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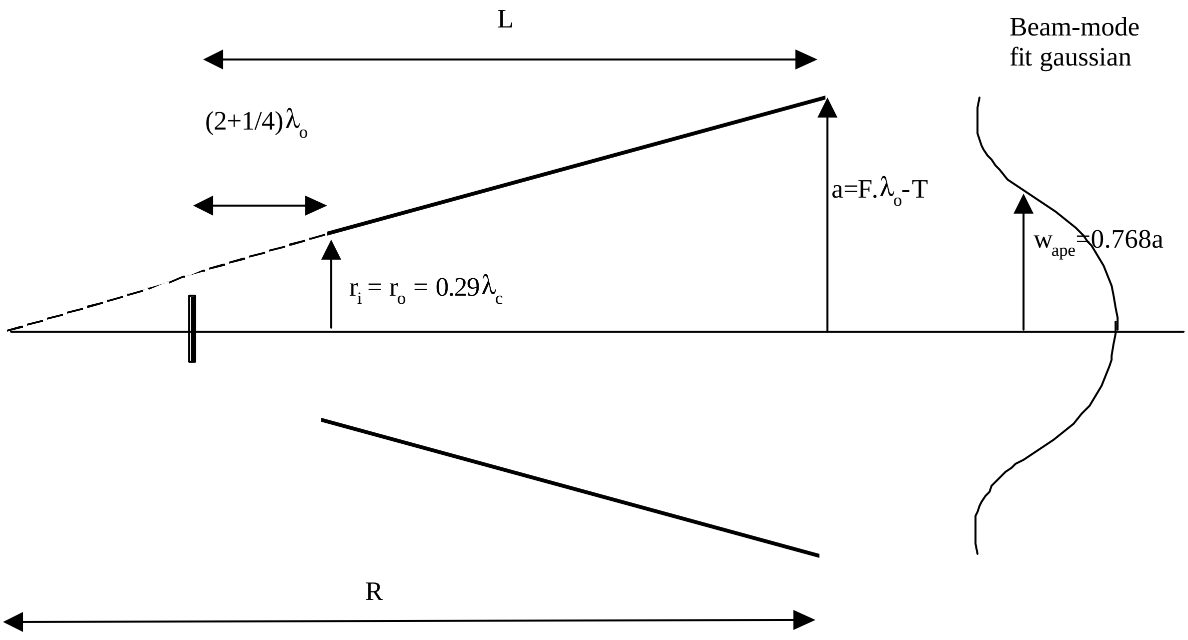
CONTENTS.

1. Geometry parameters.

Fig.1 shows the cross-section geometries for single-moded horns, both smooth & corrugated cases.



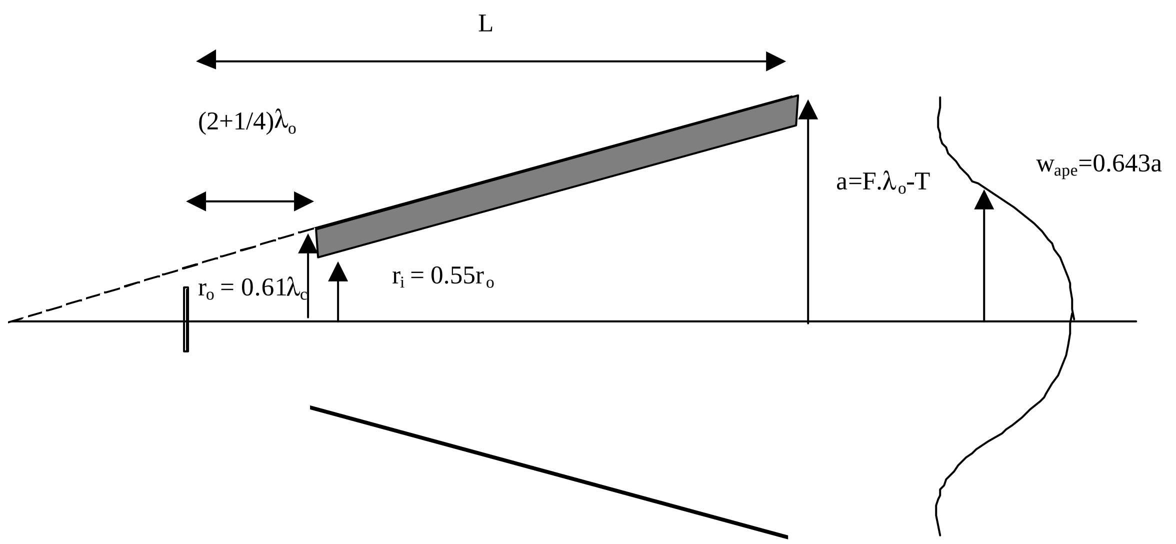


Fig.1. Horn geometries. Upper=smooth-walled, lower=corrugated. Parameter values from ref.1.

The parameters for the SW horn are (from the table of ref.1, with wall thickness revised)

Horn	**l o (mm)**	**l c (mm)**	**r_i (mm)**	**r_o (mm)**	**L (mm)**	**T (mm)**
250 SM Smooth (a)	250	292	$0.29\lambda_c$	$0.29\lambda_c$	22.68	50
250 SM Corrugated	250	292	$0.55r_o$	$0.61\lambda_c$	22.68	50

(Here 'SM' means single-mode, as for now we are considering just the photometer). The chosen cut-off wavelength determines r_o via the equation:

$$l_c = \frac{2p}{c}.r_o$$

where the χ value for the mode (either TE11 for smooth, HE11 for corrugated) is given in ref.2.

We also need the optics f-number =5, and then the optical parameters can be calculated. These are:
A: horn aperture
W_{ape}: 1/e amplitude radius of the fundamental component (gaussian) in beam-mode analysis, at horn aperture.
R: beam wavefront radius-of-curvature at horn aperture, determined by horn length L.

R is given by

$$R = (L - 2.25.\lambda_o)*a/(a-r_o)$$

Horn	a (mm)	wape (mm)	R (mm)
250 SM smooth (a)	1.2	0.9216	23.79676
250 SM corrugated	1.2	0.7716	25.97272

2. Guassian beam focus-compensation, 250um case.

In the ideal case of a horn of infinite length, the horn-mode wavefront would be flat at the aperture ($R=\infty$), and the beam-mode waist would be also at the aperture. The optimum focus position would then be with the horn aperture at the optics geometric focus (where the Airy signal beam has flat wavefront).

In the real case the finite radius of curvature R at the aperture means that the optimum focus position shifts into the horn as shown below. This position is calculated by considering only the best-fit fundamental mode in the beam-mode expansion of the horn’s pattern. This mode is gaussian and so its waist position ‘z’ in the horn can be calculated from simple equations. Since this focus correction is based on only the fundamental mode, we refer to it as the ‘gaussian beam focus compensation’.

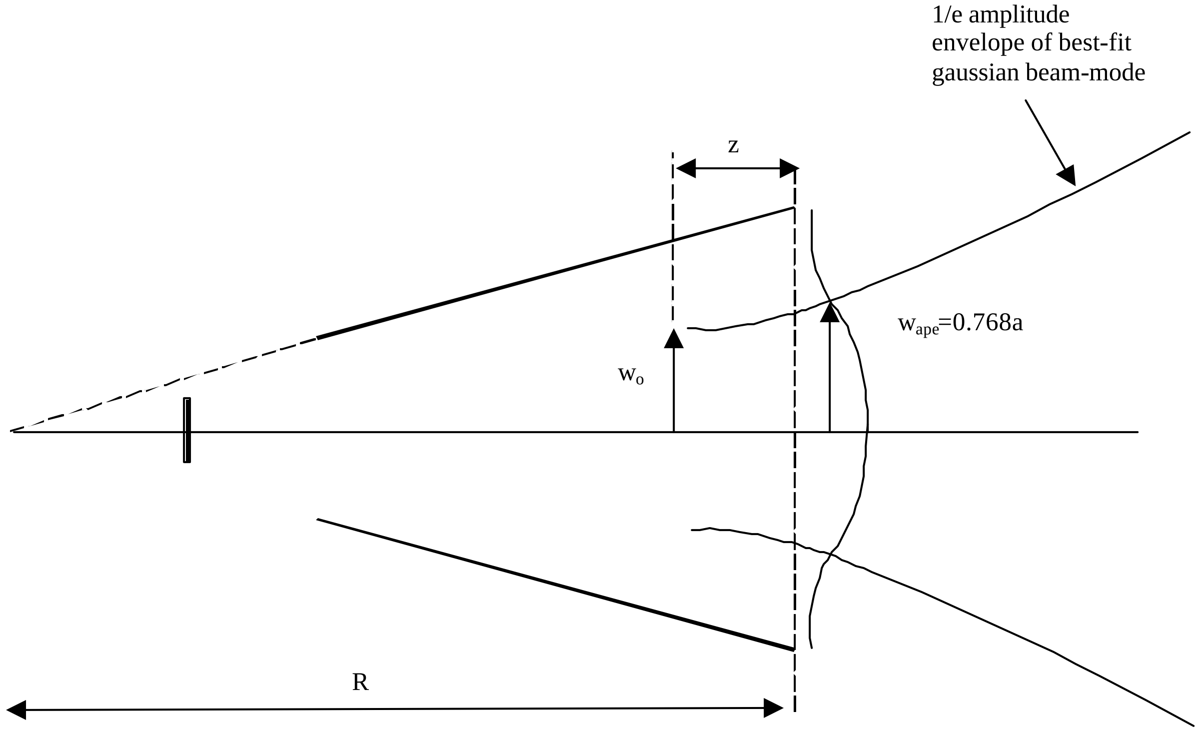


Fig.2. Gaussian beam virtual waist position in horn (ref.3)

The beam-mode waist radius w_o and position z are found from w_{ape} and R using the gaussian-beam equations:

$$w_o = \frac{w_{ape}}{\sqrt{1+\left(\frac{p w_{ape}^2}{l_o R}\right)^2}}$$

$$z_o = \frac{p w_o^2}{l_o}$$

$$z = \frac{z_o p w_{ape}^2}{l_o R}$$

Using these equations the beam parameters are:

Horn	a (mm)	wape (mm)	R (mm)	wo (mm)	zo (mm)	z (mm)
250 SM smooth (a)	1.2	0.9216	23.79676	0.840894	8.885704	**3.985372**
250 SM corrugated	1.2	0.7716	25.97272	0.741452	6.908366	**1.989996**

Table 1. 250um horn parameters.

Thus with this best-focus criterion the horn is to be moved by several mm.

3. All 3 photometer wavelengths, L=22.68mm.

To compare with the values stated e.g. in ref.1, the tables below list the values for single-moded horns at the other channel wavelengths. These assume the same ratio of λ_c/λ_o and the same length as in the 250um channel, to show the trend with wavelength.

Smooth-wall

Horn	l_o (mm)	l_c (mm)	r_i (mm)	r_o (mm)	L (mm)	T (mm)
250 SM Smooth (a)	250	292	$0.29\lambda_c$	$0.29\lambda_c$	22.68	50
350 SM smooth	350	$(29/25)*\lambda_o$	$0.29\lambda_c$	$0.29\lambda_c$	22.68	50
500 SM smooth	500	$(29/25)*\lambda_o$	$0.29\lambda_c$	$0.29\lambda_c$	22.68	50

Horn	a (mm)	wape (mm)	R (mm)	wo (mm)	zo (mm)	z (mm)
250 SM smooth (a)	1.20	0.92	23.80	0.84	8.89	**3.99**
350 SM	1.70	1.31	23.52	1.09	10.75	**6.99**
500 SM	2.45	1.88	23.14	1.36	11.56	**11.11**

Corrugated.

Horn	l_o (mm)	l_c (mm)	r_i (mm)	r_o (mm)	L (mm)	T (mm)
250 SM corrugated	250	292	$0.55r_o$	$0.29\lambda_c$	22.68	50
350 SM smooth	350	$(29/25)*\lambda_o$	$0.55r_o$	$0.29\lambda_c$	22.68	50
500 SM smooth	500	$(29/25)*\lambda_o$	$0.55r_o$	$0.29\lambda_c$	22.68	50

Horn	a (mm)	wape (mm)	R (mm)	wo (mm)	zo (mm)	z (mm)
250 SM corrugated	1.20	0.77	25.97	0.74	6.91	**1.99**
350 SM	1.70	1.09	25.63	1.01	9.13	**3.82**
500 SM	2.45	1.58	25.19	1.34	11.27	**6.98**

Table 2. Single mode horn optical parameters , all 3 channels, constant L=26.68mm.

These values for gaussian-beam focus compensation differ somewhat from those of ref.1. where the z-values were 4.3, 6.0, & 8.6 mm. The z values for the corrugated horn are lower than for the smooth because the best-fit w_{ape} is in in this case smaller (for the same ‘a’ & ‘L’).

4. All 3 channels, actual lengths.

The actual length of the horn is longer in the 350 & 500um channels. This is needed particularly at 500um as it gives significant increase in efficiency. For example the following tables show that efficiency improvements of 5 % for the smooth case and of 7% for the corrugated case occur between the proposed short & long extremes (ref.1)

Corrugated.

Channel	L=23mm	L=55mm	L= ¥
250	0.787	0.798	0.800
350	0.757	0.781	0.786
500	0.731	0.780	0.792

Smooth.

Channel	L=23mm	L=55mm	L= ¥
250	0.775	0.792	0.796
350	0.753	0.788	0.796
500	0.708	0.781	0.798

Efficiency η for different horn lengths.

The lengths are taken here as L=45.36mm in the 350um & 500um channels, giving the following gaussian-beam parameters:

Smooth

Horn	a (mm)	wape (mm)	R (mm)	wo (mm)	zo (mm)	z (mm)
250 SM (a) 22.68mm	1.20	0.92	23.80	0.84	8.89	**3.99**
350 SM 45.36mm	1.70	1.31	47.89	1.24	13.88	**4.44**
500 SM 45.36mm	2.45	1.88	47.50	1.70	18.24	**8.54**

Corrugated

Horn	a (mm)	wape (mm)	R (mm)	wo (mm)	zo (mm)	z (mm)
250 SM 22.68mm	1.20	0.77	25.97	0.74	6.91	**1.99**
350 SM 45.36mm	1.70	1.09	52.17	1.07	10.29	**2.12**
500 SM 45.36mm	2.45	1.58	51.70	1.51	14.29	**4.31**

Table 3. Horn optical parameters for the case of L=26.68mm in photometer SW & L=45.36mm in MW & LW.

Compared to section 3 it can be seen that using a longer horn for longer wavelengths reduces the size of defocus required.

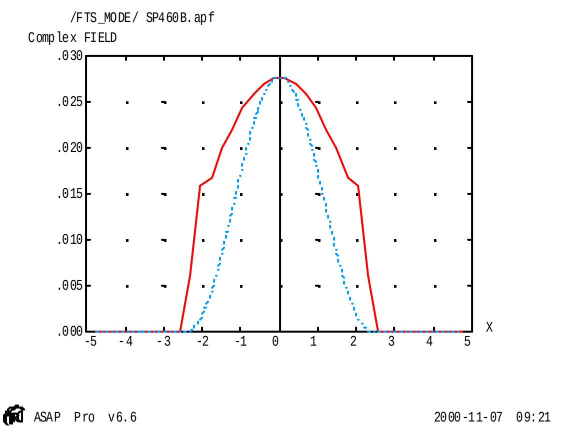
5. Effect of actual beam-patterns.

It is necessary to check to what degree the actual beam patterns conform to the gaussian-beam behavior quantified above. This is done using the ASAP optics propagation model (FTS model), and the parameters whose focus dependence we need to check are the point spread function (PSF) and the aperture efficiency η.

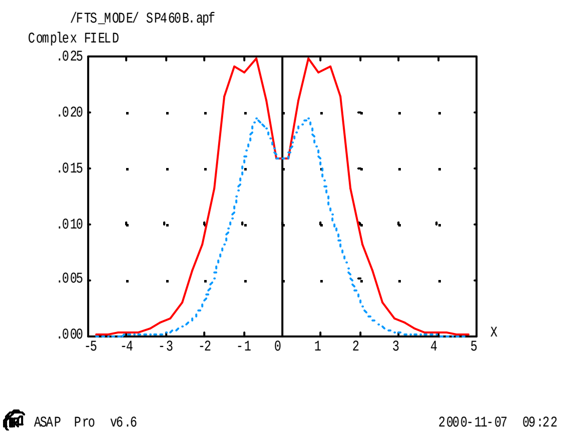
The simulations are made only for the LW channel, where the defocus effect (z) is largest, at λ=500um.

5.1 Smooth-wall horn.

Here the beam is the TE11 mode. In order to check how gaussian-like the beam is its profile (dominant polarisation only) is calculated at both the horn aperture and at the focus-compensated position within the horn. The resulting beam profiles are shown below.



(a)

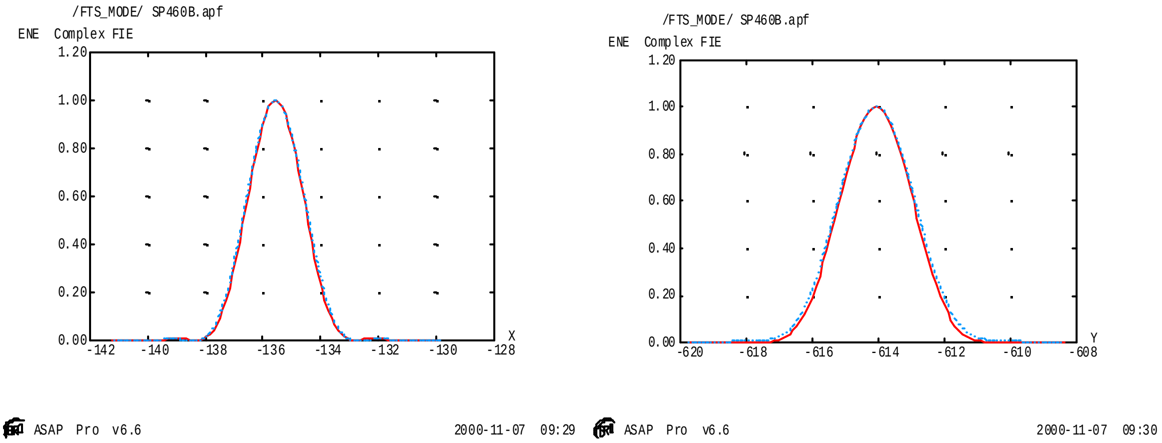


(b)

Fig.3. Beam profiles in 2 sections (a) at horn aperture, (b) at virtual waist 5mm within horn. Case of 500um horn, 0.1mm wall thickness.

The pattern within the horn is a virtual one in that it is calculated for free-space propagation backwards from the horn aperture. The plots show that the actual pattern is quite non-gaussian.

The far-field beam pattern (PSF) is calculated by propagating the mode from the detector to the first pupil (cold stop), applying the F/5 clipping there & then propagating the beam to the far-field. The figure below shoes the difference in PSF of cases with & without focus compensation.



(a) (b)

Fig.4. Far-field beam pattern for cases without focus compensation (solid line) and with (dotted line). (a) in x-section (b) in y-section.

The focus compensation is seen to have only a small effect, and actually gives a slight broadening of PSF rather than the expected narrowing.

Also the focus compensation has a small effect on aperture efficiency. This was already calculated, in RAL N-0316, App. A , where it was found that at this wavelength, in optimising the focus to z=5mm, the increase in η is by just 1 %.

The explanation for this low sensitivity to focus is:

1. The beam is quite non-gaussian.
2. The beam is clipped at the pupil, and due to the significant higher-order spatial content, this clipping is more significant than the 1/e edge taper of a gaussian.

The uncompensated case has a narrower PSF than the compensated because it ‘fills’ the pupil more (has a slightly higher edge-taper level), due to the horn being closer to the pupil in this case, as shown below.

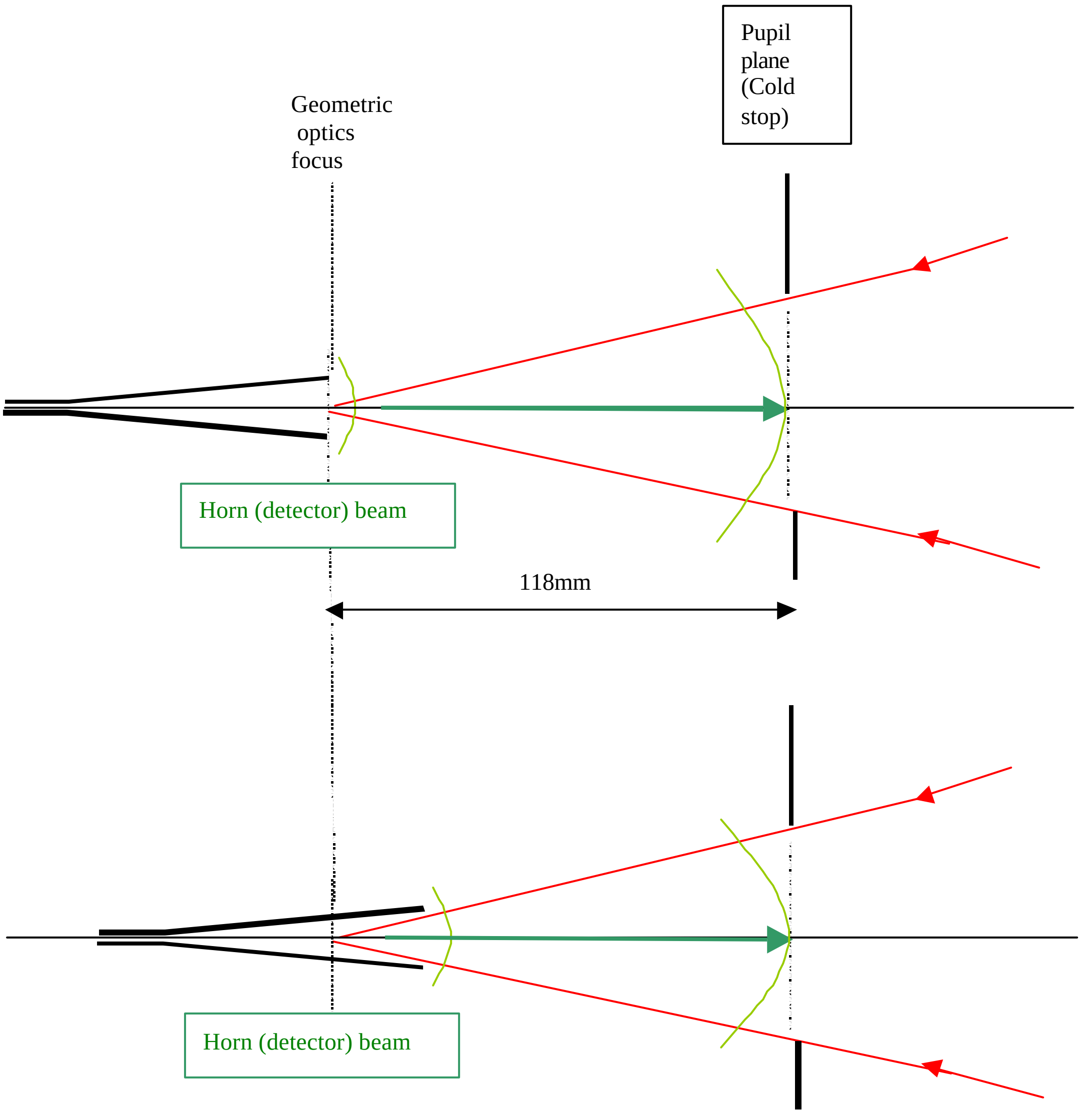


Fig.5. Difference between un-compensated horn position (upper diagram) and compensated (lower), for FTS pupil location.

5.2 Corrugated horn.

Here the beam is the HE11 mode, which is linearly polarised. In order to check how gaussian-like the beam is its profile is calculated at both the horn aperture and at the focus-compensated position within the horn. The resulting beam profile are shown below (unlike the smooth case the beam is rotationally symmetric so only one profile is needed).

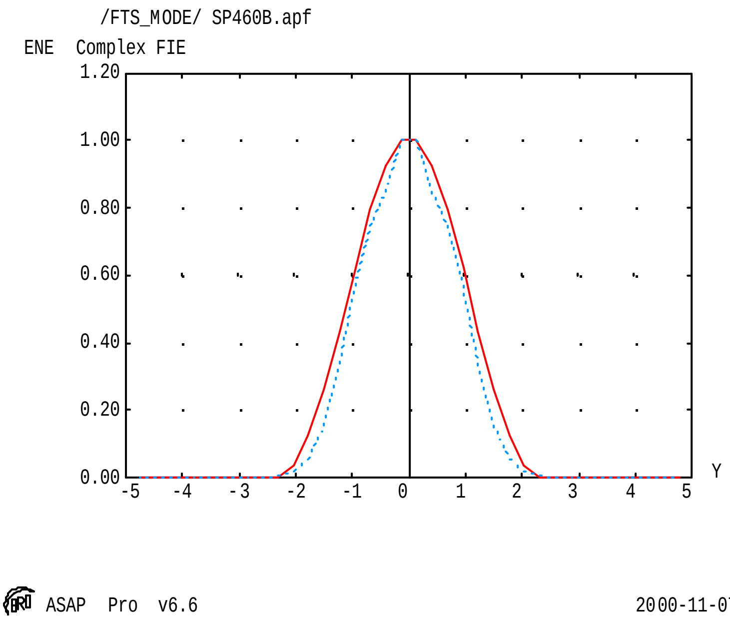


Fig.6. Beam profiles in section at horn aperture (solid line), & at virtual waist 2.6mm within horn (dotted line). Case of 500um horn, 0.1mm wall thickness.

This plot shows that the pattern is much more gaussian-like than the smooth-wall case.

The PSF pattern is shown below. There is very little difference in shape between the non-compensated & compensated cases, but the non-compensated case does have a peak intensity approx. 5% larger. This is consistent with the smooth-wall result, where the non-compensated case gives slightly narrower PSF due to it having a higher edge-taper level at the pupil.

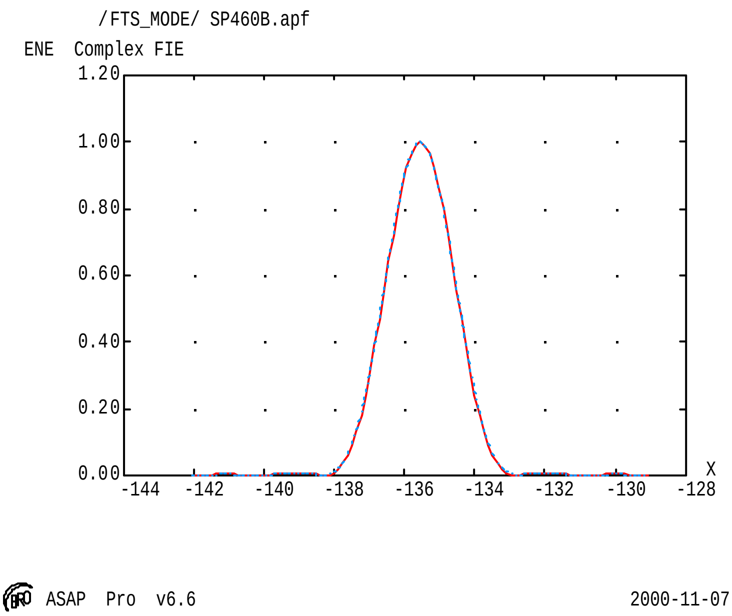


Fig.7. PSF (far-field, normalised) patterns for corrugated (he11) case, without focus compensation (solid,red plot), and with (dotted, blue).

6. Conclusions.

The effect of gaussian-beam focus compensation was checked for the λ=500um L=45.36mm case. Its use has only a small effect on the beam, in terms of both PSF full-width at half maximum, and aperture efficiency. The effect on PSF is actually a slight degradation rather than improvement (~3% in fwhm, smooth-wall horn, negligible in corrugated horn), and the effect on efficiency is an improvement of only approx. <1 % (either type of horn).

The contrary PSF effect compared to that of a gaussian beam is attributed to real beams being non-gaussian (especially true in smooth-wall case), and the presence of pupil clipping.
The conclusion is that these features effectively increase the ‘depth-of-focus’ of the beam compared to a gaussian, such that the performance is not sensitive to horn focus position within the range indicated by the gaussian beam focus offset ‘z’ in the tables above.

7. References.

Ref.1. Feedhorn working group meeting minutes (July27,2000), section 8.
Ref.2. Markuvitz *Waveguide Handbook*. Section 2.3 p.66.
Ref.3. “MM-wave gaussian beam-mode optics & corrugated feed-horns” R J Wylde IEE Proc. Vol.131,Pt.H,No.4.pp258-262.