SPIRE photometric sensitivity mathematical models

Matt Griffin **SPIRE-QMW-PRJ-000559**

24 November 2000

The sensitivities of the SPIRE photometer and spectrometer for point source and mapping observations have been computed in the attached MathCad worksheets, under the assumptions listed. These models are in draft form at present, and will be formally issued after review by the PI, Project Scientists, Instrument Scientist and other members of the SPIRE team.

The main assumptions made in estimating the scientific performance of the instrument are listed below. Additional assumptions are given in the worksheets.

Telescope

Temperature	80 K
Used diameter	3.29 m
Effective emissivity	0.04

FPU temperatures

Level 0 temperature	1.8 K
Level 1 temperature	5 K

Detectors/feedhorns

Temperature	300 mK	
Overall absorption efficiency of feedhorn plus detector	0.45 (min)	0.85 (goal)

Coupling efficiency to point source	0.7
Throughput for each photometer detector	$A\Omega = \lambda^2$
Throughput for each spectrometer detector	$A\Omega = n\lambda^2$ (where n depends on wavelength)

Detector Detective Quantum Efficiency

Photometer	250 μm	0.55 (min)	0.66 (goal)
	350 μm	0.61 (min)	0.73 (goal)
	500 μm	0.66 (min)	0.79 (goal)
FTS	SW	0.66 (min)	0.79 (goal)
	LW	0.61 (min)	0.73 (goal)

Photometer

Central wavelengths (μm)	250	350	and	500
Beam FWHM (arcsec)	17.4	24.4	and	34.6
Field of view of each array (arcmin)	4 x 8 arcminutes			
Overall instrument transmission	30%			
Filter widths ($\lambda/\Delta\lambda$)	3.3			
Observing efficiency (slewing, setup, calibration etc.)	90%			

FTS

Nominal bands (cm^{-1})	33.5 - 50 (200-300 μm)	15 - 33.5 (300-670 μm)
Numbers of pixels	37	19
Field of view	2.6 arcminutes, approx. circular	
Max. spectral resolution	0.04 cm^{-1} ($\lambda/\Delta\lambda$ = 1000 at 250 μm)	
Overall instrument transmission	15%	
Cos^2 signal modulation efficiency	0.5	
Observing efficiency	0.8	
Electrical filter efficiency	0.8	
Degradation in efficiency between 400 and 670 μm	Factor of 2	

Results are calculated in the worksheets for different values of the detector DQE and detector/ feedhorn efficiency, and are summarised in the tables below.

Photometer performance estimates

Filter pass-band ($\lambda/\Delta\lambda$)	**250 μm**	**350 μm**	**500 μm**
Point source observation: **1σ 1sec limiting flux density (mJy)**	Min: 0.61 Goal: 0.40	Min: 0.61 Goal: 0.41	Min: 0.61 Goal: 0.41
Mapping observation: **1σ 1sec limiting flux density (mJy)** **for fully sampled map of FOV**	Min: 2.3 Goal: 1.5	Min: 2.3 Goal: 1.5	Min: 2.3 Goal: 1.6

Spectrometer performance estimates

Line spectroscopy (point source observation): 1σ:1 hr limiting line flux (W m^{-2})	SW band:		Min:	8.7 x 10^{-18}
			Goal:	5.8 x 10^{-18}
	LW band:	300 μm	Min:	1.0 x 10^{-17}
			Goal:	6.4 x 10^{-18}
		400 μm	Min:	1.0 x 10^{-17}
			Goal:	6.4 x 10^{-18}
		670 μm	Min:	2.0 x 10^{-17}
			Goal:	1.3 x 10^{-17}
Line spectroscopy (mapping observation): 1 σ; 1 hr limiting line flux for fully sampled FOV map (W m^{-2})	SW band:		Min:	2.3 x 10^{-17}
			Goal:	1.5 x 10^{-17}
	LW band:	300 μm	Min:	2.6 x 10^{-17}
			Goal:	1.7 x 10^{-17}
		400 μm	Min:	2.6 x 10^{-17}
			Goal:	1.7 x 10^{-17}
		670 μm	Min:	5.2 x 10^{-17}
			Goal:	3.4 x 10^{-17}
Spectrophotometry 1 cm^{-1} resolution (point source observation) 1σ:1 hr limiting flux density (mJy)	SW band:		Min:	29
			Goal:	19
	LW band:	300 μm	Min:	32
			Goal:	21
		400 μm	Min:	32
			Goal:	21
		670 μm	Min:	64
			Goal:	42
Spectrophotometry 1 cm^{-1} resolution (mapping observation) 1 σ; 1 hr limiting flux density for fully sampled FOV map (mJy)	SW band:		Min:	77
			Goal:	51
	LW band:	300 μm	Min:	86
			Goal:	57
		400 μm	Min:	86
			Goal:	57
		670 μm	Min:	170
			Goal:	120

For the photometer, these figures are comparable to the values presented in the SPIRE proposal (but with an increased field of view). In the case of the spectrometer, the performance is slightly degraded with respect to the SPIRE proposal. This is because of the compromise between wavelength coverage and sensitivity - extending the range of the spectrometer beyond the requirement of 400 μm introduces additional photon noise which affects the sensitivity across the band. The optimum scientific trade-off between spectral coverage and sensitivity will be addressed, and changes can be made to the spectrometer feedhorn and/or filtering design to re-optimise. Such changes, if they are made, will have no system-level impact.

Photometer sensitivity model SPIRE_Phot_1.MCD 21 November 2000
for SPIRE feedhorn option

BOLPH_01.MCD 18 Sept. 1997
Modified to compute mapping sensitivity correctly following discussion with WKG

BOLPH_02.MCD 11 Oct. 1997
Telescope focal ratio changed to f/9.59
Horn outside diameter changed to 2Fλ
Hours per day changed from 20 to 22

BOLPH_03.MCD 11 Nov. 1997
Telescope focal ratio changed to f/8.68
Dtel changed to 3.285 m

BOLPH_04.MCD 26 Nov. 1997
Adjusted calculation of sensitivity for frame mapping to use factors for S/N enhancement
as in draft note on mapping speed by Griffin, Bock and Gear
NEPdet changed from 1E-17to 3E-1'7
Observing efficiency: 0.9 for point source ; 0.8 for field map

BOLPH_05.MCD 2 April 1999
Revised to include each optical element of photometer explicitly
15-K level makes significant additional contribution
Overall transmission still set at around 0.3

BOLPH_06.MCD 22 April 1999
Revised to incorporate 4 x 8 fov for deep surveys
Strong source power levels calculated
Internal calibrator requirements now included

BOLPH_07.MCD 16 May 1999
Detector sensitivity characterised in terms of DQE

BOLPH_07_revised.MCD 28 June 1999
New version incorporating Jamie's comments
in his e-mail of June 25. Revisions are noted
in purple.

BOLPH_08.MCD
Version prepared for array selection meeting
* Bands set at 250, 350, 500 mm, the nominal values used for the array selection
* Temperature table updated to reflect current optical/thermal design
* Power and NEP now referred to what is absorbed by the detector
* Only one observing efficiency factor (0.9) used for all observations
* Full NEPph calculation implemented (makes no real difference)

BOLPH_08_JPL_Spec.MCD
* QE changed to represent bolometer + horn with spec of 0.6, goal of 0.85
* DQE wrt absorbed power now used to define overall NEP using values in
JPL spec doc.

SPIRE_Phot_1.MCD 21 November 2000
* Version prepared for Systems Design Review and Toledo Meeting

Constants $h \equiv 6.626\cdot 10^{-34}$ $c \equiv 3\cdot 10^{8}$ $kb \equiv 1.38\cdot 10^{-23}$ Planck function $B(nu,T) := \dfrac{2\cdot h\cdot (nu)^{3}}{c^{2}\cdot\left[e^{\left(\frac{h\cdot nu}{kb\cdot T}\right)} - 1\right]}$

$i \equiv 1,2..3$ $origin \equiv 1$

Assumptions

Telescope	Temp.	Emissivity	Diameter	Area	Focal ratio
	$Ttel \equiv 80$	$\varepsilon tel \equiv 0.04$	$Dtel \equiv 3.285$	$Atel \equiv 0.25\cdot\pi\cdot Dtel^{2}$	$Ftel := 8.68$

Plate scale at telescope focus (arcsec/mm): $PS := \dfrac{1}{Dtel\cdot Ftel}\cdot\dfrac{360}{2\cdot\pi}\cdot 3.6$ $PS = 7.23$

Plate scale at arrays (arcsec/mm): $PSA := PS\cdot\dfrac{8.68}{5}$ $PSA = 12.56$

Beamwidths (arcsec.): $FWHM_i := \dfrac{1.22\cdot\lambda_i\cdot 10^{-6}}{Dtel}\cdot\dfrac{360}{2\cdot\pi}\cdot 3600$ $FWHM_i = \begin{pmatrix}19.2\\26.8\\38.3\end{pmatrix}$

Feedhorn point source coupling efficiency: $\eta tel \equiv 0.7$

Final optics focal ratio $Ffin := 5$

Cold stop attenuation of telescope background: $\eta cs := 0.8$

Bolometer and feedhorn properties (see BDA Subsystem Spec. Doc. SPIRE-JPL-PRJ-000456):

Overall optical efficiency of horn + bolometer combination $\eta feed\_min := 0.45$ $\eta feed\_goal := 0.85$ $\eta feed\_nom := 0.7$

DQE of horn-bolometer combination $DQE\_min_i := \begin{pmatrix}0.55\\0.61\\0.66\end{pmatrix}$ $DQE\_goal_i := \begin{pmatrix}0.66\\0.73\\0.79\end{pmatrix}$ $DQE\_nom_i := \begin{pmatrix}0.6\\0.7\\0.7\end{pmatrix}$

$\eta feed := \eta feed\_goal$

$DQE_i := DQE\_goal_i$

Bolometer yield $y\_min := 0.75$ $y\_goal := 0.9$ $y\_nom := 0.85$ $yield := y\_goal$

Chopping factor $\eta ch \equiv 0.45$

Observing efficicency (slewing, mechanism overheads, etc.): $\eta obs \equiv 0.9$

Bands: defined by central wavelengths (in μm) and resolution of the filters

$\lambda_i \equiv \begin{pmatrix}250\\350\\500\end{pmatrix}$ $R_i := \begin{pmatrix}3.3\\3.3\\3.3\end{pmatrix}$

$\nu_i := \dfrac{c}{\lambda_i\cdot 10^{-6}}$ $\lambda L_i := \lambda_i - \dfrac{\lambda_i}{2\cdot R_i}$ $\lambda U_i := \lambda_i + \dfrac{\lambda_i}{2\cdot R_i}$ $\Delta\lambda_i := \dfrac{\lambda_i}{R_i}$ $\Delta\nu_i := \dfrac{\nu_i}{R_i}$

$\nu L_i := \dfrac{c}{\lambda U_i\cdot 10^{-6}}$ $\nu U_i := \dfrac{c}{\lambda L_i\cdot 10^{-6}}$

$i =$	$\lambda_i =$	$\lambda L_i =$	$\lambda U_i =$	$\Delta\lambda_i =$	$\nu_i\cdot 10^{-9} =$	$\nu L_i\cdot 10^{-9} =$	$\nu U_i\cdot 10^{-9} =$	$\Delta\nu_i\cdot 10^{-9} =$
1	250	212	288	76	1200	1042	1414	364
2	350	297	403	106	857	744	1010	260
3	500	424	576	152	600	521	707	182

Transmission, emissivity and temperature of optical elements

$j \equiv 0,1..$ $k \equiv 0,1..12$

$Tdets \equiv 0.3$ $T2 \equiv 2.0$ $T4 \equiv 5.0$ $T4 = 5$

	$k =$	$t_k \equiv$	$\varepsilon_k \equiv$	$T_k \equiv$	$td_j =$
0 = Telescope	0	0.960	0.04	Ttel	0.301
1 = 15-K filter	1	0.900	0.100	T4	0.334
2 = M3	2	0.995	0.005	T4	0.336
3 = M4	3	0.995	0.005	T4	0.338
4 = M5	4	0.995	0.005	T4	0.339
5 = 4-K filter	5	0.900	0.100	T4	0.377
6 = M6	6	0.995	0.005	T4	0.379
7 = 2-K filter	7	0.900	0.100	T2	0.421
8 = M7	8	0.995	0.005	T2	0.423
9 = Dichroic	9	0.900	0.100	T2	0.47
10 = M8	10	0.995	0.005	T2	0.473
11 = Bandpass filter	11	0.525	0.300	Tdets	0.9
12 = Blocker	12	0.900	0.100	Tdets	

Transmission from element to detector

$$td_j \equiv \prod_{k = j+1}^{12} t_k$$

Array parameters

Detector Numbers

$Ndets_i :=$	$Ndets_i =$
$16 \cdot 5 + 16 \cdot 4$	144
$13 \cdot 4 + 12 \cdot 3$	88
$9 \cdot 3 + 8 \cdot 2$	43

Horn aperture outside dia. (mm)

$$Dhorn_i := \frac{2 \cdot Ffin \cdot \lambda_i}{1000}$$

Array dimension cente-centre (pixels):

$Nmax_i :=$	$Nmin_i :=$
15	8
12	6
8	4

Horn size projected onto telescope focus (mm):

$$Dpix_i := \left(Dhorn_i\right) \cdot \frac{Ftel}{Ffin}$$

Array dimensions at telescope focus centre-centre (mm):

$$Lmm_i := Nmax_i \cdot Dpix_i$$

$$Wmm_i := Nmin_i \cdot Dpix_i$$

Field size (arcmin):

$$Larcmin_i := \frac{Lmm_i \cdot PS}{60} \qquad Warcmin_i := \frac{Wmm_i \cdot PS}{60}$$

$Lmm_i =$	$Wmm_i =$	$Larcmin_i =$	$Warcmin_i =$	$Dhorn_i =$	$Dpix_i =$
65	35	7.8	4.2	2.5	4.3
73	36	8.8	4.4	3.5	6.1
69	35	8.4	4.2	5.0	8.7

Background power levels on the detectors

Throughput:

$$A\Omega_i := \eta cs \cdot \left(\lambda_i \cdot 10^{-6}\right)^2$$

Power contribution absorbed by detector from any element (pW)

$$Power_{i,j} := td_j \cdot \varepsilon_j \cdot 10^{12} \cdot \eta feed \cdot \int_{\nu L_i}^{\nu U_i} B\left(\nu, T_j\right) \cdot A\Omega_i \, d\nu$$

Total power **absorbed** by detector (pW)

$$Pdet_i := \sum_{n=0}^{9} Power_{i,n}$$

Note that this is totally dominated by the telescope

$Power_{1,j} =$	$Power_{2,j} =$	$Power_{3,j} =$
4.77	3.84	2.93
0.00	0.00	0.01
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.01
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	0.00

Photon noise levels and single-detector NEFD

Photon noise limited NEP (full expression)

$$NEPph_i := \left[\frac{4 \cdot A\Omega_i \cdot h^2}{c^2} \cdot \left[\int_{\nu L_i}^{\nu U_i} \frac{\varepsilon tel \cdot td_0 \cdot \eta feed \cdot nu^4}{e^{\left(\frac{h \cdot nu}{kb \cdot T_0}\right)} - 1} \cdot \left[1 + \frac{\varepsilon tel \cdot td_0 \cdot \eta feed}{e^{\left(\frac{h \cdot nu}{kb \cdot T_0}\right)} - 1} \right] dnu \right] \right]^{0.5} \cdot 10^{17}$$

Overall NEP (W Hz-1/2 x 10-17)

$$NEPtot_i := \frac{NEPph_i}{\left(DQE_i\right)^{0.5}}$$

referred to the power absorbed by the detector

Detector NEP (W Hz-1/2 x 10-17)

$$NEPdet_i := \left[\left(NEPtot_i\right)^2 - \left(NEPph_i\right)^2\right]^{0.5}$$

NEFD (mJy Hz-1/2) for point source chopped observations

$$NEFDp_i := \frac{NEPtot_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta ch \cdot \eta tel \cdot 2^{0.5} \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed}$$

Factor of SQRT(2) from pixel-pixel chopping

NEFD (mJy Hz-1/2) for chopped field mapping

$$NEFDf_i := \frac{NEPtot_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta ch \cdot \eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed}$$

No factor of SQRT(2)in the denominator as we are not pixel-pixel chopping

NEFD (mJy Hz-1/2) for scan map observations without chopping

$$NEFDs_i := \frac{NEPtot_i \cdot 10^{-17} \cdot 10^{26} \cdot 1000}{\eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu_i \cdot t_0 \cdot \eta feed} \cdot 2^{0.5}$$

Factor of SQRT(2) assumes need for background subtraction (probably pessimistic as background can be estimated by averaging a number of scan points)

1-σ; 1 sec. limiting flux densities (mJy):

$S\_1\sigma\_1s\_point_i := \frac{NEFDp_i}{2^{0.5}}$ $S\_1\sigma\_1s\_field_i := \frac{NEFDf_i}{2^{0.5}}$ $S\_1\sigma\_1s\_scan_i := \frac{NEFDs_i}{2^{0.5}}$

1-σ; 1 hr. limiting flux densities (mJy):

$Slim\_point\_1hr_i := \frac{S\_1\sigma\_1s\_point_i}{(3600\cdot\eta obs)^{0.5}}$ $Slim\_field\_1hr_i := \frac{S\_1\sigma\_1s\_field_i}{(3600\cdot\eta obs)^{0.5}}$ $Slim\_scan\_1hr_i := \frac{S\_1\sigma\_1s\_scan_i}{(3600\cdot\eta obs)^{0.5}}$

Deep mapping of one field for 1 hour:

Loss in S/N for point source due to need to make a map:

S/N improvement through pixel co-addition $SN\_imp := 1.5$

S/N reduction through decrease in integration time/point by factor of 16 $SN\_red := 4$

Overall reduction in S/N $factor := \frac{SN\_imp}{SN\_red}$ $factor = 0.375$

1 σ; 1 hr limiting flux density for field map (mJy) $\Delta S\_field\_1hr_i := \frac{Slim\_field\_1hr_i}{factor}$

Large area deep survey (nominally 100 sq. deg; 100 days):

Area of one field (sq. arcmin) taking bolometer yield into account $Afield := (4)\cdot(8)\cdot yield$ $Afield = 28.8$

Area to be surveyed (sq. deg.) $Asurv := 100$

Required overlap between fields: $overlap := 1.1$

Number of fields to be observed: $Nfields := \frac{Asurv\cdot 60^2}{Afield}\cdot overlap$ $Nfields = 13750$

Time for survey: $Tdays := 100$ $Tmonths := Tdays\cdot\frac{12}{365}$ $Tmonths = 3.3$

$Thrs := Tdays\cdot 21$ $Thrs = 2100$

Time for each field (hrs): $TField := \frac{Thrs}{Nfields}$ $TField = 0.153$

1 σ; 1 hr limiting flux density for scan map (mJy) $\Delta S\_scan\_1hr_i := \frac{Slim\_scan\_1hr_i}{factor}$

Large survey 5-σ flux density limit: $\Delta S\_surv\_5\sigma_i := \Delta S\_scan\_1hr_i\cdot\eta ch\cdot\left(\frac{1}{TField}\right)^{0.5}\cdot 5$

Summary of power loading and sensitivity calculations

		NEPs (W Hz-1/2 E-17)		NEFDs (mJy Hz-1/2)		
λ_i =	Pdet$_i$ =	NEPph$_i$ =	NEPtot$_i$ =	NEFDp$_i$ =	NEFDf$_i$ =	NEFDs$_i$ =
250	4.8	8.9	11.0	33	46	29
350	3.8	6.8	7.9	33	47	30
500	3.0	5.0	5.6	33	47	30
	pW					

	Point source (mJy)		Map (mJy)	
λ_i =	S_1σ_1s_point$_i$ =	Slim_point_1hr$_i$ =	ΔS_field_1hr$_i$ =	ΔS_surv_5σ$_i$ =
250	23	0.40	1.5	5.6
350	23	0.41	1.5	5.6
500	23	0.41	1.6	5.7

Results for various assumptions on detector/feed performance:

DQE_min; ηfeed_min; yield_min

Pdet	NEPph	Slim_point_1hr	ΔS_field_1h	ΔS_surv_5σ
2.5	6.5	0.61	2.3	9.2
2.0	4.9	0.61	2.3	9.3
1.6	3.6	0.61	2.3	9.3

DQE_nom; ηfeed_nom; yield_nom

Pdet	NEPph	Slim_point_1hr	ΔS_field_1h	ΔS_surv_5σ
3.9	8.1	0.47	1.8	6.6
3.2	6.1	0.46	1.7	6.5
2.4	4.5	0.48	1.8	6.8

DQE_goal; ηfeed_goal; yield_goal

Pdet	NEPph	Slim_point_1hr	ΔS_field_1h	ΔS_surv_5σ
4.8	8.9	0.40	1.5	5.6
3.8	6.8	0.41	1.5	5.6
3.0	5.0	0.41	1.6	5.7

Proposal values	Proposal values
Slim_point_1hr	**ΔS_field_1hr**
0.6	1.4
0.6	1.5
0.7	1.9

SPIRE FTS sensitivity model   SPIRE_FTS_1.MCD   21 November 2000

BOL_FTS4.MCD: 1 Dec. 1997:
Dtel changed to 3.285
Bands changed to allow for same array sizes as in photometer and to correct for previous excessively broad band 1 (was 25-38 cm-1 , λ/Δλ was 2.4)

BOL_FTS5.MCD: 7 Dec. 1997:
Modified to treat correctly variation of resolving power with wavelength: fixed resolution of 0.1 cm-1 now assumed
Error in treatment of electrical filtering now corrected - flux limits now worse by sqrt(0.8)
Some other changes made to simplify computation and improve tabulation of results

BOL_FTS6.MCD: 11 Jan. 1998:
Bands changed to extend upper wavelength to 15 cm-1 (667 um)
Cross-over put at 33.5 cm-1 (300 um) to give equal photon noise NEP in the two bands.
Background power from calibration source now also included in photon noise calculation.

BOL_FTS7.MCD: 29 Sept. 1998:
Revised to include full set of mirrors

BOL_FTS8.MCD: 6 April 1999:
Revised for Mach Zehnder (ADES) configuration

BOL_FTS9.MCD: 7 May 1999:
Three-band system extending to 150 um

BOL_FTS10.MCD: June 1999:
QE term taken out of denominator
Overall efficiency set at 20%
Back to 2-band system

BOL_FTS11.MCD: 2 July 1999:
Corrected for effiency of the intensity beam divider (to ~ 0.5)

SPIRE_FTS_1.MCD: 21 November 20009:
New version for Toledo meeting and System Review
* Multi-moding of both SW and LW bands now taken into account
* NEP contributions from each mode calculated separately and added in quadrature
* NEP now referred to the power absorbed by the detector
* Calculations done for the minimum and goal parameters of the detectors and feedhorns

Constants:

$h \equiv 6.626 \cdot 10^{-34}$ $kb \equiv 1.3806 \cdot 10^{-23}$

$origin := 1$ $c \equiv 2.998 \cdot 10^{8}$

$b := 1, 2 .. 2$

Plank function:

$$B(\nu, T) := \frac{2 \cdot h \cdot \nu^{3}}{c^{2} \cdot \left[\exp\left(\left(\frac{h \cdot \nu}{kb \cdot T}\right)\right) - 1\right]}$$

Assumptions

Telescope	Temp.	Emissivity	Diameter	Area	Focal ratio
	$\mathrm{Ttel} \equiv 80$	$\varepsilon\mathrm{tel} \equiv 0.04$	$\mathrm{Dtel} \equiv 3.285$	$\mathrm{Atel} \equiv 0.25 \cdot \pi \cdot \mathrm{Dtel}^2$	$\mathrm{Ftel} := 8.68$

Bolometers	NEP (*1E-17)	QE
	$\mathrm{NEPdet} \equiv 3.0$	$\eta\mathrm{b} \equiv 0.8$

Telescope coupling efficiency (point source) $\eta\mathrm{tel} \equiv 0.7$

$\eta\mathrm{obs\_m} \equiv 0.8$ (jiggle map)

Cold stop attenuation of telescope background: $\eta\mathrm{cs} := 0.8$

FTS efficiency	Observin-g efficiency	Elec. filter efficiency	Cos^2 modn efficiency
	$\eta\mathrm{obs} := 0.8$	$\eta\mathrm{elec} \equiv 0.8$	$\eta\mathrm{cosq} \equiv 0.5$

Bolometer and feedhorn properties (see BDA Subsystem Spec. Doc. SPIRE-JPL-PRJ-000456):

Overall optical efficiency of horn + bolometer combination

$\eta\mathrm{feed\_min} := 0.45$ $\quad \eta\mathrm{feed\_goal} := 0.85$ $\quad \eta\mathrm{feed\_nom} := 0.7$

DQE of horn-bolometer combination

$$\mathrm{DQE\_min}_b := \begin{pmatrix} 0.61 \\ 0.66 \end{pmatrix} \quad \mathrm{DQE\_goal}_b := \begin{pmatrix} 0.73 \\ 0.79 \end{pmatrix} \quad \mathrm{DQE\_nom}_b := \begin{pmatrix} 0.6 \\ 0.7 \end{pmatrix}$$

$\eta\mathrm{feed} := \eta\mathrm{feed\_nom}$

$\mathrm{DQE}_b := \mathrm{DQE\_nom}_b$

Beam divider reflection transmission, emissivity

$\mathrm{tbd} \equiv 0.487$ $\quad \mathrm{rbd} \equiv 0.487$ $\quad \eta\mathrm{bd1} \equiv 2 \cdot \mathrm{tbd} \cdot \mathrm{rbd}$ $\quad \eta\mathrm{bd2} \equiv \mathrm{tbd}^2 + \mathrm{rbd}^2$

$\eta\mathrm{bd1} = 0.5$ $\quad \eta\mathrm{bd2} = 0.5$ $\quad \varepsilon\mathrm{bd} \equiv 1 - (\mathrm{tbd} + \mathrm{rbd})$ $\quad \varepsilon\mathrm{bd} = 0.03$

Temperature of 4-K and 15-K levels $\quad \mathrm{T4} \equiv 5$ $\quad \mathrm{T15} \equiv 11$

Diffraction loss at each mirror $\quad \mathrm{diffraction} \equiv 0.97$

Emissivity of each mirror $\quad \varepsilon\mathrm{\_mirr} \equiv 1 - 0.995$

Effective transmission of each mirror $\quad \mathrm{t\_mirr} \equiv 0.995 \cdot \mathrm{diffraction}$ $\quad \mathrm{t\_mirr} = 1.0$

Overall diffraction loss $\quad \mathrm{diff\_loss} := \mathrm{diffraction}^{11}$ $\quad \mathrm{diff\_loss} = 0.7$

Transmission, emissivity and temperature of optical elements

$j \equiv 0, 1 .. 16 \quad k \equiv 0, 1 .. 17$

	$k =$	$t_k \equiv$	$T_k \equiv$	$\varepsilon_k \equiv$	$td_j =$
0 = Telescope	0.0	0.96	80	0.04	0.147
1 = CFI1 (15 K)	1.0	0.90	T15	0.1	0.164
2 = CFIL2 (4 K)	2.0	0.9	T4	ε_mirr	0.182
3 = CIPM (M3)	3.0	t_mirr	T4	ε_mirr	0.189
4 = CBSM (M4)	4.0	t_mirr	T4	ε_mirr	0.195
5 = CRIM (M5)	5.0	t_mirr	T4	ε_mirr	0.203
6 = SPOM (M6)	6.0	t_mirr	T4	ε_mirr	0.210
7 = SIFM	7.0	t_mirr	T4	ε_mirr	0.217
8 = SIRM	8.0	t_mirr	T4	εbd	0.225
9 = SBD_overall	9.0	ηbd1	T4	ε_mirr	0.475
10 = SCOM	10.0	t_mirr	T4	2·ε_mirr	0.492
11 = SRTM	11.0	t_mirr^2	T4	ε_mirr	0.528
12 = SDCM	12.0	t_mirr	T4	εbd	0.547
13 = SBD2	13.0	1	T4	ε_mirr	0.547
14 = SCAM	14.0	t_mirr	T4	0.1	0.567
15 = SFIL3 (2 K)	15.0	0.9	2	0.4	0.630
16 = Bandpass (0.3 K)		0.7	0.3	0.1	
17 = Blocker (2 K)		0.9	0.3		

Transmission from element to detector

$$td_j \equiv \prod_{k=j+1}^{17} t_k$$

Array parameters

SW Band (243 μm): 37-element hex array of 2.0Fλ feedhorns:

Array side:

$$Warray := 6 \cdot 2 \cdot \frac{250 \cdot 10^{-6}}{Dtel} \cdot \frac{360}{2 \cdot \pi} \cdot 60$$

$Warray = 3.1$ arcmin

LW Band (343 mm): 19-element hex array of 2.0Fλ feedhorns:

Array side:

$$Warray := 4 \cdot 2 \cdot \frac{350 \cdot 10^{-6}}{Dtel} \cdot \frac{360}{2 \cdot \pi} \cdot 60$$

$Warray = 2.9$ arcmin

Bands

SW Band: 33.5 - 50 cm^{-1} **LW Band: 15 - 33.5 cm^{-1}**

Band limits (cm-1) $\sigma L_2 \equiv 33.5 \quad \sigma U_2 \equiv 50 \qquad \sigma L_1 \equiv 15 \quad \sigma U_1 \equiv 33.5$

Band limits (mm and Hz) $\lambda L_b := \frac{10^4}{\sigma U_b} \quad \lambda U_b := \frac{10^4}{\sigma L_b} \qquad \nu L_b := c \cdot \sigma L_b \cdot 100 \quad \nu U_b := c \cdot \sigma U_b \cdot 100$

Band centre (mm and Hz) $\nu 0_b := \frac{\nu L_b + \nu U_b}{2} \qquad \lambda 0_b := \frac{c \cdot 10^6}{\nu 0_b}$

Band λ/Δλ $R_b := \frac{\sigma U_b + \sigma L_b}{2 \cdot (\sigma U_b - \sigma L_b)}$

Band limits (mm and THz)

	$\lambda L_b =$	$\lambda 0_b =$	$\lambda U_b =$	$\nu L_b \cdot 10^{-12}$	$\nu 0_b \cdot 10^{-12} =$	$\nu U_b \cdot 10^{-12} =$	$R_b =$
LW	299	412	667	0.45	0.73	1.00	1.31
SW	200	240	299	1.00	1.25	1.50	2.53

Background power levels on the detectors

Assumptions:

1. All modes carry equal background power (per unit bandwidth) from the telescope
2. All modes couple equally well to the bolometer
3. Calibrator contributes same amount of power as the telescope

Throughput per mode $A\Omega(\nu) := \left(\frac{c}{\nu}\right)^2 \cdot \eta cs$

Coupling of higher order modes to telescope: Assume 50%
(cf. Martin Caldwell note presented at Boulder Feedhorn meeting): $\eta\_higher := 0.5$

SW band (b = 2)

Designed cut-off wavelength for TE11 mode $\lambda c := 310$ $\nu o\_TE11_2 := 0.5 \cdot (\nu L_2 + \nu U_2)$

Required waveguide radius (μm) $ro := \frac{\lambda c \cdot 1.841}{2 \cdot \pi}$ $ro = 91$ $\frac{ro}{\lambda c} = 0.3$

Cut-off wavelengths of higher modes (one higher mode can propagate)

$\lambda c\_TM01 := \frac{2 \cdot \pi \cdot ro}{2.405}$ $\lambda c\_TM01 = 237$ $\nu c\_TM01 := \frac{c \cdot 10^6}{\lambda c\_TM01}$ Propagated $\nu o\_TM01_2 := \frac{\nu c\_TM01 + \nu U_2}{2}$

$\lambda c\_TE21 := \frac{2 \cdot \pi \cdot ro}{3.054}$ $\lambda c\_TE21 = 187$ $\nu c\_TE21 := \frac{c \cdot 10^6}{\lambda c\_TE21}$ Not propagated

TE11 power absorbed by detector from each element(pW)

$$P\_TE11_{2,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed \cdot 10^{12} \cdot \int_{\nu L_2}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$$

Factor of 2 accounts for same background from calib. source in 2nd port

$$Power\_TE11_2 := \sum_{n=0}^{9} P\_TE11_{2,n} \qquad Power\_TE11_2 = 4.9$$

TE11 NEPph contribution

$$NEPph\_TE11_2 := \left(2 \cdot Power\_TE11_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TE11_2\right)^{0.5} \cdot 10^{17} \qquad NEPph\_TE11_2 = 9.0$$

TM01 power absorbed by detector from each element(pW)

$$P\_TM01_{2,j} := \eta\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed \cdot 10^{12} \cdot \int_{\nu c\_TM01}^{\nu U_2} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$$

$$Power\_TM01_2 := \sum_{n=0}^{9} P\_TM01_{2,n} \qquad Power\_TM01_2 = 1.1$$

TM01 NEPph contribution

$$NEPph\_TM01_2 := \left(2 \cdot Power\_TM01_2 \cdot 10^{-12} \cdot h \cdot \nu o\_TM01_2\right)^{0.5} \cdot 10^{17} \qquad NEPph\_TM01_2 = 4.5$$

Overall power for SW band $Power_{2,j} := P\_TE11_{2,j} + P\_TM01_{2,j}$

Overall NEPph for SW band (W Hz-1/2 * 1E-17)

$$NEPph_2 := \left[\left(NEPph\_TE11_2\right)^2 + \left(NEPph\_TM01_2\right)^2\right]^{0.5}$$

$$NEPph_2 = 10.1$$

LW band (b = 1)

Designed cut-off wavelength for TE11 mode $\lambda c := 670$

Required waveguide radius $ro := \frac{\lambda c \cdot 1.841}{2 \cdot \pi}$ $ro = 196$ $\frac{ro}{\lambda c} = 0.3$ $\nu o\_TE11_1 := 0.5 \cdot (\nu L_1 + \nu U_1)$

Cut-off wavelengths of higher modes (three higher modes can propagate)

$\lambda c\_TM01 := \frac{2 \cdot \pi \cdot ro}{2.405}$ $\lambda c\_TM01 = 513$ $\nu c\_TM01 := \frac{c \cdot 10^6}{\lambda c\_TM01}$ Propagated $\nu o\_TM01_1 := \frac{\nu c\_TM01 + \nu U_1}{2}$

$\lambda c\_TE21 := \frac{2 \cdot \pi \cdot ro}{3.054}$ $\lambda c\_TE21 = 404$ $\nu c\_TE21 := \frac{c \cdot 10^6}{\lambda c\_TE21}$ Propagated $\nu o\_TE21_1 := \frac{\nu c\_TE21 + \nu U_1}{2}$

$\lambda c\_TE01 := \frac{2 \cdot \pi \cdot ro}{3.832}$ $\lambda c\_TE01 = 322$ $\nu c\_TE01 := \frac{c \cdot 10^6}{\lambda c\_TE01}$ Propagated $\nu o\_TE01_1 := \frac{\nu c\_TE01 + \nu U_1}{2}$

$\lambda c\_TE31 := \frac{2 \cdot \pi \cdot ro}{4.201}$ $\lambda c\_TE31 = 294$ $\nu c\_TE31 := \frac{c \cdot 10^6}{\lambda c\_TE31}$ Not propagated

TE11 power absorbed by detector from each element (pW)

$$P\_TE11_{1,j} := 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed \cdot 10^{12} \cdot \int_{\nu L_1}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$$

$$Power\_TE11_1 := \sum_{n=0}^{9} P\_TE11_{1,n} \qquad Power\_TE11_1 = 7.0$$

TE11 NEPph contribution

$$NEPph\_TE11_1 := \left(2 \cdot Power\_TE11_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE11_1\right)^{0.5} \cdot 10^{17}$$

$$NEPph\_TE11_1 = 8.2$$

TM01 power absorbed by detector from each element (pW)

$$P\_TM01_{1,j} := \eta\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed \cdot 10^{12} \cdot \int_{\nu c\_TM01}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$$

$$Power\_TM01_1 := \sum_{n=0}^{9} P\_TM01_{1,n} \qquad Power\_TM01_1 = 2.5$$

TM01 NEPph contribution

$$NEPph\_TM01_1 := \left(2 \cdot Power\_TM01_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TM01_1\right)^{0.5} \cdot 10^{17}$$

$$NEPph\_TM01_1 = 5.2$$

TE21 power absorbed by detector from each element (pW)

$$P\_TE21_{1,j} := \eta\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed \cdot 10^{12} \cdot \int_{\nu c\_TE21}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$$

$$Power\_TE21_1 := \sum_{n=0}^{9} P\_TE21_{1,n} \qquad Power\_TE21_1 = 1.5$$

TE21 NEPph contribution

$$NEPph\_TE21_1 := \left(2 \cdot Power\_TE21_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE21_1\right)^{0.5} \cdot 10^{17}$$

$$NEPph\_TE21_1 = 4.2$$

TE01 power absorbed by detector from each element (pW)

$$P\_TE01_{1,j} := \eta\_higher \cdot 2 \cdot td_j \cdot \varepsilon_j \cdot \eta feed \cdot 10^{12} \cdot \int_{\nu c\_TE01}^{\nu U_1} B(\nu, T_j) \cdot A\Omega(\nu)\, d\nu$$

$$Power\_TE01_1 := \sum_{n=0}^{9} P\_TE01_{1,n} \qquad Power\_TE01_1 = 0.4$$

TE01 NEPph contribution

$$NEPph\_TE01_1 := \left(2 \cdot Power\_TE01_1 \cdot 10^{-12} \cdot h \cdot \nu o\_TE01_1\right)^{0.5} \cdot 10^{17}$$

$$NEPph\_TE01_1 = 2.3$$

Overall power or LW band

$$Power_{1,j} := P\_TE11_{1,j} + P\_TM01_{1,j} + P\_TE21_{1,j} + P\_TE01_{1,j}$$

Overall NEPph for LW band

$$NEPph_1 := \left[\left(NEPph\_TE11_1\right)^2 + \left(NEPph\_TM01_1\right)^2 + \left(NEPph\_TE21_1\right)^2 + \left(NEPph\_TE01_1\right)^2\right]^{0.5}$$

$$NEPph_1 = 10.8$$

Total power on detector (pW)

$$Pdet_b := \sum_{n=0}^{9} Power_{b,n} \qquad Pdet_b = \begin{array}{|r|}\hline 11.4 \\ \hline 6.0 \\ \hline \end{array}$$

$Power_{1,j} =$	$Power_{2,j} =$
10.7	5.95
0.7	0.08
$1.26 \cdot 10^{-3}$	$1.05 \cdot 10^{-5}$
$1.3 \cdot 10^{-3}$	$1.09 \cdot 10^{-5}$
$1.35 \cdot 10^{-3}$	$1.13 \cdot 10^{-5}$
$1.4 \cdot 10^{-3}$	$1.17 \cdot 10^{-5}$
$1.45 \cdot 10^{-3}$	$1.21 \cdot 10^{-5}$
$1.5 \cdot 10^{-3}$	$1.25 \cdot 10^{-5}$
$8.08 \cdot 10^{-3}$	$6.74 \cdot 10^{-5}$
$3.28 \cdot 10^{-3}$	$2.73 \cdot 10^{-5}$
$6.79 \cdot 10^{-3}$	$5.66 \cdot 10^{-5}$
$3.65 \cdot 10^{-3}$	$3.04 \cdot 10^{-5}$
0.02	$1.64 \cdot 10^{-4}$
$3.78 \cdot 10^{-3}$	$3.15 \cdot 10^{-5}$
0.08	$6.53 \cdot 10^{-4}$
$1.62 \cdot 10^{-4}$	$5.57 \cdot 10^{-10}$

Note that total power is dominated by the telescope contribution

Photon noise levels and single-detector NEFD

Overall NEP
(W Hz-1/2 x 10-17)

$$NEPtot_b := \frac{NEPph_b}{(DQE_b)^{0.5}}$$

referred to the power absorbed by the detector

Detector NEP
(W Hz-1/2 x 10-17)

$$NEPdet_b := \left[(NEPtot_b)^2 - (NEPph_b)^2\right]^{0.5}$$

NEFD (Jy Hz-1/2)

$$NEFD_b := \frac{NEPtot_b \cdot 10^{-17} \cdot 10^{26}}{\eta elec \cdot \eta cosq \cdot \eta tel \cdot Atel \cdot td_0 \cdot \Delta\nu \cdot t_0 \cdot \eta feed}$$

Note: this is pessimistic in that the additional modes are assumed to couple to the telescope background but not to the source

$NEFD_b =$

2.0
1.7

Point source observation

Spectral resolution (cm-1 and Hz) $\Delta\sigma \equiv 1$ $\Delta\nu \equiv c \cdot \Delta\sigma \cdot 100$

Limiting flux densities (1-σ 1 hr)

$$Slim_b := \frac{1000 \cdot NEFD_b}{(2 \cdot 3600 \cdot \eta obs)^{0.5}}$$

Limiting line strengths (1-σ 1 hr)

$$Flim_b := \left(\frac{Slim_b \cdot 10^{-26}}{1000} \cdot \Delta\nu\right)$$

Deep mapping of one field for 1 hour:

Loss in S/N for point source due to need to make a map:

S/N improvement through co-addition of pixels $SN\_imp := 1.5$

S/N reduction through decrease in integration time per point by factor of 16 $SN\_red := 4$

Overall reduction in S/N $factor := \frac{SN\_imp}{SN\_red}$ $factor = 0.375$

Limiting flux density (mJy) $\Delta S\_1hr_b := \frac{Slim_b}{factor}$ $\Delta F\_1hr_b := \frac{Flim_b}{factor}$

Band centre and edges: wavlengths and resolving powers

$$ResL_b := \frac{\nu U_b}{\Delta\nu} \quad Res0_b := \frac{\nu 0_b}{\Delta\nu} \quad ResU_b := \frac{\nu L_b}{\Delta\nu}$$

	$\lambda L_b =$	$ResL_b =$	$\lambda 0_b =$	$Res0_b =$	$\lambda U_b =$	$ResU_b =$
LW	298.5	33.5	412.4	24.3	666.7	15.0
SW	200.0	50.0	239.5	41.8	298.5	33.5

Summary:		**NEPs**			**Point source**		**Map**	
		$Pdet_b$ =	$NEPph_b$ =	$NEPtot_b$ =	$Slim_b$ =	$Flim_b \cdot 10^{18}$ =	ΔS_1hr_b =	$\Delta F\_1hr_b \cdot 10^{17}$ =
	LW	11.4	10.8	13.9	26.0	7.8	69.5	2.1
	SW	6.0	10.1	12.1	22.6	6.8	60.2	1.8
		pW	W Hz-1/2 E -17		mJy	W m-2	mJy	W m-2
Goal values:		13.9	11.9	13.9	21	6.4	57	1.7
		7.3	11.1	12.5	19	5.8	51	1.5
Nominal values:		11.4	10.8	13.9	26	7.8	70	2.1
		6.0	10.1	12.1	13	6.8	60	1.8
Min values:		7.3	8.7	11.1	32	10	86	2.6
		3.9	8.1	10.0	29	8.7	77	2.3
Proposal values:					23 - 46	7 - 14	57 - 114	1.7 - 3.4
					25	7.6	59	1.8

Notes:

1. Limiting flux density Slim is inversely proporotional to spectral resolution ($\Delta\sigma$) and independent of wavelength.

2. Limiting line flux Flim is independent of spectral resolution and wavelength (for an unresolved line).

3. For wavelengths longer than 400 µm, the pixel size will be increasingly mis-matched to the diffraction spot size. This will degrade the efficiency either for the feed-horn arrays or fuilled arrays options. In addition, diffraction within the FTS will result in a loss of efficiency at the longest wavelengths. The implications for sensitivity of the FTS at wavelengths longward of 400 µm must be studied in detail. At the mopment, we estimate an effective loss of efficiency of a factor of two at 670 µm, and scale linearly for wavelengths between 400 and 670 µm.