20/11/98

Time sampling interferograms with an LVDT
Oversampling and interpolation

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1.1 Introduction :

This follows a previous report [1] on the evaluation of an LVDT transducer aimed at the sampling of long wavelength interferograms. Since the sampling accuracy we are looking for might be marginal with that kind of transducer it is important trying to improve it by oversampling and interpolation.

1.2 Experimental data available for the simulation :

This is based on the measurements performed on a prototype drive mechanism for CASSINI / CIRS available at Meudon. The drive is servoed around a 1cm scan LVDT position transducer.

This drive mechanism system has been characterized in term of position noise with a laser interferometer. The deviation from linearity has been recorded by time sampling the position. The number of samples recorded is about 16K for the 1cm range with a scan duration of 30 sec.

This file is then used for the simulation of 4cm range LVDT after multiplication of the deviation from linearity by a factor 3.5. This is to take into account the loss in sensitivity of the transducer going from 1 cm to 4 cm .

The simulation consists in generating the synthetic interferogram of a band pass filter in the spectral range 200-300 μm including the sampling errors. Then we evaluate the S/N in the spectrum.

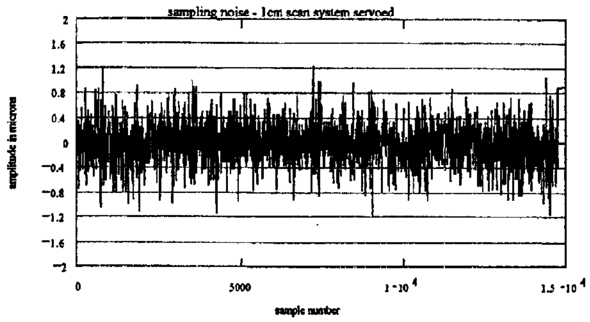

fig 1 :This is the position or jitter noise after compensation of the non-linearity. The value is .2 µm rms (scan length 1cm). The corresponding file has been used for the simulation. For a 4 cm scan this jitter figure becomes .2 x 3.5 = .7 µm rms .

1.3 Oversampling the interferogram :

The interferogram produced is highly oversampled (factor 16) , its characteristics are :

double sided			
sample numbers	16	K	(actual 15K)
spectral band	33.3 - 50	cm-1	(300 - 200 µm)
absorption line	41.7	cm-1	
OPD	9.15	cm	
sampling interval	6.1	µm	
free spectral range	819	cm-1	
resolution (no apodization)	.1	cm-1	
scan duration	30	s	
modulation frequency @ 33.3 cm -1	15.2	Hz	
modulation frequency @ 50 cm-1	10.2	Hz	

To verify the oversampling effect on the S/N, we split the 16K interferogram into 8 interferograms of each 2K samples. These interferograms are FFT transformed and apodized with a simple cosine window . The results are shown in the next figures .

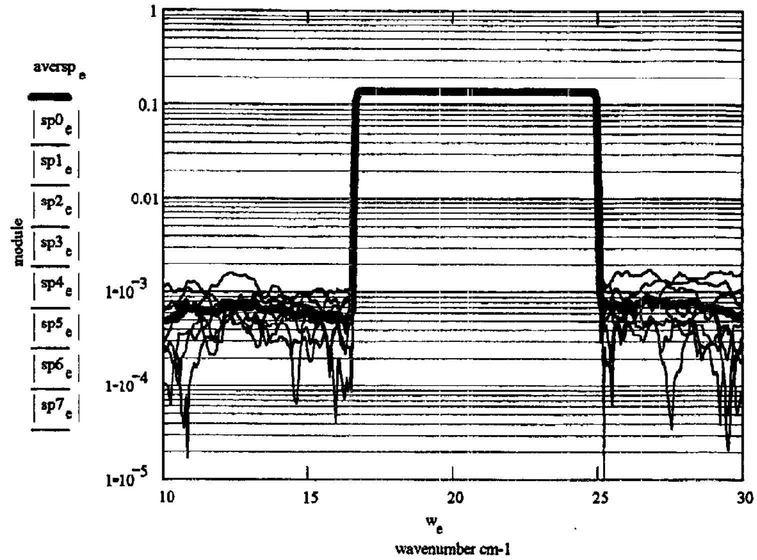

fig 2 : sp0..7 are the modules of the spectra of the 2K interferograms and aversp is the average of the 2K interferograms. The mean S/N of the spectra sp0..7 is 521 and that of the average of the 8 spectra is 992.

The oversampling by a factor 8 leads to an improvement of the S/N by 2 instead of 2.8 (√ 8) expected. This is of course the result of a single scan .The statistic would tend to √ 8 by considering multiple scans.

For the real handling of the interferograms we will proceed to the data compression with the following steps :

1/ numerical filtering to extract the spectral band of interest (33-50 cm-1).

2/ down-sampling the interferogram (by a factor 8 in the case of the previous simulation leading to a 2K interferogram).

In that case the S/N obtained with the 2K interferogram is identical to the S/N obtained by transforming directly the original 16K interferogram.

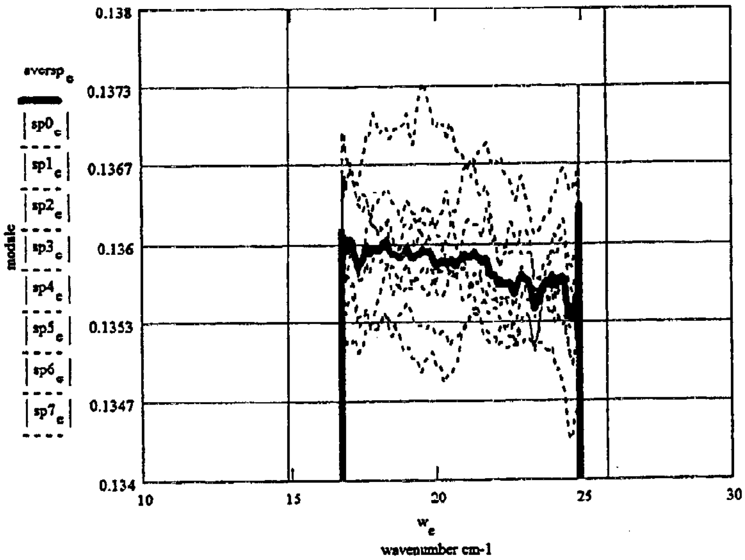

fig 3 : Same conditions as in the previous figure . The noise in the band pass is shown here with a linear scale.

As expected the oversampling improves significantly the S/N . For SPIRE the limitation will come from the data acquisition and real time processing load. At the moment a factor of 5 seems practical, leading to a potential $\sqrt{5} = 2.2$ factor improvement of the sampling accuracy.

The results of this simulation can be extrapolated for the different configuration of interferometer :

Michelson	Single Path	Single Path	Double Path
Interferogram	Double Sided	Single Sided	Single Sided
Oversampling factor	16	16	16
LVDT	4cm	2cm	1cm
Resolution (unapodized)	.1 cm-1	.1 cm-1	.1 cm-1
S/N	1000	2000	4000

1.4 Improvement of the accuracy by interpolation :

Interpolation could be an additional mean of reducing the sampling error in the case we have access the position [2]. This technique is illustrated in the next figure.

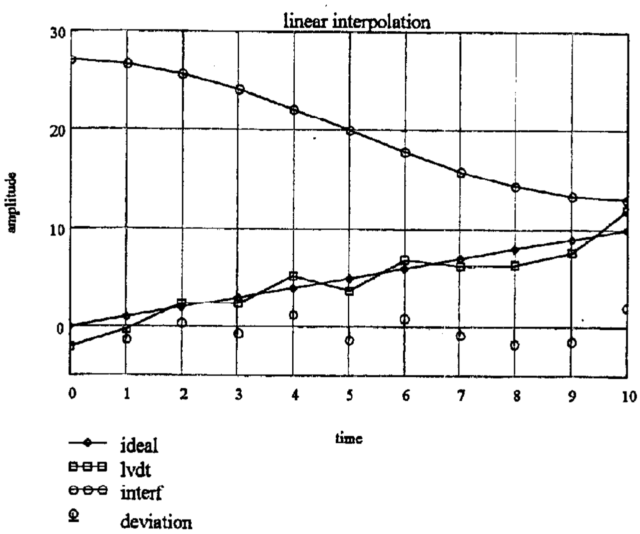

fig 4 : Principle of the interpolation. The curves shown are the interferogram, the ideal linear displacement, the lvdt actual position measurement and the deviation from ideal.
The correction consists in resampling the interferogram according to the amount of deviation from the ideal linear ramp.

To demonstrate this technique we can use the simulated interferogram and the sampling noise file and proceed to a simple linear interpolation which could be easily implemented in real time.
At this point, it is important to note that we have here a perfect correlation between the perturbated interferogram and the sampling noise.
In the real case the correlation factor might be low and that kind of correction useless (at the output of the transducer conditioner we have of course the information on the position plus electrical noise which is not related to the position).
Anyway what follows illustrate what we get with a correlation of 100 %.This gives an indication of the upper limit of the gain to be expected from that kind of interpolation.

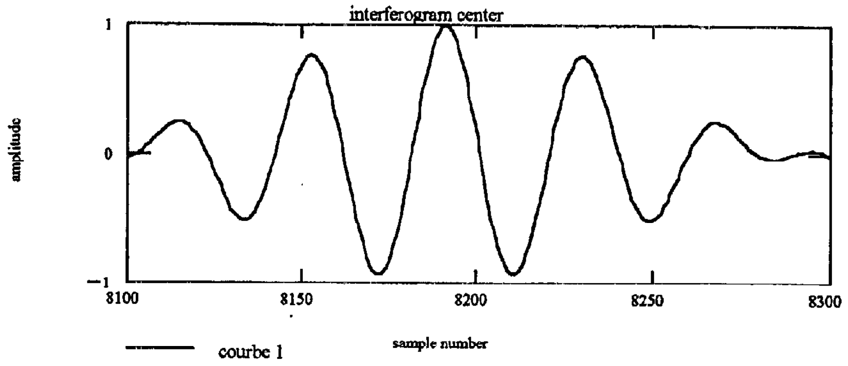

fig 5 : This is the center of the simulated interferogram including sampling errors. The error is more important at the center because since it is proportional to the derivative. This is shown in the next figure where we have the difference between the perturbated and ideal interferogram.

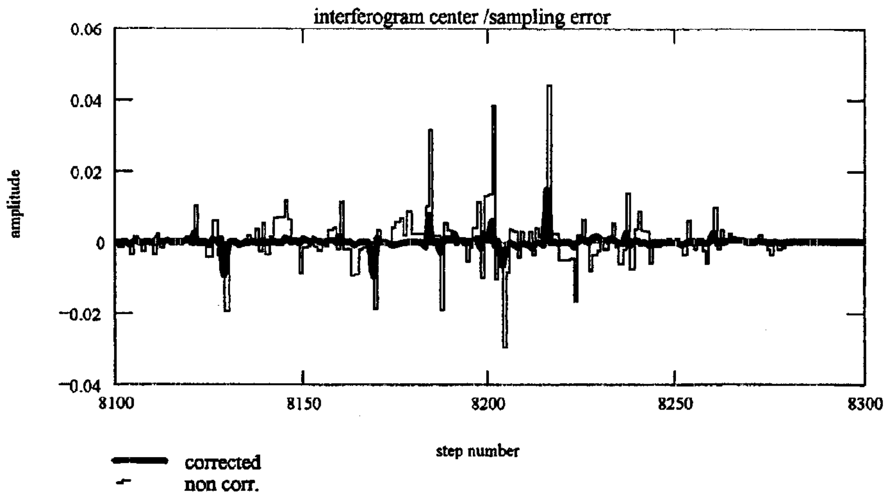

fig 6 : Here we can see the sampling errors of the non corrected and corrected interferogram. The improvement is quite significant.

Another way of showing the improvement is to compute the sampling noise over the free spectral range window.

$$wn_q := q \cdot \frac{fsr}{8192}$$

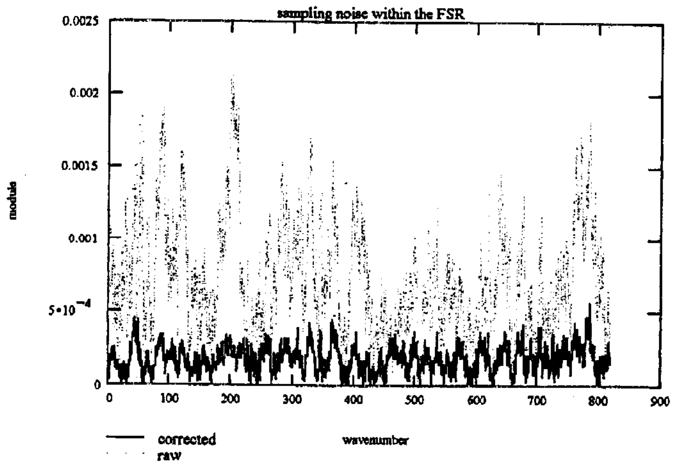

fig 7 :This shows the result of the sampling noise on the spectrum over the full spectral range with and without linear interpolation. There is a factor 4 improvement. This is the best we can get with a simple linear interpolation under the conditions of the simulation.

For the real case we can think of digitizing at the same time the interferogram and the LVDT output. It is probably equivalent to digitize the difference between the command ramp and the LVDT output i.e. the **error signal** (this will have the advantage of minimizing the dynamic range to get a good resolution with a 12 bits ADC).

The efficiency of that correction will depend on the degree of correlation between the **error signal** and the **actual jitter** as measured with the laser interferometer. To simulate this effect one can degrade the correlation index by adding white noise to the deviation from the ideal function cf (fig 4). The results are :

correlation index	S/N in the 33-50 cm-1 spectral range	gain factor corrected / non corrected
1	4108	4
.9	3045	3
.8	2451	2.4
.7	1971	1.9
.6	1600	1.5
.5	985	.9

This correlation measured on the CIRS prototype system is of the order of **.4** . Under this condition the interpolation would do more harm than good.

1.5 Conclusion :

As predictable, oversampling the interferogram is the very simple way of improving the sampling accuracy. As mentioned before the limitation will be the real time computation load. Altogether a factor 2 improvement over the jitter noise seems very realistic.

The interpolation can certainly be investigated on the prototype to be build. It seems that we will be strongly limited by the LVDT noise (the part non connected to actual displacement). In presence of vibrations the situation might be different and the interpolation become more effective.
At the moment the benefit of that kind of correction seems very unlikely.

1.6 References :

[1] G.Michel The sampling of interferograms with an LVDT transducer
Spire team meeting 15/5/98

[2] J.C. Brasunas and G.M. Cushman
Uniform time-sampling Fourier Transform Spectroscopy
APPLIED OPTICS / Vol. 36,No 10 / 1 April 1997