SPIRE sensitivity to variations in the temperature distribution over the FIRST primary mirror

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1 Background

This is an updated version of the note by Matt Griffin of 5 June 1998, *Simple assessment of SPIRE sensitivity to variations in the temperature gradient across the FIRST primary mirror*. Besides making some minor changes to the original, we have added Section 5 on the analysis of the telescope temperature map provided by JPL, and have updated the conclusions (Section 6).

Original Message from Göran Pilbratt

Dear Matt, Albrecht, and Thijs,

I managed to catch all three of you one way or another last week to discuss the issue of gradients in the telescope background. I send this note to record our discussions.

During in-orbit operations the FIRST telescope will be subjected to an environment which is neither uniform nor stationary. The front side of the primary mirror will primarily see cold space (and the focal plane reflected by the secondary mirror), while the backside of the primary will see the cryostat vacuum vessel, or any shielding preventing it from doing so. On one side of the telescope there is the inside of the sunshade, on the other side cold space.

The telescope boresight is the x axis, the direction to the sun is the z axis, the y axis completes an orthogonal cartesian system. The spacecraft is constrained to stay within the following solar aspect angles (SAA):

SAA x in the range 60-105 deg; i.e. the telescope optical axis may never point closer than 60 deg to the sun, and never further away than 105 deg.

SAA y in the range 85-95 deg; i.e. the z axis must never be more than 5 deg away from the direction to the sun.

In a "nominal" steady state position where the z axis points to the sun and the x axis at right angle from it, you expect a temperature gradient from the +z to the -z end of the telescope. You also expect a temperature gradient through the primary from the backside to the front side. The magnitudes of these gradients in the past were a fraction of a K across the primary, and a few K through it. In the present more open design which offers a much colder telescope these gradients are expected to be larger, perhaps substantially so. In addition, when the boresight angle to the sun changes, the magnitude of the gradient across the primary will change in time, and when/if the z axis points (slightly) away from the sun the gradient will have a component which is no longer (fully) orthogonal to the chopping direction of the instruments.

The acceptable telescope temperature gradients are constrained by the WFE specification; the telescope has to perform at all times. The question which now needs to be addressed is whether additional requirements are necessary from an acceptable (change in) instrumental background point of view. Three sources must be considered: (i) spatial temperature gradients; (ii) temporal temperature gradients; (iii) non-uniformity in emissivity.

As we discussed last week we need your input on these parameters. If the instrumental background requirements are more stringent than the optical performance requirements it is important that we find out now so that these requirements can be taken into account in the telescope design.

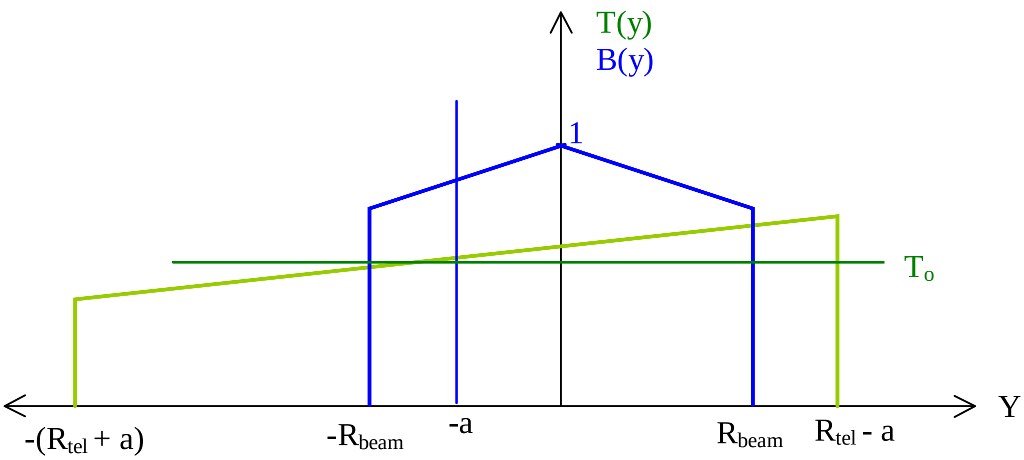
Please, when you give your input also tell me what your assumptions are, e.g. wrt type of observation, chopping schemes, etc. For "average" telescope temperature you can assume 70 +- 10 K. Since there is a preliminary telescope design review in June, it would be useful (necessary I can hear some people saying) to have your (first) input before the end of this month.

Best regards,   G"oran

2 Assumptions

1. Wavelength = 250 or 500 µm.
2. Nominal telescope temperature T_o = 70 K. The telescope emission is not quite in the Rayleigh-Jeans regime. For a fractional change in temperature α, the change in brightness is 1.6α at 200 µm.
3. Looking at a simple 1-D case can give us a fairly accurate idea of the magnitude of the effect.
4. The primary is represented by a straight line segment representing the projection of the curved mirror onto a plane perpendicular to the optical axis (i.e., the Y-Z plane).
5. The effects of the central obscuration and the secondary supports are neglible.
6. The temperature gradient across the primary in the chop direction (Y) is linear.
7. We are insensitive to temperature gradients in the direction orthogonal to the chop (Z).
8. The primary and secondary have equal emissivities and nominal temperatures.
9. The secondary is isothermal.
10. The illumination pattern on the secondary has a linear taper from the centre and is truncated at the edge of the seconday (in fact the illumination profile is irrelevant, as shown below).
11. The effects of diffraction at the secondary are negligible (see Appendix 1). Therefore, the illumination pattern on the primary is the same as on the secondary, with a sharp truncation at a diameter of 3.28 m.
12. The offset between the centre of the beam and the centre of the mirror is never so large that the beam overspills the physical extent of the primary (see Appendix 2).
13. Simple two position chopping.
14. No use of pixel-pixel correlations in noise to reduce effects of temperature gradient fluctuations.

3 Calculation of power offset



R_{beam} = radius at which beam is truncated
R_{tel} = physical radius of primary

a = offset of beam centre from telescope centre (we assume that $R_{beam} + a < R_{rel}$)
m = telescope temperature gradient
q = gradient of beam roll-off
P(a) = power from telescope for offset between beam centre and telescope centre = a

Telescope temperature profile in chop direction: $T(y) = T_o + m(y + a)$

Telescope brightness profile in chop direction: $B(y) = B_o + 1.6m(y + a)$

Illumination pattern on primary: $I(y) = 1 + qy \quad (-R_{beam} < y < 0)$
$= 1 - qy \quad (0 < y < R_{beam})$

$$P(a) = \int_{-R_{beam}}^{R_{beam}} I(y)B(y)\,dy = \int_{-R_{beam}}^{0} [1+qy][T_o + 1.6m(y+a)]dy + \int_{0}^{R_{beam}} [1-qy][T_o + 1.6m(y+a)]dy$$

This result of this integral is $P(a) = R_{beam}(2 - qR_{beam})(T_o + 1.6ma)$.

The power offset for a beam displacement = a is therefore given by

$$\frac{\Delta P}{P(0)} = \frac{P(a) - P(0)}{P(0)} = \frac{1.6ma}{T_o}$$

Note: The offset is independent of the illumination pattern. So, to first order, it makes no difference whether we have a flat-topped or tapered illumination of the primary mirror.

4 Putting limits on acceptable fluctuation in temperature gradient

Let ΔT be the temperature difference between the two sides of the primary in the chopping direction.

Therefore $m = \Delta T/(2R_{beam})$.

The offset signal corresponding to a total chop throw of 2a (±a around the central position) is

$$P_{off} = 1.6P(0)\frac{\Delta T}{T_o}\frac{a}{R_{beam}}$$

Note:

- For simple two-position chopping with no nodding, P_{off} is indistinguishable from a genuine signal.
- To first order, P_{off} is the same for all pixels on the array.
- If P_{off} never varied, it could be measured once, by observing blank sky, and subtracted from all source observations.
- Fluctuations in P_{off} are equivalent to excess sky noise in the case of a ground-based telescope. To reduce or eliminate the effects, one must either nod the telescope faster than the variations and/or exploit the correlation in the noise between different pixels.

- If the array field contains “blank” sky, then P_{off} could be measured during the observation by using pixels which are off-source.

In practice, the variation of the temperature gradient is much more likely to be a drift rather than in the form of random fluctuations. If the telescope has recently slewed to a new position, then the temperature distribution on the primary will be settling down to some new equilibrium. Even if the gradient were constant for a given observation, it would probably vary from one observation to another – so taking it out by nodding or doing something equivalent like 3-position chopping may be necessary anyway. Assume:

- Two position chopping
- No use of correlated noise
- Change in the temperature gradient is linear in time
- Basic measurement time = τ (either the interval between nods or the total integration time if we are not nodding)

Let’s adopt the following criterion to set a limit on the change in the temperature gradient: the maximum change in ΔT during the interval τ should correspond to less than some fraction, say $1/\beta$, of the noise level to which we can integrate down in that time:

$$\delta P_{off} \leq \frac{NEP_{ph}}{\beta \tau^{0.5}} \qquad \text{(an acceptable figure for } \beta \text{ would be} \sim 3\text{).}$$

Letting $\delta(\Delta T)$ be the maximum allowed excursion in ΔT, we have

$$\delta P_{off} = 1.6P(0)\frac{\delta(\Delta T)}{T_o}\frac{a}{R_{beam}} \leq \frac{NEP_{ph}}{\beta\tau^{0.5}} \qquad \Rightarrow \qquad \delta(\Delta T) \leq \frac{T_o R_{beam} NEP_{ph}}{1.6aP(0)\beta\tau^{0.5}}$$

Plugging in some representative values:

R_{beam}	=	1.64 m	
a	=	1.33 cm for 4 arcminute chop. [= (26.6 mm)/2 – see note from Martin Caldwell, Appendix 2]	
T_o	=	70 K	
β	=	3	
P(0)	=	(0.5)(7.5) = 3.8 pW	(250 µm)
		(0.5)(5.0) = 2.5 pW	(500 µm)

Note: the factor of 0.5 accounts for the fact that primary contributes only half of the total background on the detector)

NEP_{ph}	=	12.5×10^{-17} W Hz$^{-1/2}$	(250 µm)
	=	7.7×10^{-17} W Hz$^{-1/2}$	(500 µm)

$$\Rightarrow \quad \delta(\Delta T) \leq \frac{59 \text{ mK}}{\tau^{0.5}} \quad (250\ \mu\text{m}) \qquad \delta(\Delta T) \leq \frac{55 \text{ mK}}{\tau^{0.5}} \quad (500\ \mu\text{m})$$

No nodding: τ could be up to 1 hour $\Rightarrow$ **dT = 0.9 mK on a timescale of 1 hour**

Nodding with a period of, say, 2 minutes: $\Rightarrow$ **dT = 5 mK on a time scale of 2 minutes**

5 Analysis of the JPL telescope temperature map

The nominal telescope temperature map provided by JPL has been used to determine the offset signals generated by motion of the SPIRE beam across the primary when chopping. This map is for the case of zero roll angle about the X axis so that the solar input is symmetrical. To simulate the effect of non-zero roll angle, we have merely rotated this pattern. In reality, the temperature pattern would be different (and this will need to be calculated explicitly at some stage).

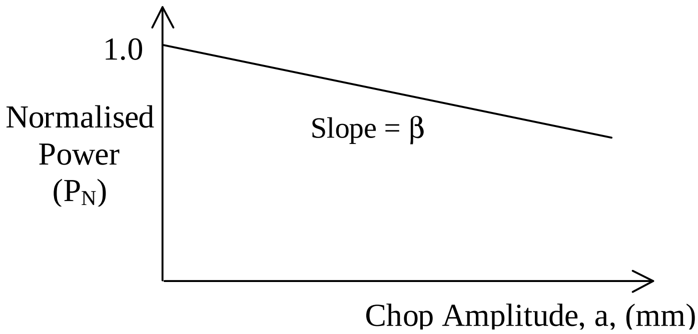
5.1 Assumptions

- Illumination pattern on the primary is a top hat (the results are not very sensitive to departures from this assumption, which is the worst case)
- Chopping along the Y-axis
- Telescope temperature pattern is rotated by an angle θ with respect to the chop direction, with $\theta = 0 - 6^o$.

5.2 Results

5.2.1 Magnitude of the power offset due to chopping

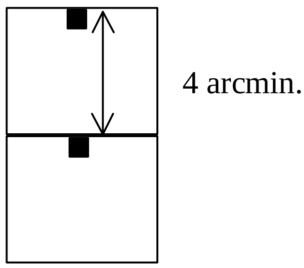
The power collected by the beam, normalised to the maximum value (with the beam centred on the telescope axis) was computed as a function of chop amplitude (in mm from the central position) for different values of θ.



The results may be summarised as follows:

$P_N = 1 - \beta a$

The SPIRE maximum chop (4 arcmin. on the sky) corresponds to a = 13.3 mm. The worst case is for a detector in the centre of the field which is chopped by the full amount.



So the maximum normalised power offset is $P_{Noff} = 13.3\beta$.

Offset in terms of detector NEP (500 μm): $P(0) = 2.5 \times 10^{-12}$ W
$NEPph = 7.1 \times 10^{-17}$ W Hz$^{-1/2}$

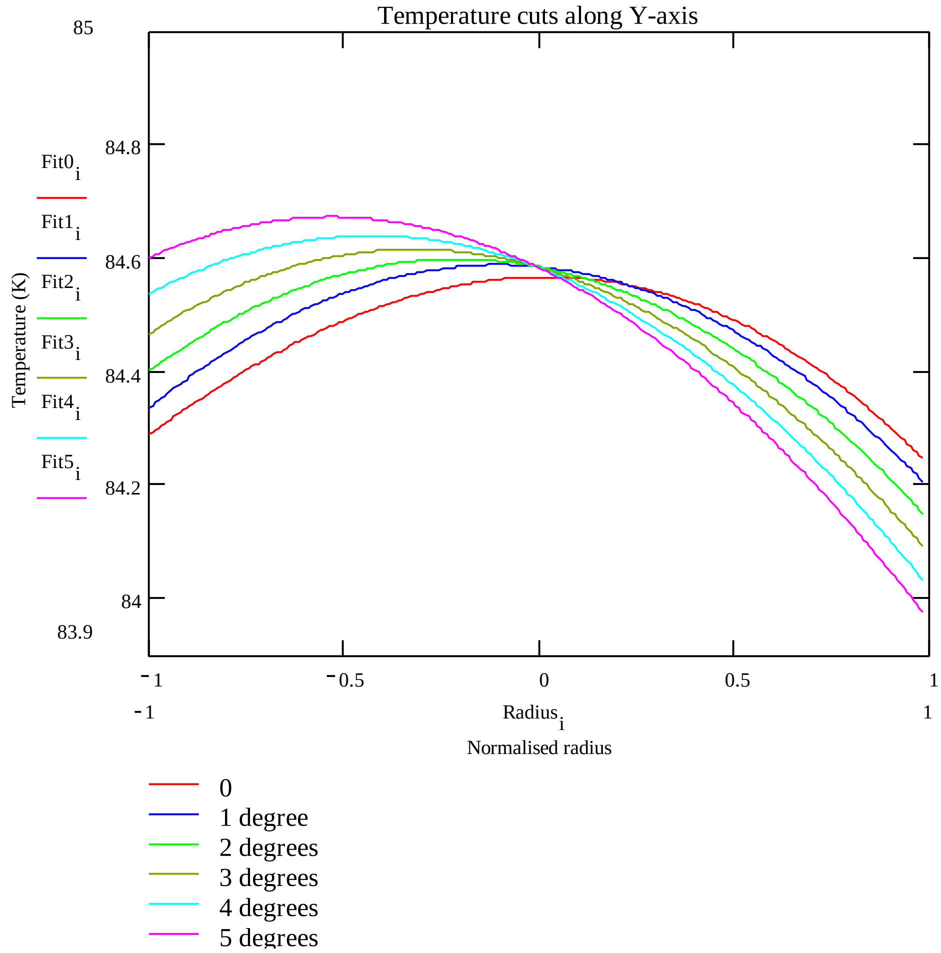
$P_{off} = 13.3\beta(2.5 \times 10^{-12}) = (3.3 \times 10^{-11})\beta$

The table below summarises the offset due to chopping for various values of θ:

Angle	θ	Deg.	1	2	3	4	5	6
	β	mm^{-1}	4.80E-07	8.00E-07	1.28E-06	1.56E-06	2.00E-06	2.20E-06
Normalised offset	P_{Noff}		6.38E-06	1.06E-05	1.70E-05	2.08E-05	2.66E-05	2.93E-05
Absolute offset at 500 um	P_{off}	W	1.60E-17	2.66E-17	4.26E-17	5.19E-17	6.65E-17	7.32E-17
Offset as fraction of NEP			0.23	0.38	0.60	0.73	0.94	1.03

5.2.2 Constraints on the thermal time constant of the telescope

In the simple analysis presented above, a limit of 2.5 mK/minute was placed on the temperature difference across the primary in the Y direction. The diagram below shows polynomial fits to temperature cuts along the Y direction for different values of the roll angle θ (note that the hole in the primary is ignored).



We can place a preliminary requirement on the time constant as follows:

- Assume as a worst case that θ undergoes a step change from 0 to 5°.
- Let the equilibrium temperature difference along the Y axis $= \Delta T_{max} \approx 0.7$ K (it's not a linear gradient, but this is OK as a rough number)
- Let τ be the thermal time constant describing the relaxation of this temperature difference.
- Let R be the maximum allowed rate of change of temperature = 2.5 mK/minute
- Then $R = \Delta T_{max}/\tau$ gives $\tau \geq 280$ minutes (4.7 hours)

6. Conclusions and comments

1. If we nod with a period of about 2 minutes, the required stability of the temperature gradient along the Y-direction is about 2.5 mK per minute. It would be preferable to nod less frequently than this (or not at all).

2. Even for the extreme 5^o roll angle, the offset introduced by chopping is very small - comparable to the overall NEP. The impact on dynamic range is therefore negligible. This conclusion will apply to any similar temperature distribution.

3. In principle, the temperature discontinuities at the gaps between the petals do not produce any undesirable effects.

4. Variability in the offset over the timescale of an observation is what is important, as this dictates how often we have to nod. A very preliminary estimate of the thermal time constant of the telescope gives a figure of about 5 hours. This will need to be confirmed by analysis of a more realistic thermal model.

5. To update the constraint on the time variability of the temperature distribution we would need to have the equilibrium telescope temperature maps for $\theta = 0$ and $\theta = 5^o$ (and intermediate values if possible) for the segmented telescope.

6. It would be useful to have any information that becomes available on the thermal time constant associated with a step change in θ.

Appendix 1: Estimate of beam broadening due to diffraction at the secondary

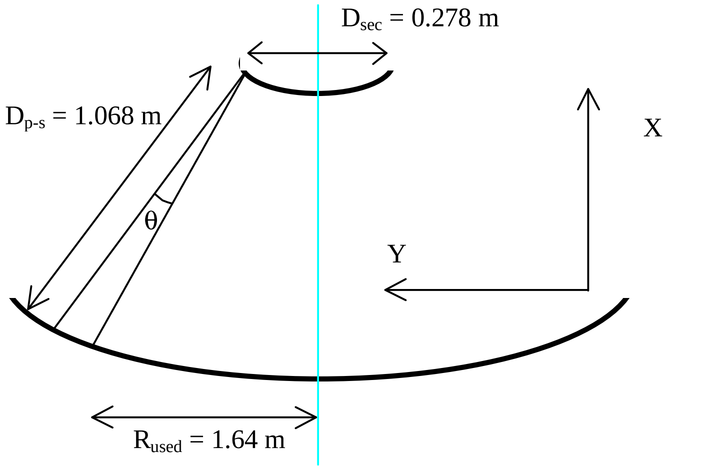
Assume the beam is truncated sharply at the secondary. Diffraction at the secondary will create a wing on the primary illumination pattern with the increase in radius in the y-direction, ΔR, approximately given by

$$\Delta R \approx D_{p\text{-}s} \frac{\lambda}{2D_{sec}} \frac{1}{\cos(27^{o})}$$

where

$D_{p\text{-}s}$ = Distance between secondary and primary

D_{sec} = Diameter of secondary



Taking λ = 500 μm gives ΔR = 1.1 mm.

This is small in comparison with the amount by which the primary is oversized – a radius increment of (3500-3280)/2 = 110 mm. We therefore assume that overspill past the edge of the primary can be neglected.

Appendix 2: Estimate of beam motion on the primary due to chopping

From Martin.Caldwell@rl.ac.uk Thu May 21 14:56:06 1998
Date: Thu, 21 May 1998 14:46:30 +0100
From: Martin E Caldwell <Martin.Caldwell@rl.ac.uk>
To: M.J.Griffin@qmw.ac.uk
Cc: E.Atad@roe.ac.uk, agr@rl.ac.uk, bms87@ssdnt01.bnsc.rl.ac.uk, p.gray@rl.ac.uk
Subject: FIRST telescope, chopped beam motion on M1

Hello Matt,

I said I'd work out the motion of SPIRE's beam across FIRST's M1 due to the chopping.

The ray-trace model I'm using is Eli's CodeV model of 7/11/97. (There is a newer version but I've checked that the telescope & field angles are the same.)

The FOV in the model X-direction (that in which the chopping motion occurs), is +/- 18.22 mm at the telescope focal plane. The chopping is by an amount +/- 22mm at this plane (Eli's e-mail of 3-12-97), so each point in the FOV moves by 44 mm in this plane. From the ray-trace, the corresponding motion of the geometric beam across M1 is by 26.6 mm.

Note that this is the motion in the X-direction, perpendicular to the telescope axis. The large curvature on mirror M1 means that at its edge the angle-of-incidence is approx 27 degrees, so the beam motion measured along the mirror surface would be 1/sin(90-27) = 1.12 times larger than that above.

Regards,

Martin