SPIRE	Technical Note Feedhorn focus positions	Ref: Note RAL-NOT-000566 Issue: 1 Date: 10-1-2001 Page: 1 of 10

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CONTENTS.

Summary.
We consider the final selection of SPIRE focus, which is set by the axial positioning of the feed-horn apertures (mouths) in the optical system. The focus criterion is that of maximising the point-source aperture efficiency. This efficiency , and its sensitivity to defocus, is calculated for the actual beam pattern of the smooth-wall horn (TE11 mode), using numerical analysis. The previous gaussian beam analysis is also included, for reference & as a check of the real-beam behaviour.

The results for recommended defocus 'dz' relative to the geometric optics focus are :-

Photometer.

Channel (centre wavelength)	250um	350um	500um
Effective length	22.68	45.36	45.36
Gaussian beam defocus dz (dimension 'a')	-3.99	-4.44	-8.54
Recommended dz	-1.6	-2*	-4

* inferred from the trends of the other 2 channels

The effect of not applying these defocus changes (i.e. making dz=0), would be a loss of ~1% in each case.

FTS

Channel (centre wavelength)	250um		500um
Effective length	22.68		45.36
Gaussian beam defocus dz (dimension ‘a’)	-3.99		-8.54
Recommended dz	0 *		0 *

The proposed FTS defocus is zero because (a) the non-telecentricity would otherwise cause a change in optical design or plate scale (b) the loss of single-mode efficiency for dz=0 is small at < 1%.
The effect of multi-moding is not included here, but is known to also drive the system behaviour more towards the GO case , i.e. to dz=0. We recommend that whatever tolerance can be allowed for dz is retained , e.g. an adjustment range of dz = + - 1mm.

1. Introduction.

In SPIRE the focus is to be optimised for detection efficiency of a point-source, and this is formally expressed in terms of the *aperture efficiency* of the point source *signal beam* (Airy pattern at the image plane) with the *detector beam* (the feed-horn’s antenna pattern, ref.1 & 2).

In the short-wavelength (geometric optics, GO) limit the optimum axial position for the GO focus is at the horn entrance aperture. At long-wavelength two effects lead to this optimum shifting some distance into the horn. These are :

1. Horn antenna pattern shape. As this is approximately gaussian, it has been modelled using gaussian beam imaging (N-0430). If this is done through the whole optical chain then it includes the effect of the powered field mirrors (for pupil imaging), which are present in SPIRE (N-0269 iss.5).
2. The finite horn length, which adds some wavefront curvature to the horn beam pattern at its entrance aperture.

The currently defined horn focus positions relative to GO (dimension ‘*a*’ in feedhorn meeting minutes ref.3) were created from a parametric gaussian-beam analysis of the feedhorns (with differing results for the smooth-wall and corrugated designs), and the details of this have been reproduced in N-0528. In that analysis the optical system isn’t included, and it was only the finite-length effect 2 above that led to non-zero values of *a*.

We have previously also investigated the optimum focusing & aperture efficiencies using the real detector beam (horn aperture) patterns in numerical calculations. This was mainly needed to allow the different horn designs (including lengths) to be compared, and especially for the longer wavelengths (e.g. N-0316).

In this note the numerical method is used to determine focus for the photometer (single mode) SW & LW channels, and the gaussian beam analysis is maintained for comparison purposes.

2. Comparison of SPIRE & gaussian beams.

In the analytic treatment of ref.1, the gaussian detector beam is considered coupling to the point-source beam, at the system pupil (in ref.1 this is a cassegrain system secondary mirror). It is found that an optimum aperture efficiency of 81% occurs when at this pupil : (a) the beams' wavefront curvatures are matched and (b) the ratio of gaussian 1/e radius w to pupil edge radius r is $w/r \sim 1/1.1$.

The 2^{nd} condition is close to that of a pupil edge-taper of 1/e , and to achieve this the 1/e diameter of the detector beam should be

$$2w_o = (2/\pi).2F\lambda_o \qquad (1)$$

In SPIRE, with the horn apertures sized to

$$D \sim 2F\lambda_o \qquad (2)$$

(λ_o = centre wavelength). The **corrugated** single-mode pattern (HE11) has a best-fit gaussian with a 1/e diameter slightly smaller than the optimum of equation (1). As a result the efficiency peaks at a somewhat lower wavelength than the channel centre. The peak is also higher than the pure gaussian case, at 83% cf. 81%.

For the **smooth-wall** single-mode horn (TE11) the efficiency peaks close to the centre wavelength (gaussian beam model), but with a lower value (80% cf. 81%). Here the best-fit gaussian is more difficult to consider because the mode is quite asymmetric. The pupil edge taper is 25% and 12% in the symmetry-plane directions, i.e. higher & lower than the optimum 1/e = 13.5 %. Therefore if the edge-taper were averaged around the pupil rim it would be relatively close to the optimum value. This is probably why the peak efficiency remains centred on λ_o despite the non-gaussian shape.

Since the smooth-wall case is the likely horn choice for SPIRE, and its efficiency behaviour isn't too far removed from that of a gaussian detector beam of size optimised according to equation (1), it is then justifiable to use the gaussian beam analysis of ref.1 to model the SPIRE focus effects.

3. Defocus for Gaussian beams at L=¥.

The optimum position for the horn mouth (infinite length horn) is to be taken as the waist position of the gaussian beam which is

1. Matched to the system pupil as per ref.1.
2. wavelength λ_o.

The pupil chosen to define this gaussian is the telescope secondary. This is done because we can then propagate the beam through the whole system , and so investigate the effects of field mirrors. In fact it is found that the same result is obtained if the analysis is made using the system exit pupil, and that this alternative should correctly include the field mirror effects, since the field mirror determines the location of this pupil. However in SPIRE the exit pupil positions are not all well-defined, e.g. in PHOT it is tilted & aberrated.

The analysis uses the 'unfolded' form of the optics, so that each powered element can be represented as an ideal thin-lens. The effective focal lengths for these lenses are calculated from the distances of image-planes from & to the relevant mirror in the optical design. These distances and the lens separations are measured along the on-axis chief-ray for each instrument (system files sp460b.inr, bolpht153.inr). The focal length for the field mirror M6 is likewise calculated from the distance from &

to the relevant pupil planes. The exact focal lengths & separations are not critical because we are not calculating the absolute focus position, only the position relative to that of GO.

The important parameter for defining the optimum gaussian beam is the secondary mirror aperture size. We determine this from the telescope f-number F=8.68, plus the distance of the M2 pole from the focal surface FP along SPIRE's on-axis chief-ray D_{M2FP}=2613.31mm. This gives

Pupil radius $r=D_{M2FP}/(2F)$
$= 150.5\text{mm}$

Optimum $w_s/r = \sqrt{1.2}$
$\Rightarrow\ w_s = 137.4\text{mm}$

Since all beams pass through the centre of M2, the M2 focal length is taken as equal to half its ROC at the pole, whereas for M1 an effective focal length is derived based on its distance from M2 along the chief ray. This gives f=1750.45mm as compared to the value f=1750.0mm if SPIRE were to lie along the telescope axis.

The tables below show the gaussian beam parameters throughout each instrument for this choice of beam size (the 1/e radius at the secondary mirror is highlighted in the last column), at the longest centre-wavelength λ_o=0.5mm.

The optimum value for the defocus dimension '*a*' can be determined by comparing the GO image position and the beam image (waist) position. These items are highlighted for each image plane, and the values at the final powered optic are differenced to give the parameter dz reported on the last line.

mode_im3.pro								
instrument: PHOT			**wavelength: 0.500000mm**					
lens name	**axial-posn**	**focal length**	**GO image posn**	**roc before lens R1**	**roc after lens R2**	**beam image posn**	**1/e radius at waist**	**1/e radius at lens**
object	-1000000000000.00	0.00	-1000000000000.00	0.00	0.00	-4201.83	1485.00	0.00
PM	-4201.83	1750.45	-2451.38	-1000000000000.00	1750.45	-2451.38	0.19	1485.00
SM	-2613.31	-172.63	-0.06	161.94	-83.56	-1.25	3.03	137.38
FP	0.00	1000000000000.00	-0.06	-0.06	0.06	-1.25	3.03	3.03
m3	97.30	204.26	-88.72	-97.36	65.93	-2.85	5.14	6.00
m5	508.20	146.17	701.78	-596.93	117.42	702.91	1.87	16.65
m6	701.80	-153.75	701.78	-0.02	0.02	699.76	1.87	1.88
M7	904.69	152.19	1513.49	-202.91	86.96	1431.33	4.98	17.56
M8	1092.69	-106.74	949.67	420.80	-85.14	952.07	1.90	11.92
M9	1292.69	165.55	1612.68	-343.02	111.66	1612.20	1.78	28.56
Det	1612.68	0.00	0.00	0.00	0.00	0.00	0.00	1.78
dz	-0.479004	final waist	posn from GO focus (beam image)-(detr posn)					

mode_im3.pro								
instrument: FTS			wavelength: 0.500000mm					
lens name	**axial-posn**	**focal length**	**GO image posn**	**roc before lens R1**	**roc after lens R2**	**beam image posn**	**1/e radius at waist**	**1/e radius at lens**
object	-1000000000000.00	0.00	-1000000000000.00	0.00	0.00	-4201.83	1485.00	0.00
PM	-4201.83	1750.45	-2451.38	-1000000000000.00	1750.45	-2451.38	0.19	1485.00
SM	-2613.31	-172.63	-0.06	161.94	-83.56	-1.25	3.03	137.38
FP	0.00	1000000000000.00	-0.06	-0.06	0.06	-1.25	3.03	3.03
m3	94.76	196.21	-88.73	-94.82	63.93	2.10	5.14	5.89
m5	512.96	143.40	701.23	-601.70	115.80	702.84	1.83	16.63
m6s	701.25	250.00	701.23	-0.02	0.02	704.55	1.81	1.83
Rin	900.96	100.26	1102.27	-199.72	66.75	1101.17	1.85	17.36
Coll	1250.28	148.22	-104819.00	-148.01	74.06	1441.00	12.77	12.99
Cam	1644.92	148.22	1793.34	-106464.00	148.01	1794.29	1.84	13.02
Rout	1994.29	100.24	2194.29	-200.95	66.88	2190.83	1.81	17.36
Det	2194.29	0	0	0	0	0	0	1.8375
dz	-3.46533	final waist	posn from GO focus (beam image)-(detr posn)					

Table 1. Dimensions of unfolded SPIRE optics systems and gaussian-beam parameters (all dimensions are mm), listed in order of passage from the ‘object’ at axial position= minus infinity, through to the LW detector. The position origin is at the FP.

Notes: Only powered elements are listed, except for FP and Det.

Repeating this analysis for all channels in the instrument, the following results are obtained. NB for the beam 1/e size constant at the telescope secondary, it changes with wavelength at the other components.

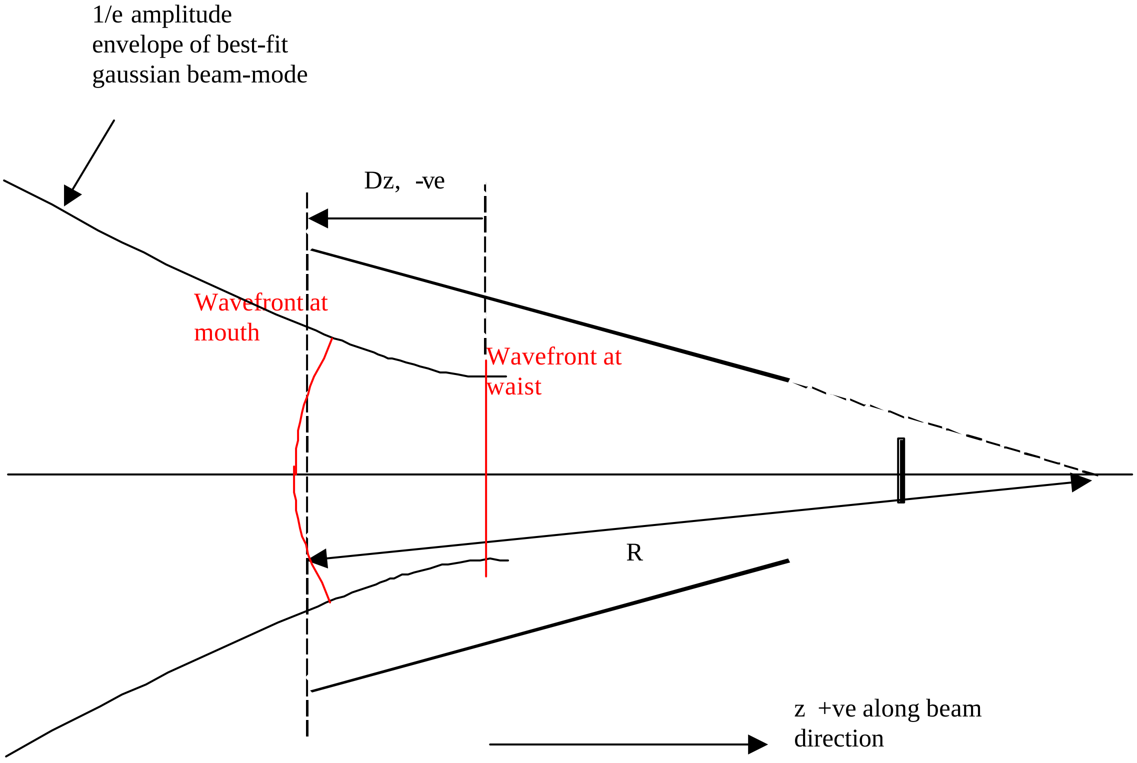
Channel centre wavelengths	250um	350um	500um
PHOT dz values	-0.12	-0.23	-0.48
FTS dz values	-0.88	-	-3.46

Table 2. Optimum positions dz for gaussian detector beam, from GO focus. This corresponds to optimum horn mouth position in case of L=∞.

It is useful to see how sensitive the efficiency is to this focussing. For the FTS long-wavelength case when the dz is not applied, the beam mis-match at the FTS exit pupil is ~3.5mm in wavefront ROC (i.e. approx equal to dz), and ~3% in beam 1/e size. The efficiency is then 80 % as compared to the optimum 81 %. Therefore if this defocus is not implemented the loss of efficiency is by only 1 %.

4. Effect of finite horn length L.

The finite length L adds to the antenna pattern at the mouth of the horn a wavefront curvature, of radius approx. equal to L (N-0528). For a gaussian beam this would be equivalent to a shift in the waist position into the horn as shown in fig.1, plus a small reduction in waist size.



This effect can be calculated analytically, and is given in N-0528, for both corrugated & smooth-wall cases (with a certain best-fit gaussian-beam for each case). For smooth-walled the results are:

Wavelength	250um	350um	500um
Length	22.68	45.36	45.36
dz	-3.99	-4.44	-8.54

If the horn is changed from L=∞ to finite L, it should be re-focused by the dz shown.

In fig.2. the effect of finite-L is included in the numerical aperture-integration of a gaussian beam with signal beam defined at the exit pupil. The results then include both the L=∞ defocus (from the last section) plus that due to finite L.

It can be seen from fig.2 (example of 500um case) that with finite-L plus re-focus the new peak efficiency is lower than the L=∞ case, and this is because the mode size is changed. In order to avoid this loss it would be necessary to in addition change the horn mouth aperture, but this is assumed fixed (as per equation 2). If the above defocus were not applied (dz=0), the drop in efficiency is approx. 1% (fig.2).

4.1 The real mode.

The real mode calculation is so far made for the photometer (single-mode) only, because in the FTS no re-focus is proposed (next section). The exit pupil is taken to be at z=–1000mm, and non-aberrated. The numerical calculation is not made at the pupil in this case because its diameter in wavelengths (~440), is too large to allow reasonable calculation time. Instead the coupling is calculated at 1/10 of this distance from the horn, where its size is ~1/10 as large.

Figs 2 & 3 shows the resulting sensitivity to defocus, plotted along with the gaussian beam results, for the λ=250 & 500um cases

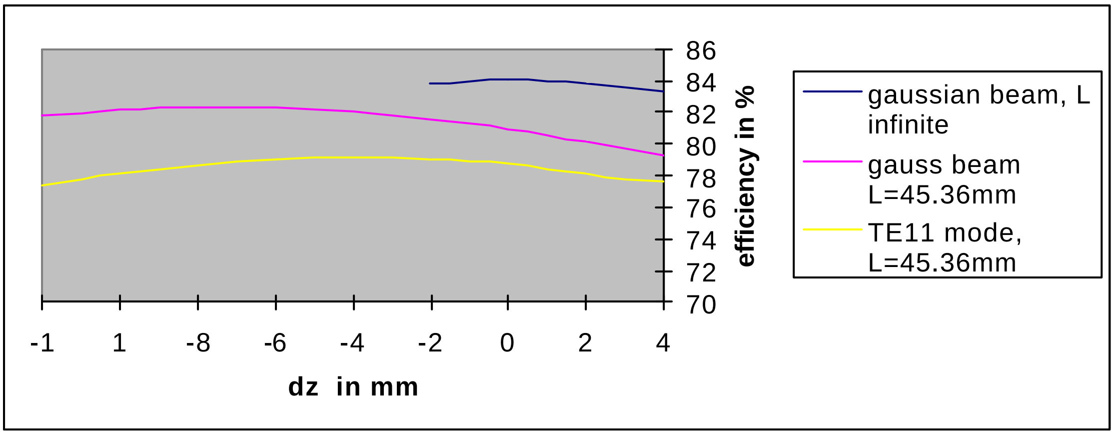


Fig.2. Efficiency versus defocus for PHOT, LW channel at λ=500um.

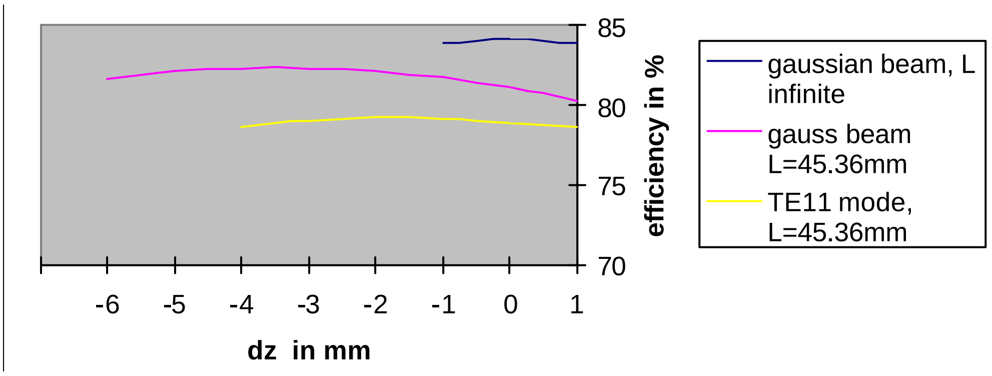


Fig.3. Efficiency versus defocus for PHOT, SW channel at λ=250um.

It can be seen that in each case the dz required for the TE11 mode is approx. ½ of that from the gaussian beam model, and the effect of not introducing this defocus is likewise smaller than it would be for gaussian beams, with the drop in efficiency for dz=0 being only < 1 %. The difference in shape between the real mode & a gaussian mode is illustrated in fig.4.

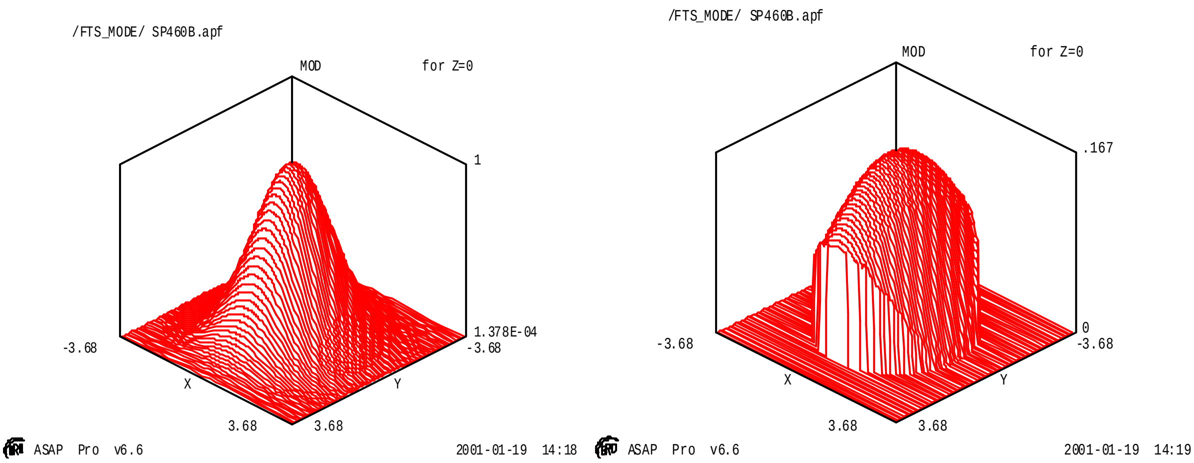


Fig.4. Guausian & TE11 modes, at PLW horn aperture plotted in amplitude (main polarisation), versus position in mm.

5. Focus recommendations.

5.1 Photometer.

Since the photometer is telecentric (approx., exit pupil is at –1000mm) the defocus (relative to GO) can be applied with little effect on plate scale nor the GO design. From the above figures the recommended values of dz to be applied are :

Wavelength	250um	350um	500um
Length	22.68	45.36	45.36
Gaussian beam defocus dz (dimension 'a')	-3.99	-4.44	-8.54
Recommended dz	-1.6	-2*	-4

* inferred from the trends of the other 2 results

The effect of not applying these defocus changes (i.e. making dz=0), would be a loss of ~1% in each case.

5.2 FTS.

Here the system is quite non-telecentric, with exit pupil distance Lex = -118.766mm. This means that the defocus relative to GO cannot be applied without also increasing the plate-scale of the instrument, as well as the GO field-of-view angles (& therefore optic prescription), by a factor (Lex+dz)/Lex (dz negative). To avoid this the array size could be changed, but this would necessitate a change in the horn mouth aperture (equation 2).

The above analysis has not been repeated for the FTS case, as here it is complicated by the use of multi-mode horns. The extra modes should shift the results towards the GO (i.e. many-moded) case, i.e. to reduce dz.

Since the efficiency loss due to keeping dz=0 is small at < 1%, and efficiency is not as big a driver for the FTS as for the photometer. It is proposed that dz=0 for the FTS.

6. References.

Ref.1. “Quasi-optical coupling of gaussian beam systems to large Cassegrain antennas” J.W.Lamb. Int. J of IR & MM-waves Vol.7, pp1511-1535 (1986).
Ref.2. Spire tech.note RAL-N-0316, App.A Horn aperture efficiency calculation method.
Ref.3. Minutes of feedhorn working group meeting, Boulder, 27-28 July 2000.