SPIRE Sensitivity Models

SPIRE-QMW-NOT-000642 Issue 2.0

Matt Griffin

22 January 2002

This is an update to the note issued for the SPIRE IIDR (April 6 2001). Changes with respect to that version are relatively minor and are indicated in blue.

The attached MathCad models describe the computed sensitivities of the SPIRE photometer and spectrometer for point source and mapping observations. The main assumptions made in estimating the scientific performance of the instrument are listed below. Additional assumptions are given in the worksheets.

Telescope				
Telescope temperature (K)		80		
Telescope emissivity		0.04		
Telescope used diameter (m)	(1)	3.29		
No. of observable hours per 24-hr period		21		
Photometer				
Bands (µm)		250	363	517
Numbers of detectors		139	88	43
Beam FWHM (arcsec.)		17	24	35
Bolometer DQE	(2)	0.73	0.68	0.61
Feed-horn/cavity efficiency	(3)	0.60	0.65	0.70
Filter widths $(\lambda/\Delta\lambda)$		3.0	3.2	3.0
Throughput		λ^2		
Bolometer yield		0.8		
Field of view (arcmin.)	Scan mapping Field mapping	4 x 8 4 x 4		
Overall instrument transmission		0.3		
Observing efficiency (slewing, setting up, etc.)		0.9		
Chopping efficiency factor		0.45		
Reduction in telescope background by cold stop	(4)	0.8		
FTS spectrometer				
Bands (µm)		200-315	315-670	
Numbers of detectors		37	19	
Bolometer DQE		0.70	0.65	
Feed-horn/cavity efficiency		0.70	0.65	
Usable field of view diameter (arcmin.)		2.6		
Max. spectral resolution (cm^{-1})		0.04		
Overall instrument transmission		0.15		
Signal modulation efficiency		0.5		
Observing efficiency		0.8		
Electrical filter efficiency		0.8		

Notes:

1. The telescope secondary mirror is the pupil stop for the system, so that the outer edges of the primary mirror are not seen by the detectors. This is important to make sure that radiation from highly emissive elements beyond the primary reflector does not contribute stray light.

2. The bolometer DQE (Detective Quantum Efficiency) is defined as $\left[\frac{NEP_{ph}}{NEP_{Total}}\right]^2$ where NEP_{ph} is the photon noise NEP due to the absorbed radiant power and NEP_{Total} is the overall NEP including the contribution from the bolometer noise.

3. This is the overall absorption efficiency of the combination of feed-horn, cavity and bolometer element.

4. A fraction of the feedhorn throughput falls outside the solid angle defined by the photometer 2-K cold stop and is thus terminated on a cold (non-emitting) surface rather than on the 4% emissive 80-K telescope. This reduces the background power on the detector.

Summary:

		Photometer band (mm)			**FTS band (mm)**	
		250	350	500	200-315	315-670
Background power/detector	pW	3.8	3.0	2.6	11	11
Background-limited NEP	W Hz$^{-1/2}$ x 10^{-17}	7.9	5.9	4.6	11	14
Overall NEP (inc. detector)	W Hz$^{-1/2}$ x 10^{-17}	9.3	7.1	5.9	13	16

Table Error! No text of specified style in document.**-1 -** Background power and photon noise-limited NEPs for SPIRE.

Photometry					
λ	um		250	350	500
ΔS(5-σ; 1-hr)	mJy	Point source (7-point mode)	2.4	2.8	3.1
		4’ x 4’ jiggle map	8.5	9.3	9.7
		4’ x 8’ scan map	6.8	7.4	7.7
Time (days) to map 1 deg.^{2} to 3 mJy 1-σ		1^o x 1^o scan map	1.7	2.0	2.1

Line spectroscopy Ds **= 0.04 cm^{-1}**					
λ	um		200 - 315	315 - 500	500-670
ΔS (5-σ; 1-hr)	W m^{-2} x 10^{-17}	Point source	4.7	4.0	4.0 - 5.6
		2.6’ map	13	11	11 - 15

Low-resolution spectrophotometry Ds **= 1 cm^{-1}**					
λ	um		200 - 315	315 - 500	500-670
ΔS (5-σ; 1-hr)	mJy	Point source	160	140	140 - 190
		2.6’ map	430	360	360-500

Photometer sensitivity model SPIRE_Phot_5_IBDR.MCD 22 Jan. 2002

SPIRE_Phot_5_IBDR.MCD 22 Jan 2002

* Prepared for SPIRE IBDR
* Includes estimation of sensitivity to temperatures of telescope and SPIRE instrument stages
* Filter bands updated to reflect filter DDR and horn specification definition (now 250, 363, 517 µm)
* Feedhorn efficiencies and DQE values updated according to the BDA SSSD *SPIRE-JPL-PRJ-000456 Working Version,* 7 November 2001
* Revised analysis of PCAL requirements

Previous versions:

SPIRE_Phot_1.MCD 21 November 2000

* Version prepared for Systems Design Review and Toledo Meeting

SPIRE_Phot_2_IIDR.MCD April 2001

* Version prepared for IIDR

SPIRE_Phot_3_SCM.MCD April 2001

* Version prepared for SPIRE Consortium meeting July 2001
* Proposed 0.56-m central obscuration included

Constants $h \equiv 6.626\cdot 10^{-34}$ $c \equiv 3\cdot 10^{8}$ $kb \equiv 1.38\cdot 10^{-23}$

$i \equiv 1, 2 .. 3$ $origin \equiv 1$

Planck function

$$B(nu, T) := \frac{2\cdot h\cdot (nu)^3}{c^2\cdot\left[e^{\left(\frac{h\cdot nu}{kb\cdot T}\right)} - 1\right]}$$

Assumptions

Telescope	Temp.	Emissivity	Diameter	Focal ratio	Diameter of central obscuration (m)
	$\mathrm{Ttel} \equiv 80$	$\varepsilon\mathrm{tel} \equiv 0.04$	$\mathrm{Dtel} \equiv 3.285$	$\mathrm{Ftel} := 8.68$	$\mathrm{Dobs} \equiv 0.56$

Area (m^2) $\mathrm{Atel} := 0.25 \cdot \pi \cdot \left(\mathrm{Dtel}^2 - \mathrm{Dobs}^2\right)$ $\mathrm{Atel} = 8.23$

Percentage obscuration $\mathrm{Obs\_percent} := \left(1 - \frac{8.229}{8.475}\right) \cdot 100$

$\mathrm{Obs\_percent} = 2.9$

Plate scale at telescope focus (arcsec/mm): $\mathrm{PS} := \frac{1}{\mathrm{Dtel} \cdot \mathrm{Ftel}} \cdot \frac{360}{2 \cdot \pi} \cdot 3.6$ $\mathrm{PS} = 7.23$

Plate scale at arrays (arcsec/mm): $\mathrm{PSA} := \mathrm{PS} \cdot \frac{8.68}{5}$ $\mathrm{PSA} = 12.6$

Beamwidths (arcsec.): $\mathrm{FWHM}_i := \frac{1.11 \cdot \lambda_i \cdot 10^{-6}}{\mathrm{Dtel}} \cdot \frac{360}{2 \cdot \pi} \cdot 3600$ $\mathrm{FWHM}_i = \begin{pmatrix} 17.4 \\ 25.3 \\ 36 \end{pmatrix}$

Feedhorn point source coupling efficiency: $\eta\mathrm{tel} \equiv 0.7$

Final optics focal ratio $\mathrm{Ffin} := 5$

Cold stop attenuation of telescope background: $\eta\mathrm{cs} := 0.8$

Bolometer and feedhorn properties (see BDA Subsystem Spec. Doc. SPIRE-JPL-PRJ-000456):

Overall optical efficiency of horn + bolometer combination $\eta\mathrm{feed\_min} := 0.45$ $\eta\mathrm{feed\_goal} := 0.85$

Assume intermediate value with slightly better performance for the larger horns. Note: Mapping speed scales with ηfeed and with sqrt(DQE)

$\lambda_i = \begin{pmatrix} 250 \\ 363 \\ 517 \end{pmatrix}$ $\eta\mathrm{feed\_nom}_i := \begin{pmatrix} 0.60 \\ 0.65 \\ 0.70 \end{pmatrix}$

DQE of horn-bolometer combination

$\lambda_i = \begin{pmatrix} 250 \\ 363 \\ 517 \end{pmatrix}$ $\mathrm{DQE\_min}_i := \begin{pmatrix} 0.67 \\ 0.62 \\ 0.56 \end{pmatrix}$ $\mathrm{DQE\_goal}_i := \begin{pmatrix} 0.80 \\ 0.74 \\ 0.67 \end{pmatrix}$ $\mathrm{DQE\_nom}_i := \begin{pmatrix} 0.73 \\ 0.68 \\ 0.61 \end{pmatrix}$

$\eta\mathrm{feed}_i := \eta\mathrm{feed\_nom}_i$

$\mathrm{DQE}_i := \mathrm{DQE\_nom}_i$

Bolometer yield $\mathrm{y\_min} := 0.75$ $\mathrm{y\_goal} := 0.9$ $\mathrm{y\_nom} := 0.8$ $\mathrm{yield} := \mathrm{y\_nom}$

Chopping efficiency factor $\eta\mathrm{ch} \equiv 0.45$

Observing efficicency $\eta\mathrm{obs} \equiv 0.9$

Bands: defined by central wavelengths (in mm) and resolution of the filters

$\lambda_i \equiv \begin{pmatrix} 250 \\ 363 \\ 517 \end{pmatrix}$ $R_i := \begin{pmatrix} 3.00 \\ 3.18 \\ 3.00 \end{pmatrix}$

$\nu_i := \frac{c}{\lambda_i \cdot 10^{-6}}$ $\lambda L_i := \lambda_i - \frac{\lambda_i}{2 \cdot R_i}$ $\lambda U_i := \lambda_i + \frac{\lambda_i}{2 \cdot R_i}$ $\Delta\lambda_i := \frac{\lambda_i}{R_i}$ $\Delta\nu_i := \frac{\nu_i}{R_i}$

$\nu L_i := \frac{c}{\lambda U_i \cdot 10^{-6}}$ $\nu U_i := \frac{c}{\lambda L_i \cdot 10^{-6}}$

$i =$	$\lambda_i =$	$\lambda L_i =$	$\lambda U_i =$	$\Delta\lambda_i =$	$\nu_i \cdot 10^{-9} =$	$\nu L_i \cdot 10^{-9} =$	$\nu U_i \cdot 10^{-9} =$	$\Delta\nu_i \cdot 10^{-9} =$
1	250	208.3	291.7	83	1200	1029	1440	400
2	363	305.9	420.1	114	826	714	981	260
3	517	430.8	603.2	172	580	497	696	193

Waveguide diameters (µm): $\mathrm{D\_wg}_i := \frac{\lambda U_i \cdot 1.841}{\pi}$ $\mathrm{D\_wg}_i = \begin{pmatrix} 171 \\ 246 \\ 353 \end{pmatrix}$

Transmission, emissivity and temperature of optical elements $j \equiv 0,1..11$ $k \equiv 0,1..12$

$Tdets \equiv 0.3$ $T2 \equiv 2.0$ $T4 \equiv 5.0$

	$k =$	$t_k \equiv$	$\varepsilon_k \equiv$	$T_k \equiv$	$td_j =$
0 = Telescope	0	0.960	0.04	Ttel	0.30
1 = input filter	1	0.900	0.100	T4	0.33
2 = M3	2	0.995	0.005	T4	0.34
3 = M4	3	0.995	0.005	T4	0.34
4 = M5	4	0.995	0.005	T4	0.34
5 = 4-K filter	5	0.900	0.100	T4	0.38
6 = M6	6	0.995	0.005	T4	0.38
7 = 2-K filter	7	0.900	0.300	T2	0.42
8 = M7	8	0.995	0.005	T2	0.42
9 = Dichroic	9	0.900	0.100	T2	0.47
10 = M8	10	0.995	0.005	T2	0.47
11 = Bandpass filter	11	0.525	0.100	Tdets	0.90
12 = Blocker	12	0.900	0.100	Tdets	

Transmission from element to detector

$$td_j \equiv \prod_{k = j+1}^{12} t_k$$

Note:
The 2-K filter is located at the 2-K pupil stop. An emissivity of 0.3 is ascribed to this component: 0.1 for the filter itself plus 0.2 for the spillover onto the cold-stop

Overall transmission of filter stack

$t\_filters := t_1 \cdot t_5 \cdot t_7 \cdot t_9 \cdot t_{11} \cdot t_{12}$

$t\_filters = 0.31$

Array parameters

Detector Numbers

$Ndets_i :=$	$Ndets_i =$
$15 \cdot 5 + 16 \cdot 4$	139
$13 \cdot 4 + 12 \cdot 3$	88
$9 \cdot 3 + 8 \cdot 2$	43

Horn aperture outside dia. (mm)

$$Dhorn_i := \frac{2 \cdot Ffin \cdot \lambda_i}{1000}$$

$Dhorn_i =$
2.5
3.6
5.2

Array dimension cente-centre (pixels):

$Nmax_i :=$	$Nmin_i :=$
15	8
12	6
8	4

Horn size projected onto telescope focus (mm):

$$Dpix_i := (Dhorn_i) \cdot \frac{Ftel}{Ffin}$$

Array dimensions at telescope focus centre-centre (mm):

$$Lmm_i := Nmax_i \cdot Dpix_i$$

$$Wmm_i := Nmin_i \cdot Dpix_i$$

Field size (arcmin):

$$Larcmin_i := \frac{Lmm_i \cdot PS}{60} \qquad Warcmin_i := \frac{Wmm_i \cdot PS}{60}$$

$Dhorn_i =$	$Dpix_i =$	$Lmm_i =$	$Wmm_i =$	$Larcmin_i =$	$Warcmin_i =$
2.5	4.3	65	35	7.8	4.2
3.6	6.3	76	38	9.1	4.6
5.2	9	72	36	8.7	4.3

Background power levels on the detectors

Throughput:

$$A\Omega_i := \eta cs \cdot \left(\lambda_i \cdot 10^{-6}\right)^2$$

Power contribution absorbed by detector from any element (pW)

$$Power_{i,j} := td_j \cdot \varepsilon_j \cdot 10^{12} \cdot \eta feed_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_j) \cdot A\Omega_i \, d\nu$$

Total power **absorbed** by detector (pW)

$$Pdet_i := \sum_{n=0}^{9} Power_{i,n}$$

$Power_{1,j} =$	$Power_{2,j} =$	$Power_{3,j} =$
3.75	2.98	2.62
0.00	0.00	0.01
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.01
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00

Note that the background power on the detectors is totally dominated by the telescope

Photon noise levels and single-detector NEFD

Photon noise limited NEP (full expression)

$$NEPph_i := \left[\frac{4 \cdot A\Omega_i \cdot h^2}{c^2} \cdot \left[\int_{\nu L_i}^{\nu U_i} \frac{\varepsilon tel \cdot td_0 \cdot \eta feed_i \cdot nu^4}{e^{\left(\frac{h \cdot nu}{kb \cdot T_0}\right)} - 1} \cdot \left[1 + \frac{\varepsilon tel \cdot td_0 \cdot \eta feed_i}{e^{\left(\frac{h \cdot nu}{kb \cdot T_0}\right)} - 1} \right] dnu \right] \right]^{0.5} \cdot 10^{17}$$

Overall NEP (W Hz-1/2 x 10-17)

$$NEPtot_i := \frac{NEPph_i}{\left(DQE_i\right)^{0.5}}$$

referred to the power absorbed by the detector

Detector NEP (W Hz-1/2 x 10-17)

$$NEPdet_i := \left[\left(NEPtot_i\right)^2 - \left(NEPph_i\right)^2 \right]^{0.5}$$

NEFD (mJy Hz-1/2) for point source chopped observations

$$NEFDp_i := \frac{NEPtot_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta ch \cdot \eta tel \cdot 2^{0.5} \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed_i}$$

Factor of SQRT(2) from pixel-pixel chopping

NEFD (mJy Hz-1/2) for field mapping (jiggle mode)

$$NEFDf_i := \frac{NEPtot_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta ch \cdot \eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed_i}$$

No factor of SQRT(2) in the denominator as we are not pixel-pixel chopping

NEFD (mJy Hz-1/2) for scan map observations without chopping

$$NEFDs_i := \frac{NEPtot_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed_i} \cdot 2^{0.5}$$

Factor of SQRT(2) assumes need for background subtraction (probably pessimistic as background can be estimated by averaging a number of scan points)

1-σ ; 1 sec. limiting flux densities (mJy):

$$S\_1\sigma\_1s\_point_i := \frac{NEFDp_i}{2^{0.5}} \qquad S\_1\sigma\_1s\_field_i := \frac{NEFDf_i}{2^{0.5}} \qquad S\_1\sigma\_1s\_scan_i := \frac{NEFDs_i}{2^{0.5}}$$

1-σ ; 1 hr. limiting flux densities (mJy):

$$Slim\_point\_1hr_i := \frac{S\_1\sigma\_1s\_point_i}{(3600\cdot\eta obs)^{0.5}} \qquad Slim\_field\_1hr_i := \frac{S\_1\sigma\_1s\_field_i}{(3600\cdot\eta obs)^{0.5}} \qquad Slim\_scan\_1hr_i := \frac{S\_1\sigma\_1s\_scan_i}{(3600\cdot\eta obs)^{0.5}}$$

Point source photometry in 7-point mode:

Loss in S/N for point source due to need to make a 7-point map:

$$loss_i := \begin{array}{|c|}\hline 0.06 \\ \hline 0.13 \\ \hline 0.20 \\ \hline \\ \hline\end{array} \qquad Slim\_point\_1hr_i = \begin{array}{|c|}\hline 0.453 \\ \hline 0.494 \\ \hline 0.514 \\ \hline\end{array}$$

5-σ 1 hr flux density limit (mJy) in 7-point mode:

$$Slim\_7\_pt\_5\_\sigma\_1hr_i := 5\cdot Slim\_point\_1hr_i\cdot(1 + loss_i) \qquad Slim\_7\_pt\_5\_\sigma\_1hr_i = \begin{array}{|c|}\hline 2.4 \\ \hline 2.8 \\ \hline 3.1 \\ \hline\end{array}$$

Deep mapping of one field for 1 hour in jiggle-map mode:

Loss in S/N for point source due to need to make a map:

S/N improvement through pixel co-addition $SN\_imp := 1.5$

S/N reduction through decrease in integration time/point by factor of 16 $SN\_red := 4$

Overall reduction in S/N $factor := \frac{SN\_imp}{SN\_red}$ $factor = 0$

1 σ; 1-hr limiting flux density for field map (mJy)

$$\Delta S\_field\_1hr_i := \frac{Slim\_field\_1hr_i}{factor} \qquad \Delta S\_field\_1hr_i = \begin{array}{|c|}\hline 1.7 \\ \hline 1.9 \\ \hline 1.9 \\ \hline\end{array}$$

Deep mapping of one field for 1 hour in scan-map mode:

Note: this would not be done in practice as the telescope turn-around overhead would be unacceptable. But the calculation allows the sensitivity for large-scale maps to be estimated.

1 σ; 1 hr limiting flux density for scan map (mJy)

$$\Delta S\_scan\_1hr_i := \frac{Slim\_scan\_1hr_i}{factor} \qquad \Delta S\_scan\_1hr_i = \begin{array}{|c|}\hline 1.1 \\ \hline 1.2 \\ \hline 1.2 \\ \hline\end{array}$$

5-σ flux density limit (mJy) for 4 x 8 arcminute field (allow 25% margin)

$margin := 1.25$

$$\Delta S\_scan\_5\_\sigma\_1hr_i := \Delta S\_scan\_1hr_i\cdot 5\cdot margin \qquad \Delta S\_scan\_5\_\sigma\_1hr_i = \begin{array}{|c|}\hline 6.8 \\ \hline 7.4 \\ \hline 7.7 \\ \hline\end{array}$$

Time to map 1 sq.deg. to the confusion limit in scan-map mode

Confusion limit for 1 source per 40 beams (mJy) using source count models of Rowan-Robinson (2000)

$$\Delta\mathrm{Sconf\_MRR}_i := \begin{array}{|c|}\hline 19\\ \hline 20\\ \hline 15\\ \hline\end{array}$$

Take 15 mJy as required 5-σ limit for all three bands

$$\Delta\mathrm{Sconf}_i := \begin{array}{|c|}\hline 15\\ \hline 15\\ \hline 15\\ \hline\end{array}$$

Time to reach confusion limit for one field at 5-σ (minutes)

$$\mathrm{T\_1\_field}_i := \left(\frac{\Delta\mathrm{S\_scan\_5\_\sigma\_1hr}_i}{\Delta\mathrm{Sconf}_i}\right)^2\cdot 60$$

$$\mathrm{T\_1\_field}_i = \begin{array}{|c|}\hline 12.3\\ \hline 14.6\\ \hline 15.9\\ \hline\end{array}$$

Required overlap between fields:

$$\mathrm{overlap} := 1.2$$

Number of fields to be mapped for 1 sq. deg.

$$\mathrm{Nfields} := \frac{60^2}{4\cdot 8}\cdot\frac{\mathrm{overlap}}{\mathrm{yield}}$$

Time needed (days)

$$\mathrm{T\_1\_sq\_deg}_i := \mathrm{Nfields}\cdot\mathrm{T\_1\_field}_i\cdot\frac{1}{60\cdot 21}$$

$$\mathrm{T\_1\_sq\_deg}_i = \begin{array}{|c|}\hline 1.7\\ \hline 2.0\\ \hline 2.1\\ \hline\end{array}$$

Note: It is assumed (pessimistically) that the overlap between fields does not lead to any S/N enhancement

Large area deep survey (nominally 100 sq. deg; 180 days):

Area of one field (sq. arcmin) taking bolometer yield into account

$$\mathrm{Afield} := (4)\cdot(8)\cdot\mathrm{yield} \qquad \mathrm{Afield} = 26$$

Area to be surveyed (sq. deg.)

$$\mathrm{Asurv} := 100$$

Number of fields to be observed:

$$\mathrm{Nfields} := \frac{\mathrm{Asurv}\cdot 60^2}{\mathrm{Afield}}\cdot\mathrm{overlap} \qquad \mathrm{Nfields} = 16875$$

Time for survey:

$$\mathrm{Tdays} := 180 \qquad \mathrm{Tmonths} := \mathrm{Tdays}\cdot\frac{12}{365} \qquad \mathrm{Tmonths} = 5.9$$

$$\mathrm{Thrs} := \mathrm{Tdays}\cdot 21 \qquad \mathrm{Thrs} = 3780$$

Time for each field (hrs):

$$\mathrm{TField} := \frac{\mathrm{Thrs}}{\mathrm{Nfields}} \qquad \mathrm{TField} = 0.224$$

1-σ; 1-hr limiting flux density for scan map (mJy)

$$\Delta\mathrm{S\_scan\_1hr}_i := \frac{\mathrm{Slim\_scan\_1hr}_i\cdot\mathrm{margin}}{\mathrm{factor}}$$

Large survey 5-σ flux density limit (mJy):

$$\Delta\mathrm{S\_surv\_5\sigma}_i := \Delta\mathrm{S\_scan\_1hr}_i\cdot\left(\frac{1}{\mathrm{TField}}\right)^{0.5}\cdot 5$$

$$\Delta\mathrm{S\_surv\_5\sigma}_i = \begin{array}{|c|}\hline 14.4\\ \hline 15.6\\ \hline 16.3\\ \hline\end{array}$$

Summary of power loading and sensitivity calculations

	Pdet absorbed (pW)	NEPs (W Hz-1/2 E-17)		NEFDs (mJy Hz-1/2)		
λ_i =	Pdet_i =	NEPph_i =	NEPtot_i =	NEFDp_i =	NEFDf_i =	NEFDs_i =
250	3.8	7.9	9.3	36	52	33
363	3.0	5.9	7.1	40	56	36
517	2.6	4.6	5.9	41	59	37

	Point source 7-point (mJy) 5 s 1 hr	Field Map (mJy 5 s 1 hr)
λ_i =	$\mathrm{Slim\_7\_pt\_5\_\sigma\_1hr}_i$ =	$\Delta\mathrm{S\_field\_1hr}_i \cdot 5$ =
250	2.4	8.5
363	2.8	9.3
517	3.1	9.7

	Scan Map (mJy 5 s 1 hr)	1 sq.deg. (15 mJy 5s) Days	100 sq.deg. 180 day survey (mJy 5s)
λ_i =	$\Delta\mathrm{S\_scan\_5\_\sigma\_1hr}_i$ =	$\mathrm{T\_1\_sq\_deg}_i$ =	$\Delta\mathrm{S\_surv\_5\sigma}_i$ =
250	6.8	1.7	14.4
363	7.4	2.0	15.6
517	7.7	2.1	16.3

Temperature stability analysis

1. Telescope

Power from telescope absorbed by detector for nominal telescope temp.

$$\mathrm{Ptel\_nom}_i := \mathrm{td}_0 \cdot \varepsilon_0 \cdot 10^{12} \cdot \eta\mathrm{feed}_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_0) \cdot A\Omega_i \, d\nu$$

Power from telescope absorbed by detector for telescope temp. 1 K higher

$$\mathrm{Ptel\_plus}_i := \mathrm{td}_0 \cdot \varepsilon_0 \cdot 10^{12} \cdot \eta\mathrm{feed}_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_0 + 1) \cdot A\Omega_i \, d\nu$$

Rate of change of Ptel with Ttel (pW K-1)

$$\mathrm{dPteldT}_i := \mathrm{Ptel\_plus}_i - \mathrm{Ptel\_nom}_i$$

Maximum telescope temperature noise (mK Hz-1/2) based on criterion: ΔPtel < NEP/3

$$\mathrm{Tn\_tel\_max}_i := \frac{\frac{1}{3} \cdot \mathrm{NEPtot}_i \cdot 10^{-17}}{\mathrm{dPteldT}_i \cdot 10^{-12}} \cdot 1000$$

Summary

$\mathrm{Ptel\_nom}_i =$	$\mathrm{Ptel\_plus}_i =$	$\mathrm{dPteldT}_i =$	$\mathrm{Tn\_tel\_max}_i =$
3.755	3.821	0.067	0.46
2.981	3.029	0.048	0.50
2.619	2.658	0.039	0.51
pW	pW	pW K-1	mK Hz-1/2

2. 4-K and 2-K stages

Elements 1 - 6 are at the "4-K" stage

$\mathrm{o\_min} := 1$ $\quad \mathrm{o\_max} := 6$ $\quad o := \mathrm{o\_min}, \mathrm{o\_min} + 1 .. \mathrm{o\_max}$

Elements 7 - 10 are at the "2-K" stage

$\mathrm{p\_min} := 7$ $\quad \mathrm{p\_max} := 10$ $\quad p := \mathrm{p\_min}, \mathrm{p\_min} + 1 .. \mathrm{p\_max}$

$k =$	$T_k =$
0	80
1	5
2	5
3	5
4	5
5	5
6	5
7	2
8	2
9	2
10	2
11	0
12	0

Power contributions (pW) from each 4-K and 2 K element at nominal temp.

$$\mathrm{P4K\_nom}_{i,o} := \mathrm{td}_o \cdot \varepsilon_o \cdot 10^{12} \cdot \eta\mathrm{feed}_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_o) \cdot A\Omega_i \, d\nu$$

$$\mathrm{P2K\_nom}_{i,p} := \mathrm{td}_p \cdot \varepsilon_p \cdot 10^{12} \cdot \eta\mathrm{feed}_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_p) \cdot A\Omega_i \, d\nu$$

Total power contributions (pW) from 4-K and 2-K at nominal temp.

$$\mathrm{Ptot4K\_nom}_i := \sum_{o = \mathrm{o\_min}}^{\mathrm{o\_max}} \mathrm{P4K\_nom}_{i,o}$$

$$\mathrm{Ptot2K\_nom}_i := \sum_{p = \mathrm{p\_min}}^{\mathrm{p\_max}} \mathrm{P2K\_nom}_{i,p}$$

Power contributions (pW) absorbed by detector from each 4-K and 2-K element at nominal temps. + 0.1 K

$$\mathrm{P4K\_plus}_{i,o} := \mathrm{td}_o \cdot \varepsilon_o \cdot 10^{12} \cdot \eta\mathrm{feed}_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_o + 0.1) \cdot A\Omega_i \, d\nu$$

$$\mathrm{P2K\_plus}_{i,p} := \mathrm{td}_p \cdot \varepsilon_p \cdot 10^{12} \cdot \eta\mathrm{feed}_i \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, T_p + 0.1) \cdot A\Omega_i \, d\nu$$

Total power contributions (pW) from 4-K and 2-K stages at nominal temps. + 0.1 K

$$\mathrm{Ptot4K\_plus}_i := \sum_{o = \mathrm{o\_min}}^{\mathrm{o\_max}} \mathrm{P4K\_plus}_{i,o}$$

$$\mathrm{Ptot2K\_plus}_i := \sum_{p = \mathrm{p\_min}}^{\mathrm{p\_max}} \mathrm{P2K\_plus}_{i,p}$$

Rate of change of P4K and P2K with T4K and T2K (pW K-1)

$$\mathrm{dP4KdT}_i := 10 \cdot \left(\mathrm{Ptot4K\_plus}_i - \mathrm{Ptot4K\_nom}_i\right)$$

$$\mathrm{dP2KdT}_i := 10 \cdot \left(\mathrm{Ptot2K\_plus}_i - \mathrm{Ptot2K\_nom}_i\right)$$

Maximum Level-1 temperature noise (mK Hz-1/2) based on criterion: ΔP4K < NEP/3

$$\mathrm{Tn\_4K\_max}_i := \frac{\frac{1}{3} \cdot \mathrm{NEPtot}_i \cdot 10^{-17}}{\mathrm{dP4KdT}_i \cdot 10^{-12}} \cdot 1000$$

Maximum Level-0 temperature noise (mK Hz-1/2) based on criterion: ΔP2K < NEP/3

$$\mathrm{Tn\_2K\_max}_i := \frac{\frac{1}{3} \cdot \mathrm{NEPtot}_i \cdot 10^{-17}}{\mathrm{dP2KdT}_i \cdot 10^{-12}} \cdot 1000$$

Summary

$\mathrm{Ptot4K\_nom}_i =$	$\mathrm{Ptot4K\_plus}_i =$	$\mathrm{dP4KdT}_i =$	$\mathrm{dP2KdT}_i =$	$\mathrm{Tn\_4K\_max}_i =$	$\mathrm{Tn\_2K\_max}_i =$
$3 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	$6 \cdot 10^{-4}$	$2 \cdot 10^{-9}$	48.9	$2 \cdot 10^{7}$
$4 \cdot 10^{-3}$	$5 \cdot 10^{-3}$	$7 \cdot 10^{-3}$	$2 \cdot 10^{-6}$	3.5	$1 \cdot 10^{4}$
0	0	0	$1 \cdot 10^{-4}$	0.7	137
pW	pW	pW K-1	pW K-1	mK Hz-1/2	mK Hz-1/2

Chopped observations:
Assume chopping at 2 Hz, so one cycle = 0.5 sec.
Assume drift must correspond to less than NEPtot/3 over this timescale of 0.5 sec

Allowed telescope temperature drift rate (mK s-1)

$$\mathrm{Drift\_tel\_max}_i := \frac{\mathrm{Tn\_tel\_max}_i}{0.5}$$

$\mathrm{Drift\_tel\_max}_i =$
0.92
0.99
1.01

Allowed 4-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_4K\_max}_i := \frac{\mathrm{Tn\_4K\_max}_i}{0.5}$$

$\mathrm{Drift\_4K\_max}_i =$
97.85
6.98
1.41

Allowed 2-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_2K\_max}_i := \frac{\mathrm{Tn\_2K\_max}_i}{0.5}$$

$\mathrm{Drift\_2K\_max}_i =$
$3 \cdot 10^{7}$
$3 \cdot 10^{4}$
274

Scan-map observations: point source extraction:
Assume timescale corresponds to ~ 100 mHz (cf mapping speed note) or 10 seconds
Assume drift must correspond to less than NEPtot/3 over this timescale of 10 sec.

Note: this is pessimistic in that temperature fluctuations of the telescope will produce a correlated signal across the arrays which can be used to subtract the drift signal from those detectors involved in the point source detection

Allowed telescope temperature drift rate (mK s-1)

$$\mathrm{Drift\_tel\_max}_i := \frac{\mathrm{Tn\_tel\_max}_i}{10}$$

$$\mathrm{Drift\_tel\_max}_i = \begin{array}{|c|}\hline 0.046 \\ \hline 0.050 \\ \hline 0.051 \\ \hline\end{array}$$

Allowed 4-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_4K\_max}_i := \frac{\mathrm{Tn\_4K\_max}_i}{10}$$

$$\mathrm{Drift\_4K\_max}_i = \begin{array}{|c|}\hline 4.893 \\ \hline 0.349 \\ \hline 0.070 \\ \hline\end{array}$$

Allowed 2-K temperature drift rate (mK s-1)

$$\mathrm{Drift\_2K\_max}_i := \frac{\mathrm{Tn\_2K\_max}_i}{10}$$

$$\mathrm{Drift\_2K\_max}_i = \begin{array}{|r|}\hline 1.7\cdot 10^{6} \\ \hline 1.5\cdot 10^{3} \\ \hline 13.7 \\ \hline\end{array}$$

Conclusions:

The strongest requirement is set by point source extraction from scan map observations.
Required temperature stabilities (for 5-K Level-1 temperature) are:

Telescope: Better than 50 μK s-1 (180 mK hr-1)
4-K: Better than 70 μK s-1 (290 mK hr-1)
(decreases to 42 μK s-1 if the Level-1 stage rises to 6 K)
2-K: Better than 14 mK s-1 - should be no problem

Comparison with Jamie Bock's calculations (SPIRE-JPL-NOT-000623 "Temperature Stability Requirements for SPIRE"):

Jamie's results 4 K: 260 mK hr-1
Telescope: 50 mK hr-1 *

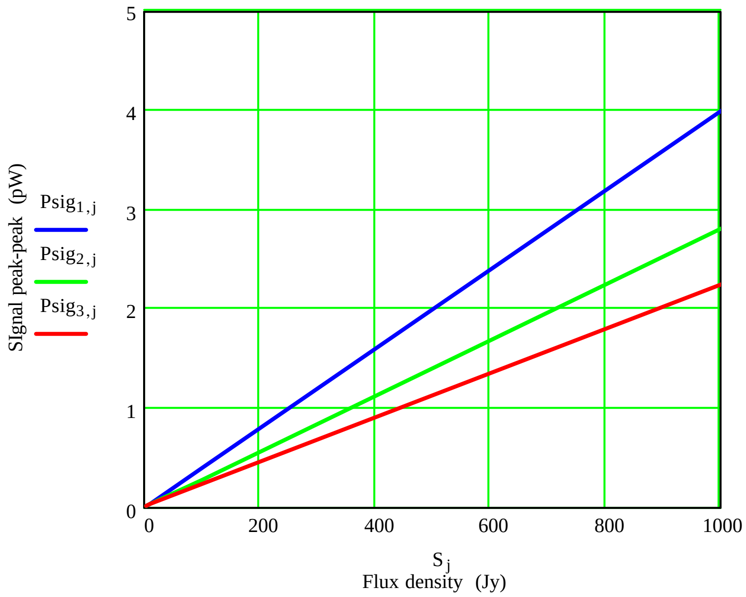
* Jamie assumed Δν = ν, leading to factor of three greater background so a factor of three greater sensitivity to telescope temperature fluctuations

Large astronomical signals

$j := 1, 2 .. 11 \quad S_j := 10^{j-7}$

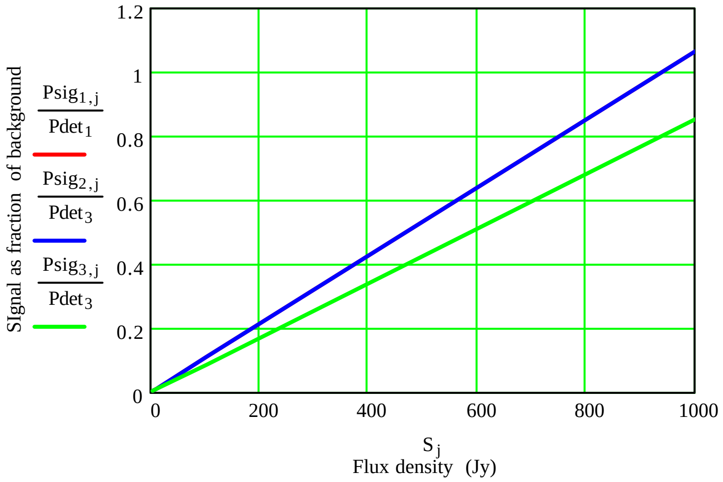
Signal power absorbed by detector

$Psig_{i,j} := S_j \cdot 10^{-26} \cdot 10^{12} \cdot \eta tel \cdot \eta feed_i \cdot Atel \cdot td_0 \cdot t_0 \cdot \Delta\nu_i$ pW



Typical 350-mm flux densities (factor of 2 higher or lower for 250 and 500 for planets):

OMC1	**1500**
W3(OH)	**680**
K3-50	**320**
W75N	**650**
Neptune	**100**
Uranus	**250**
Saturn	**7300**
Jupiter	**24000**



$Pdet_i =$	$\lambda_i =$
3.8	250
3.0	363
2.6	517

Comments and conclusions:

* We define a signal power equivalent to 20% of the background to correspond to a high level of loading (where one would expect corrections for detector non-linearity to become problematic).

* This corresponds to a flux densithy level of aroud 200 Jy for all bands. This means that Uranus will be rather too bright to use as a primary calibrator, but Neptune shold be usable.

* Sources in the 1E5 Jy range will result in signal powers more than 20 times the telescope background. This would completely swamp the detectors such that observations would be impossible to carry out or to calibrate.

* A reasonable limit to adopt at present to define observability is 1000 Jy. Sources near this limit will be difficult to calibrate (achievable accuracy will need to be assessed in ground calibration and PV phase).

Requirements for internal calibrator (PCAL)

Assumptions: Calibrator = black body located at centre of M4

M4 diameter and area (mm; mm^2) $DM4 := 26 \qquad AM4 := \frac{\pi \cdot DM4^2}{4}$

PCAL size and area (mm; mm^2) $DCal := 1 \qquad ACal := DCal^2$ Square emitting area assumed

Required instantaneous S/N $SN\_Req := 500$

Overall NEPs (W Hz-1/2 E-17) for the three bands, referred to power absorbed by detector)

$\lambda_i =$	$NEPtot_i =$
250	9.3
363	7.1
517	5.9

Required calibrator power absobed by detector (pW)

$$PCal\_Req_i := NEPtot_i \cdot SN\_Req \cdot 10^{-17} \cdot 10^{12}$$

$PCal\_Req_i =$
0.046
0.036
0.030

Effect of 8-dB Gaussian illumination profile on the pupil

The pupil is illuminated by the photometer feedhorns with a nominal edge taper of 8 dB. The telescope power received from the pupil is therefore weighted according to this illumination profile. PCAL is at the centre of the pupil, and so is viewed more efficiently than the pupil as a whole. Let PCAL be ascribed an efficiency of unity. The relative efficiency of the whole pupil illumination is calculated as follows.

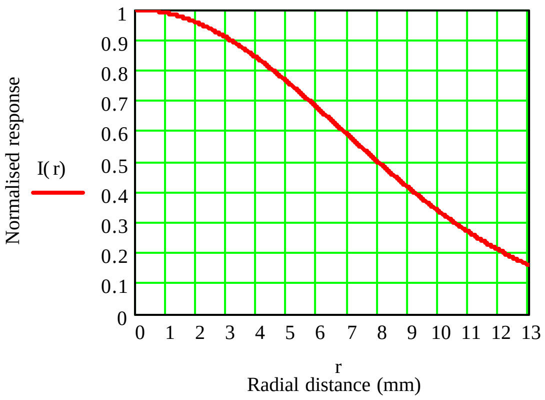
Pupil diameter and radius (mm) $D := 26 \qquad R := 0.5 \cdot D$

Pupil edge taper (dB and linear) $taper\_dB := 8 \qquad taper\_lin := 10^{\frac{taper\_dB}{10}} \qquad taper\_lin = 6.3$

Define Gaussian illumination pattern

$$ro := \frac{R}{\left(-\ln\left(\frac{1}{taper\_lin}\right)\right)^{0.5}} \qquad ro = 9.58 \qquad I(r) := \exp\left[-\left(\frac{r}{ro}\right)^2\right]$$

$$dB(r) := 10\log(I(r))$$



Relative efficiency factor for illumination of telescope pupil

$$\eta\_taper := \frac{\int_0^R I(x) \cdot 2 \cdot \pi \cdot x\,dx}{\pi \cdot R^2}$$

$$\eta\_taper = 0.457$$

Brightness temperature range for calibrator (K)

$k := 10, 11 .. 100 \qquad TCal_k := k$

Ratio at detector of calibrator power to telescope power

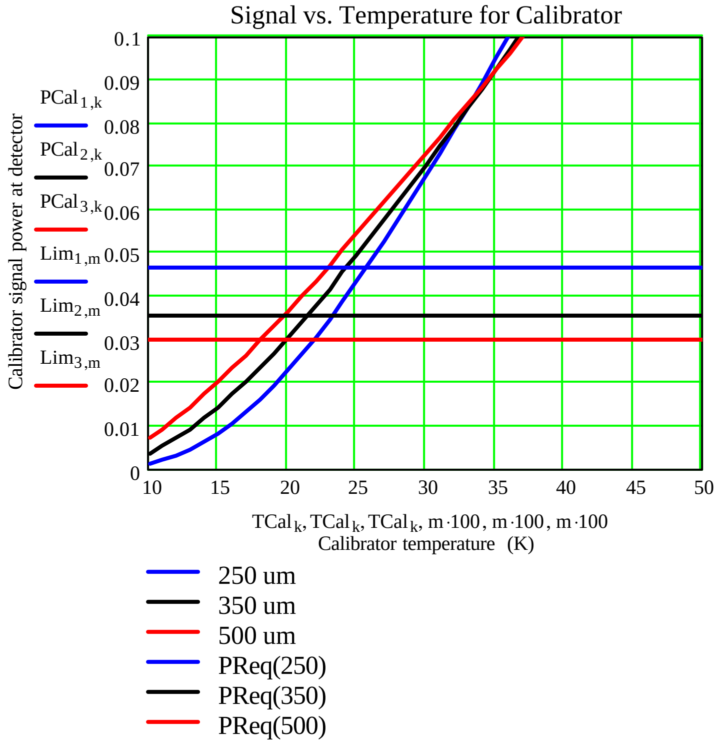
$$Ratio_{i,k} := \frac{ACal \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, TCal_k)\, d\nu}{AM4 \cdot \varepsilon tel \cdot \eta\_taper \cdot \int_{\nu L_i}^{\nu U_i} B(\nu, Ttel)\, d\nu}$$

Calibrator power at detector (pW)

$PCal_{i,k} := Ratio_{i,k} \cdot Pdet_i$

Plot calibrator power vs. calibrator temperature:

$m := 0, 1 .. 1 \qquad Lim_{i,m} := PCal\_Req_i$



Horizontal lines show the required powers for S/N = 500 in the three bands

A brightness temperature of 26 K provides S/N > 500 in all three bands. The figure is 29 K if the emitting area is circular with 1 mm diameter.

SPIRE FTS sensitivity model SPIRE_FTS_3_IBDR.mcd 22 January 2002

This version:

- * Prepared for SPIR IBDR
- * Filter bands updated to reflect filter DDR and finalised horn specifications as in ECR on BDA SSSD 11 October 2002
- * Feed and bolomneter parameters from *SPIRE-JPL-PRJ-000456 Working Version,* 7 November 2001

Previous versions:

SPIRE_FTS_1.MCD: 21 November 2000:

Version for Toledo meeting and System Review

- * Multi-moding of both SW and LW bands now taken into account
- * NEP contributions from each mode calculated separately and added in quadrature
- * NEP now referred to the power absorbed by the detector
- * Calculations done for the minimum and goal parameters of the detectors and feedhorns

SPIRE_FTS_2_IIDR.MCD: 6 April 2001:

- * Updated for IIDR

Constants: $h \equiv 6.626\cdot 10^{-34}$ $kb \equiv 1.3806\cdot 10^{-23}$

$origin := 1$ $c \equiv 2.998\cdot 10^{8}$

$b := 1, 2 .. 2$

Planck function:

$$B(\nu, T) := \frac{2\cdot h\cdot \nu^3}{c^2\cdot\left[\exp\left(\left(\frac{h\cdot\nu}{kb\cdot T}\right)\right) - 1\right]}$$

Assumptions

Telescope	Temp.	Emissivity	Diameter	Diameter of central obscuration	Focal ratio
	$\mathrm{Ttel} \equiv 80$	$\varepsilon\mathrm{tel} \equiv 0.04$	$\mathrm{Dtel} \equiv 3.285$	$\mathrm{Dobs} \equiv 0.56$	$\mathrm{Ftel} := 8.68$

Area $\mathrm{Atel} := 0.25\cdot\pi\cdot\left(\mathrm{Dtel}^2 - \mathrm{Dobs}^2\right)$ $\mathrm{Atel} = 8.2$

Percentage obscuration $\mathrm{Obs\_percent} := \left(1 - \dfrac{8.229}{8.475}\right)\cdot 100$

$\mathrm{Obs\_percent} = 2.9$

Observing efficiency $\eta\mathrm{obs\_m} \equiv 0.8$ (jiggle map)

Cold stop attenuation of telescope background: $\eta\mathrm{cs} := 0.8$

FTS efficiency	Observing efficiency	Elec. filter efficiency	Cos^2 modn efficiency
	$\eta\mathrm{obs} := 0.8$	$\eta\mathrm{elec} \equiv 0.8$	$\eta\mathrm{cosq} \equiv 0.5$

Bolometer and feedhorn properties (see BDA Subsystem Spec. Doc. SPIRE-JPL-PRJ-000456):

Overall optical efficiency of horn + bolometer combination $\eta\mathrm{feed\_min} := 0.45$ $\eta\mathrm{feed\_goal} := 0.75$ $\eta\mathrm{feed\_nom}_b :=$

$$\begin{array}{ll} \mathrm{LW} & \boxed{0.65} \\ \mathrm{SW} & \boxed{0.70} \end{array}$$

DQE of bolometer (wrt absorbed power)

$$\begin{array}{llll} & \mathrm{DQE\_min}_b := & \mathrm{DQE\_goal}_b := & \mathrm{DQE\_nom}_b := \\ \mathrm{LW} & 0.60 & 0.71 & 0.65 \\ \mathrm{SW} & 0.67 & 0.80 & 0.70 \end{array}$$

$\eta\mathrm{feed}_b := \eta\mathrm{feed\_nom}_b$

$\mathrm{DQE}_b := \mathrm{DQE\_nom}_b$

Beam divider reflection transmission, emissivity $\mathrm{tbd} \equiv 0.487$ $\mathrm{rbd} \equiv 0.487$ $\eta\mathrm{bd1} \equiv 2\cdot\mathrm{tbd}\cdot\mathrm{rbd}$ $\eta\mathrm{bd2} \equiv \mathrm{tbd}^2 + \mathrm{rbd}^2$

$\eta\mathrm{bd1} = 0.47$ $\eta\mathrm{bd2} = 0.47$ $\varepsilon\mathrm{bd} \equiv 1 - (\mathrm{tbd} + \mathrm{rbd})$ $\varepsilon\mathrm{bd} = 0.03$

Temperatures of filters $\mathrm{T4} \equiv 5$

Diffraction loss at each mirror $\mathrm{diffraction} \equiv 0.97$

Emissivity of each mirror $\varepsilon\mathrm{\_mirr} \equiv 1 - 0.995$

Effective transmission of each mirror $\mathrm{t\_mirr} \equiv 0.995$ $\mathrm{t\_mirr} = 0.99$

Overall diffraction loss $\mathrm{diff\_loss} := \mathrm{diffraction}^{11}$

$\mathrm{diff\_loss} = 0.72$ This is applied only to the signal, not to the background (see below)

Transmission, emissivity and temperature of optical elements $j \equiv 0, 1 .. 16 \quad k \equiv 0, 1 .. 17$

0 = Telescope
1 = CFI1 (4 K)
2 = CFIL2 (4 K)
3 = CIPM (M3)
4 = CBSM (M4)
5 = CRIM (M5)
6 = SPOM (M6)
7 = SIFM
8 = SIRM
9 = SBD_overall
10 = SCOM
11 = SRTM
12 = SDCM
13 = SBD2
14 = SCAM
15 = SFIL3 (2 K)
16 = Bandpass (0.3 K)
17 = Blocker (2 K)

$k =$	$t_k \equiv$	$T_k \equiv$	$\varepsilon_k \equiv$	$td_j =$
0.0	0.96	80	0.04	0.206
1.0	0.90	T4	0.1	0.229
2.0	0.9	T4	ε_mirr	0.255
3.0	t_mirr	T4	ε_mirr	0.256
4.0	t_mirr	T4	ε_mirr	0.257
5.0	t_mirr	T4	ε_mirr	0.258
6.0	t_mirr	T4	ε_mirr	0.260
7.0	t_mirr	T4	ε_mirr	0.261
8.0	t_mirr	T4	εbd	0.262
9.0	ηbd1	T4	ε_mirr	0.553
10.0	t_mirr	T4	2·ε_mirr	0.556
11.0	t_mirr^2	T4	ε_mirr	0.561
12.0	t_mirr	T4	εbd	0.564
13.0	1	T4	ε_mirr	0.564
14.0	t_mirr	T4	0.1	0.567
15.0	0.9	2	0.4	0.630
	0.7	0.3	0.1	
	0.9	0.3		

Transmission from element to detector

$$td_j \equiv \prod_{k=j+1}^{17} t_k$$

td is the transmission efficiency for the telescope background power. The diffraction loss factor is also applied below for the source signal.

Array parameters

Horn external diameter (µm) (internal diameters are 100 µm smaller)

$d\_horn_b :=$

LW	390
SW	225

The horns are thus 2Fλ at 225 and 390 µm.

Array side (centre-centre)

$$Warray_1 := 4 \cdot 2 \cdot \frac{d\_horn_1 \cdot 10^{-6}}{Dtel} \cdot \frac{360}{2 \cdot \pi} \cdot 60 \qquad Warray_2 := 6 \cdot 2 \cdot \frac{d\_horn_2 \cdot 10^{-6}}{Dtel} \cdot \frac{360}{2 \cdot \pi} \cdot 60$$

$Warray_b =$

LW	3.3
SW	2.8

arcmin

Bands

1. Old SPIRE bands

OldBands :=

Array	Design	Horn	Waveguide	Horn	Waveguide	Defocus	No. of	lL	lU	lo	Res.
	l	Length	Length	Dia.	Diameter		horns				
	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)					
S/SW	275	**TBD**	550	2.15	185	0	37	200	315	258	2.24
S/LW	450	45.36	900	3.78	393	0	19	305	670	488	1.34

2. New bands

NewBands :=

Array	Design	Horn	Waveguide	Horn	Waveguide	Defocus	No. of	lL	lU	lo	Res.
	l	Length	Length	Dia.	Diameter		horns				
	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)					
S/SW	275	23.68	575	2.15	190	0	37	190	325	258	1.91
S/LW	450	46.36	925	3.80	393	0	19	300	670	485	1.31

Note: Waveguide diameter is given by λL*(1.841/π)

Select new or old bands for calculation (Index = 1 for new and 0 for old) $\text{Index} := 1$

Extract band limits from the tables above

$$\lambda L_2 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{3,8}, \text{OldBands}_{3,8}\right) \qquad \lambda L_2 = 190$$

$$\lambda U_2 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{3,9}, \text{OldBands}_{3,9}\right) \qquad \lambda U_2 = 325$$

$$\lambda L_1 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{4,8}, \text{OldBands}_{4,8}\right) \qquad \lambda L_1 = 300$$

$$\lambda U_1 := \text{if}\left(\text{Index} = 1, \text{NewBands}_{4,9}, \text{OldBands}_{4,9}\right) \qquad \lambda U_1 = 670$$

Crossover wavelength (μm)

$$\lambda\_\text{cross} := 0.5 \cdot \left(\lambda L_1 + \lambda U_2\right) \qquad \lambda\_\text{cross} = 313$$

Waveguide radii (µm) $\quad roS := \frac{\lambda U_2 \cdot 1.841}{2 \cdot \pi} \qquad roL := \frac{\lambda U_1 \cdot 1.841}{2 \cdot \pi}$

Waveguide diameter (µm) $\quad doS := 2 \cdot roS \quad doS = 190 \qquad doL := 2 \cdot roL \quad doL = 393$

Band limits (cm-1) $\quad \sigma L_1 := \frac{10000}{\lambda U_1} \quad \sigma U_1 := \frac{10000}{\lambda L_1} \qquad \sigma L_2 := \frac{10000}{\lambda U_2} \quad \sigma U_2 := \frac{10000}{\lambda L_2}$

Band limits (mm and Hz) $\quad \nu L_b := c \cdot \sigma L_b \cdot 100 \qquad \nu U_b := c \cdot \sigma U_b \cdot 100$

Band centre (mm and Hz) $\quad \nu 0_b := \frac{\nu L_b + \nu U_b}{2} \qquad \lambda 0_b := \frac{c \cdot 10^6}{\nu 0_b}$

Band λ/Δλ $\quad R_b := \frac{\sigma U_b + \sigma L_b}{2 \cdot (\sigma U_b - \sigma L_b)}$

Band limits (mm and THz)

	$\lambda L_b =$	$\lambda 0_b =$	$\lambda U_b =$	$\sigma L_b =$	$\sigma U_b =$	$\nu L_b \cdot 10^{-12} =$	$\nu 0_b \cdot 10^{-12} =$	$\nu U_b \cdot 10^{-12} =$	$R_b =$
LW	300	414	670	14.9	33.3	0.45	0.72	1.00	1.31
SW	190	240	325	30.8	52.6	0.92	1.25	1.58	1.91

Aperture and cavity efficiencies

Jason Glenn's caluclations (as summarised in his note of July 2 2001):

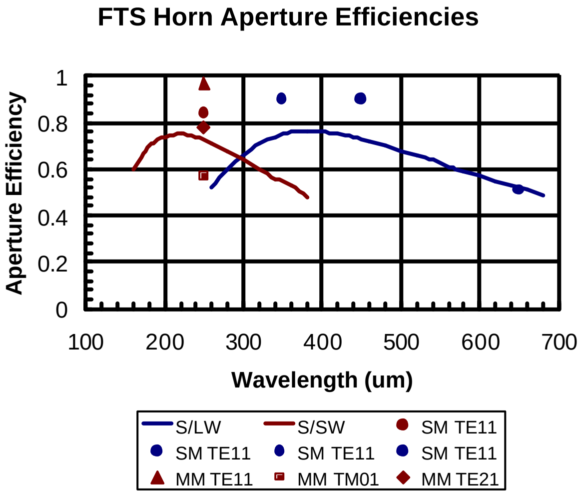
Efficiencies :=

Worksheet

$o := 0, 1 .. 11 \qquad p := 0, 1 .. 21$

$\lambda\_SW := Efficiencies^{\langle 0 \rangle} \qquad \eta app\_SW := Efficiencies^{\langle 1 \rangle}$

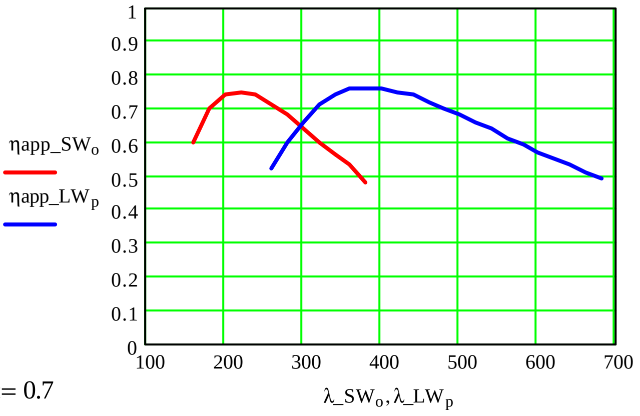
$\lambda\_LW := Efficiencies^{\langle 2 \rangle} \qquad \eta app\_LW := Efficiencies^{\langle 3 \rangle}$



Telescope coupling efficiency (point source)

Assume this is the same as the horn aperture efficiency

Simplified wavelength dependence:
- * Constant at 0.7 from 200 - 500 µm
- * Declines linearly from 0.7 to 0.5 from 500 to 670 µm (factor of 0.7/0.5 = 1.4 assumed below for degradation in sensitivity)



$\eta tel := 0.7$

Background power levels on the detectors and photon noise limited NEPs

Assumptions:

1. All modes carry equal background power (per unit bandwidth) from the telescope

2. Calibrator contributes same amount of power as the telescope

3. Efficiencies of higher order modes are as in Jason's note (independent of crossover wavelength)

Throughput per mode $A\Omega(\nu) := \left(\frac{c}{\nu}\right)^2 \cdot \eta cs$

SW horn (multimoded)

$$\eta\_SW\_TE11(\nu) := 0.84$$

$$\eta\_SW\_TM01(\nu) := 0.57$$

$$\eta\_SW\_TE21(\nu) := 0.78$$

These values are used in calculating the background, but a pessimistic value of ηfeed_nom is assumed for TE11 in calculating the NEFD

$$\eta feed\_nom = \begin{pmatrix} 0.0 \\ 0.7 \\ 0.7 \end{pmatrix}$$

LW horn (multi-moded)

Fundamental mode efficiency vs. frequency

$$q := 0, 1 .. 2$$

$$\lambda LW_q := \begin{array}{|c|}\hline 650 \\ \hline 450 \\ \hline 350 \\ \hline \end{array} \qquad \nu LW_q := \frac{c \cdot 10^6}{\lambda LW_q} \qquad \eta LW_q := \begin{array}{|c|}\hline 0.51 \\ \hline \eta feed\_nom_1 \\ \hline \eta feed\_nom_1 \\ \hline \end{array} \qquad \eta LW_q = \begin{array}{|c|}\hline 0.51 \\ \hline 0.65 \\ \hline 0.65 \\ \hline \end{array}$$

$$\eta feed\_nom_1 = 0.65$$

$$\eta\_LW\_TE11(nu) := \mathrm{linterp}(\nu LW, \eta LW, nu) \qquad \eta\_LW\_TE11\left(\frac{c \cdot 10^6}{650}\right) = 0.51$$

LW horn higher order modes:
(not modelled yet so assume 50%)

$$\eta\_LW\_higher := 0.5$$

SW band (b = 2)

Cut-off wavelength for TE11 mode

$$\lambda cSW := \lambda U_2 \qquad \lambda cSW = 325 \qquad \nu o\_TE11_2 := 0.5 \cdot (\nu L_2 + \nu U_2)$$

Waveguide radius (µm)

$$roS = 95$$

Cut-off wavelengths of higher modes

$$\lambda c\_TM01 := \frac{2 \cdot \pi \cdot roS}{2.405} \qquad \lambda c\_TM01 = 249 \qquad \nu c\_TM01 := \frac{c \cdot 10^6}{\lambda c\_TM01} \qquad \nu o\_TM01_2 := \frac{\nu c\_TM01 + \nu U_2}{2}$$

$$\lambda c\_TE21 := \frac{2 \cdot \pi \cdot roS}{3.054} \qquad \lambda c\_TE21 = 196 \qquad \nu c\_TE21 := \frac{c \cdot 10^6}{\lambda c\_TE21} \qquad \nu o\_TE21_2 := \frac{\nu c\_TE21 + \nu U_2}{2}$$

Power absorbed by detector from each element and for each mode (pw)

Note: 1. Factor of 2 accounts for same background from calib. source in 2nd port
2. Power is set to zero if the mode is not propagated

TE11
$$P\_TE11_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_2 \cdot 10^{12} \cdot \int_{\nu L_2}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_SW\_TE11(\nu)\, d\nu$$

$$P\_TE11_{2,j} := \mathrm{if}\left(P\_TE11_{2,j} < 0, 0, P\_TE11_{2,j}\right) \qquad Power\_TE11_2 := \sum_{n=0}^{9} P\_TE11_{2,n}$$

TM01
$$P\_TM01_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_2 \cdot 10^{12} \cdot \int_{\nu c\_TM01}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_SW\_TM01(\nu)\, d\nu$$

$$P\_TM01_{2,j} := \mathrm{if}\left(P\_TM01_{2,j} < 0, 0, P\_TM01_{2,j}\right) \qquad Power\_TM01_2 := \sum_{n=0}^{9} P\_TM01_{2,n}$$

TE21
$$P\_TE21_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_2 \cdot 10^{12} \cdot \int_{\nu c\_TE21}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_SW\_TE21(\nu)\, d\nu$$

$$P\_TE21_{2,j} := \mathrm{if}\left(P\_TE21_{2,j} < 0, 0, P\_TE21_{2,j}\right) \qquad Power\_TE21_2 := \sum_{n=0}^{9} P\_TE21_{2,n}$$

NEPph contributions from each mode

TE11
$$NEPph\_TE11_2 := \left(2 \cdot Power\_TE11_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TE11_2\right)^{0.5} \cdot 10^{17}$$

TM01
$$NEPph\_TM01_2 := \left(2 \cdot Power\_TM01_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TM01_2\right)^{0.5} \cdot 10^{17}$$

TE21
$$NEPph\_TE21_2 := \left(2 \cdot Power\_TE21_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TE21_2\right)^{0.5} \cdot 10^{17}$$

Summary of power (pW)
and NEPph contributions
(W Hz-1/2 E-17) in SW band

$$Power_{2,j} := P\_TE11_{2,j} + P\_TM01_{2,j} + P\_TE21_{2,j}$$

$$NEPph_2 := \left[\left(NEPph\_TE11_2\right)^2 + \left(NEPph\_TM01_2\right)^2 + \left(NEPph\_TE21_2\right)^2\right]^{0.5}$$

$$Ptot_2 := Power\_TE11_2 + Power\_TM01_2 + Power\_TE21_2$$

LW band (b = 1)

Cut-off wavelength for TE11 mode (μm) $\lambda cLW := \lambda U_1$ $\lambda cLW = 670$

Waveguide radius (mm) $roL = 196$ $\nu o\_TE11_1 := 0.5 \cdot (\nu L_1 + \nu U_1)$

Cut-off wavelengths of higher modes

$\lambda c\_TM01 := \frac{2 \cdot \pi \cdot roL}{2.405}$ $\lambda c\_TM01 = 513$ $\nu c\_TM01 := \frac{c \cdot 10^6}{\lambda c\_TM01}$ $\nu o\_TM01_1 := \frac{\nu c\_TM01 + \nu U_1}{2}$

$\lambda c\_TE21 := \frac{2 \cdot \pi \cdot roL}{3.054}$ $\lambda c\_TE21 = 404$ $\nu c\_TE21 := \frac{c \cdot 10^6}{\lambda c\_TE21}$ $\nu o\_TE21_1 := \frac{\nu c\_TE21 + \nu U_1}{2}$

$\lambda c\_TE01 := \frac{2 \cdot \pi \cdot roL}{3.832}$ $\lambda c\_TE01 = 322$ $\nu c\_TE01 := \frac{c \cdot 10^6}{\lambda c\_TE01}$ $\nu o\_TE01_1 := \frac{\nu c\_TE01 + \nu U_1}{2}$

$\lambda c\_TE31 := \frac{2 \cdot \pi \cdot roL}{4.201}$ $\lambda c\_TE31 = 294$ $\nu c\_TE31 := \frac{c \cdot 10^6}{\lambda c\_TE31}$ $\nu o\_TE31_1 := \frac{\nu c\_TE31 + \nu U_1}{2}$

TE11 $P\_TE11_{1,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu L_1}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu) \cdot \eta\_LW\_TE11(\nu)\, d\nu$

$P\_TE11_{1,j} := if(P\_TE11_{1,j} < 0, 0, P\_TE11_{1,j})$ $Power\_TE11_1 := \sum_{n=0}^{9} P\_TE11_{1,n}$

TM01 $P\_TM01_{1,j} := \eta\_LW\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu c\_TM01}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$

$P\_TM01_{1,j} := if(P\_TM01_{1,j} < 0, 0, P\_TM01_{1,j})$ $Power\_TM01_1 := \sum_{n=0}^{9} P\_TM01_{1,n}$

TE21 $P\_TE21_{1,j} := \eta\_LW\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu c\_TE21}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$

$P\_TE21_{1,j} := if(P\_TE21_{1,j} < 0, 0, P\_TE21_{1,j})$ $Power\_TE21_1 := \sum_{n=0}^{9} P\_TE21_{1,n}$

TE01 $P\_TE01_{1,j} := \eta\_LW\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed_1 \cdot 10^{12} \cdot \int_{\nu c\_TE01}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$

$P\_TE01_{1,j} := if(P\_TE01_{1,j} < 0, 0, P\_TE01_{1,j})$ $Power\_TE01_1 := \sum_{n=0}^{9} P\_TE01_{1,n}$

NEPph contributions from each mode

TE11 $NEPph\_TE11_1 := \left(2 \cdot Power\_TE11_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE11_1\right)^{0.5} \cdot 10^{17}$

TM01 $NEPph\_TM01_1 := \left(2 \cdot Power\_TM01_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TM01_1\right)^{0.5} \cdot 10^{17}$

TE21 $NEPph\_TE21_1 := \left(2 \cdot Power\_TE21_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE21_1\right)^{0.5} \cdot 10^{17}$

TE01 $NEPph\_TE01_1 := \left(2 \cdot Power\_TE01_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE01_1\right)^{0.5} \cdot 10^{17}$

Summary of power (pW) and NEPph contributions (W Hz-1/2 E-17) in LW band

$$Power_{1,j} := P\_TE11_{1,j} + P\_TM01_{1,j} + P\_TE21_{1,j} + P\_TE01_{1,j}$$

$$NEPph_1 := \left[\left(NEPph\_TE11_1\right)^2 + \left(NEPph\_TM01_1\right)^2 + \left(NEPph\_TE21_1\right)^2 + \left(NEPph\_TE01_1\right)^2\right]^{0.5}$$

$$Ptot_1 := Power\_TE11_1 + Power\_TM01_1 + Power\_TE21_1 + Power\_TE01_1$$

Summary

SW band

$Power\_TE11_2 = 7.6$ $NEPph\_TE11_2 = 11.2$

$Power\_TM01_2 = 2.8$ $NEPph\_TM01_2 = 7.2$

$Power\_TE21_2 = 0.5$ $NEPph\_TE21_2 = 3.1$

$Ptot_2 = 10.8$ $NEPph_2 = 13.6$

LW band

$Power\_TE11_1 = 5.2$ $NEPph\_TE11_1 = 7.1$

$Power\_TM01_1 = 3.1$ $NEPph\_TM01_1 = 5.7$

$Power\_TE21_1 = 1.9$ $NEPph\_TE21_1 = 4.6$

$Power\_TE01_1 = 0.5$ $NEPph\_TE01_1 = 2.5$

$Ptot_1 = 10.6$ $NEPph_1 = 10.5$

Note that total power is dominated by the telescope contribution

SW band	LW band
$Power_{2,j} =$	$Power_{1,j} =$
10.8	10.6
$4.6\cdot10^{-4}$	0
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$2.6\cdot10^{-5}$	$1.1\cdot10^{-3}$
$1.4\cdot10^{-4}$	$5.7\cdot10^{-3}$
$5.5\cdot10^{-5}$	$2.3\cdot10^{-3}$
$1.1\cdot10^{-4}$	$4.7\cdot10^{-3}$
$5.6\cdot10^{-5}$	$2.3\cdot10^{-3}$
$2.9\cdot10^{-4}$	0
$5.7\cdot10^{-5}$	$2.4\cdot10^{-3}$
$1.1\cdot10^{-3}$	0
$3.1\cdot10^{-9}$	$8.5\cdot10^{-5}$

Photon noise levels and single-detector NEFD

Overall NEP
(W Hz-1/2 x 10-17)

$$\mathrm{NEPtot}_b := \frac{\mathrm{NEPph}_b}{\left(\mathrm{DQE}_b\right)^{0.5}}$$

referred to the power absorbed by the detector

Detector NEP
(W Hz-1/2 x 10-17)

$$\mathrm{NEPdet}_b := \left[\left(\mathrm{NEPtot}_b\right)^2 - \left(\mathrm{NEPph}_b\right)^2\right]^{0.5}$$

NEFD (Jy Hz-1/2)

Diffraction loss is taken into account here.

$$\mathrm{NEFD}_b := \frac{\mathrm{NEPtot}_b \cdot 10^{-17} \cdot 10^{26}}{\eta\mathrm{elec} \cdot \eta\mathrm{cosq} \cdot \eta\mathrm{tel} \cdot \mathrm{Atel} \cdot \mathrm{td}_0 \cdot \Delta\nu \cdot t_0 \cdot \eta\mathrm{feed}_b \cdot \mathrm{diff\_loss}}$$

$$\mathrm{NEFD}_b = \begin{array}{|c|}\hline 2.04 \\ \hline 2.38 \\ \hline \end{array}$$

Note: this is pessimistic in that the higher order modes are assumed to couple to the telescope background but not to the source - ηtel is taken to be the same value as for single-mode coupling

Point source observation

Spectral resolution (cm-1 and Hz) $\Delta\sigma \equiv 1$ $\Delta\nu \equiv c \cdot \Delta\sigma \cdot 100$

Limiting flux density
(mJy 5-σ 1-hr)

$$\mathrm{Slim}_b := \frac{1000 \cdot \mathrm{NEFD}_b \cdot 5}{\left(2 \cdot 3600 \cdot \eta\mathrm{obs}\right)^{0.5}}$$

Limiting line strength
(mJy 5-σ 1-hr)

$$\mathrm{Flim}_b := \left(\frac{\mathrm{Slim}_b \cdot 10^{-26}}{1000} \cdot \Delta\nu\right)$$

Deep mapping of one field for 1 hour:

Loss in S/N for point source due to need to make a map:

S/N improvement through co-addition of pixels $\mathrm{SN\_imp} := 1.5$

S/N reduction through decrease in integration time per point by factor of 16 $\mathrm{SN\_red} := 4$

Overall reduction in S/N

$$\mathrm{factor} := \frac{\mathrm{SN\_imp}}{\mathrm{SN\_red}} \qquad \mathrm{factor} = 0.375$$

Limiting flux density (mJy)

$$\Delta\mathrm{S\_1hr}_b := \frac{\mathrm{Slim}_b}{\mathrm{factor}} \qquad \Delta\mathrm{F\_1hr}_b := \frac{\mathrm{Flim}_b}{\mathrm{factor}}$$

Band centre and edges: wavlengths and resolving powers

$$\mathrm{ResL}_b := \frac{\nu\mathrm{U}_b}{\Delta\nu} \qquad \mathrm{Res0}_b := \frac{\nu 0_b}{\Delta\nu} \qquad \mathrm{ResU}_b := \frac{\nu\mathrm{L}_b}{\Delta\nu}$$

	$\lambda\mathrm{L}_b =$	$\mathrm{ResL}_b =$	$\lambda 0_b =$	$\mathrm{Res0}_b =$	$\lambda\mathrm{U}_b =$	$\mathrm{ResU}_b =$
LW	300.0	33.3	414.4	24.1	670.0	14.9
SW	190.0	52.6	239.8	41.7	325.0	30.8

Summary:

Crossover wavelength $\lambda\_cross = 313$ $\Delta\sigma = 1.0$

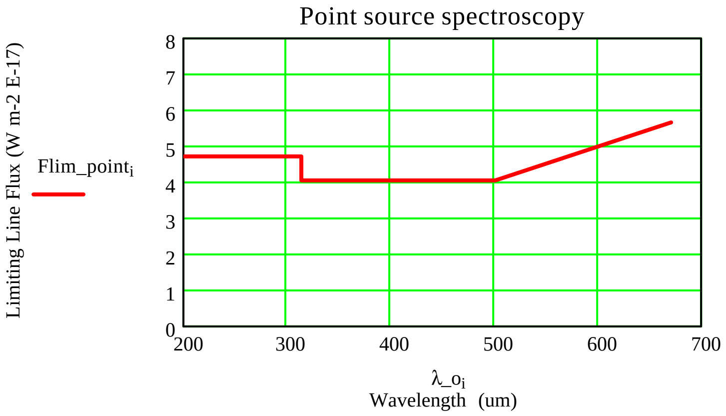
	Pdet absorbed (pW)	**NEPs (W Hz-1/2 E-17)**	
	$Ptot_b =$	$NEPph_b =$	$NEPtot_b =$
LW	10.6	10.5	13.0
SW	10.8	13.6	16.3

	Spectrophotometry (mJy 5-s;1-hr) Point source	Map	**Spectroscopy (W m-2 5-s;1-hr)** Point source	Map
	$Slim_b =$	$\Delta S\_1hr_b =$	$Flim_b \cdot 10^{17} =$	$\Delta F\_1hr_b \cdot 10^{17} =$
LW	135	359	4.0	10.8
SW	157	418	4.7	12.5

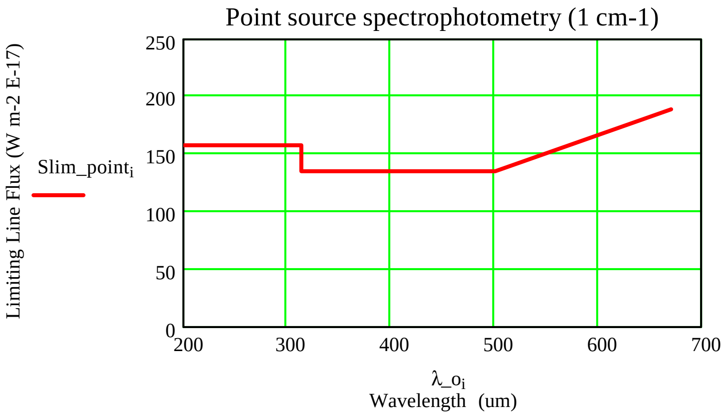
$i := 0, 1 .. 4$

$$\lambda\_o_i := \begin{pmatrix} \lambda L_2 \\ \lambda\_cross - 0.1 \\ \lambda\_cross + 0.1 \\ 500 \\ 670 \end{pmatrix} \quad Flim\_point_i := \begin{pmatrix} Flim_2 \cdot 10^{17} \\ Flim_2 \cdot 10^{17} \\ Flim_1 \cdot 10^{17} \\ Flim_1 \cdot 10^{17} \\ Flim_1 \cdot 10^{17} \cdot 1.4 \end{pmatrix} \quad Slim\_point_i := \begin{pmatrix} Slim_2 \\ Slim_2 \\ Slim_1 \\ Slim_1 \\ Slim_1 \cdot 1.4 \end{pmatrix}$$

$\lambda\_o_i =$	$Flim\_point_i =$	$Slim\_point_i =$
190	4.7	157
312	4.7	157
313	4.0	135
500	4.0	135
670	5.6	188



Factor of 0.7/0.5 = 1.4 degradation in telescope coupling efficiency between 500 and 670 µm



Notes:
1. Limiting flux density Slim is inversely proporotional to spectral resolution (Δσ) and independent of wavelength except for LW band longer than 500 µm
2. For an unresolved line, limiting line flux Flim is independent of spectral resolution and wavelength within the SW band. In the LW band it is constant up to 500 µm.
3. Beyond 500 µm, sensitivity declines linearly, by a factor of 5/7 out to 670 µm