b =$	$Flim_b \cdot 10^{17} =$	$\Delta F\_1hr_b \cdot 10^{17} =$
LW	10.6	10.5	13.0	135	359	4.0	10.8
SW	10.8	13.6	16.3	157	418	4.7	12.5

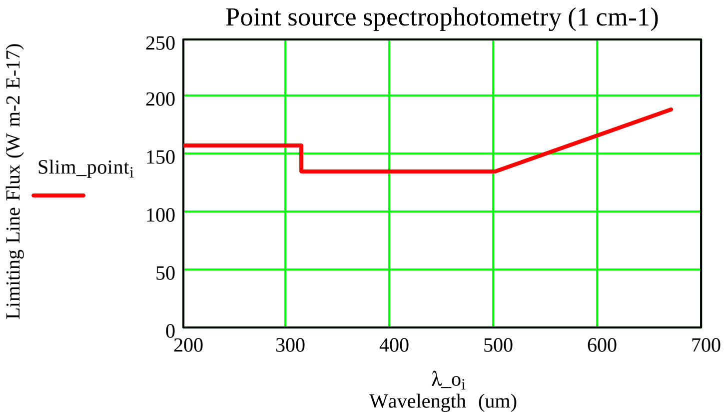
$i := 0, 1 .. 4$

$\lambda\_o_i :=$	$Flim\_point_i :=$	$Slim\_point_i :=$
λL_2	$Flim_2 \cdot 10^{17}$	$Slim_2$
$\lambda\_cross - 0.1$	$Flim_2 \cdot 10^{17}$	$Slim_2$
$\lambda\_cross + 0.1$	$Flim_1 \cdot 10^{17}$	$Slim_1$
500	$Flim_1 \cdot 10^{17}$	$Slim_1$
670	$Flim_1 \cdot 10^{17} \cdot 1.4$	$Slim_1 \cdot 1.4$

$\lambda\_o_i =$	$Flim\_point_i =$	$Slim\_point_i =$
190	4.7	157
312	4.7	157
313	4.0	135
500	4.0	135
670	5.6	188



Factor of 0.7/0.5 = 1.4 degradation in telescope coupling efficiency between 500 and 670 µm



Notes:

1. Limiting flux density Slim is inversely proporotional to spectral resolution (Δσ) and independent of wavelength except for LW band longer than 500 µm
2. For an unresolved line, limiting line flux Flim is independent of spectral resolution and wavelength within the SW band. In the LW band it is constant up to 500 µm.
3. Beyond 500 µm, sensitivity declines linearly, by a factor of 5/7 out to 670 µm