FIRST-SPIRE
Impact of telescope defocus

Kjetil Dohlen

dohlen@observatoire.cnrs-mrs-fr

Laboratoire d'Optique, Observatoire de Marseille
2 Place Le Verrier, 13248 Marseille Cedex 4, France

1. Introduction

ESA has requested an evaluation of the impact of telescope defocus from each of the instrument groups (meeting at ESTEC 30/9/98). Two issues should be evaluated: degradation of instrument performance and ability to measure the amount of defocus present. We treat the two questions separately, considering for the former a source barely visible above the noise, and for the latter a strong source several orders of magnitude stronger than the noise.

Telescope aberrations are represented by 6 μm RMS of spherical aberration, and the performance is calculated for 10, 20 , and 30 μm RMS defocus.

The analysis offered is only valid for detectors with π steradians field of view. It is not valid for Gaussian beams.

2. Performance criteria

Adding defocus or any other aberration to a system decreases the intensity of the central peak of a star image and heightens the level of the diffraction rings, finally blurring them into a halo. This outward movement of energy may be studies by calculating the point-spread function (PSF) of the system.

2.1 Weak source detectivity

Detectivity (D) of a weak, non-resolved stellar source may be described by the ratio of power in the PSF peak (Pp) over power in the background noise just under the peak (Pn). Since the noise level in this case is comparable to the peak of the PSF, it is much higher than the level of the diffraction rings which we may therefore ignore.

Apart from factors of proportionality we have, approximately:

$$\mathrm{Pp} \propto \mathrm{W}^2\,\mathrm{S}$$

and:

$$\mathrm{Pn} \propto \mathrm{sqrt}(\mathrm{W}^2) = \mathrm{W}$$

where W is full-width at half maximum (FWHM) and S is the Strehl ratio (ratio of the actual peak PSF intensity to the theoretical, diffraction-limited peak PSF intensity). Hence, for detectivity:

$$\mathrm{D} = \mathrm{Pp}/\mathrm{Pn} \propto \mathrm{W}\,\mathrm{S} \qquad (1)$$

It is therefore fairly easy to determine the effect upon detectivity of small imaging perturbations. Wetherell [1, p. 303] gives the following model for Strehl ratio:

$$S \approx e^{-\left[(2pw)^2+e^2+(2.1s)^2\right]} \qquad (2)$$

$$\text{for } w < 0.12,\ e < 0.6,\ s < 0.6, \text{ and } S < 0.4$$

where ω is RMS wavefront error in units of wavelength, ε is linear central obscuration ratio, and σ is standard deviation of the image point motion normalized to the diffraction PSF:

$$s = s_m D/\,l \qquad (3)$$

where σ_m is standard deviation of the image point motion in angular units, D is telescope aperture diameter and λ is wavelength. The image point motion is modeled by:

$$I_m(r) = e^{\left(-r^2/2s_m{}^2\right)} \qquad (4)$$

For small wavefront errors and central obscurations, energy is moved from the central peak into the PSF wings without changing the width of the PSF peak. This is not true for image motion however, whose effect is to redistribute the energy within the central peak by widening it. The resulting FWHM may be approximated by:

$$W \approx W_0 \sqrt{1+\left(\frac{W_m}{W_0}\right)^2} \approx W_0 \sqrt{1+\left(\frac{2.36\sigma_m}{\lambda/D}\right)^2} = W_0\sqrt{1+(2.36\sigma)^2} \tag{5}$$

where W_0 is the unperturbed FWHM and $W_m = 2.36\ \sigma_m$ is the FWHM of the image point motion.

Detectivity as defined in Eq. 1 may then be expressed as:

$$D \propto \exp\left[-\left\{(2\pi\omega)^2 + \varepsilon^2 + (2.1\sigma)^2\right\}\right]\sqrt{1+(2.36\sigma)^2} \tag{6}$$

Total RMS wavefront error due to spherical aberration and defocus is given by:

$$\omega = (\omega_D)^2 + (\omega_S)^2$$

where ω_D is RMS wavefront error due to defocus and ω_S is RMS wavefront error due to spherical aberration. The telescope primary is expected to have 6 μm RMS of spherical aberration due to its method of fabrication. At 200 μm this corresponds to $\omega_S = 0.030$.

The obscuration ratio of the the current FIRST telescope is $\varepsilon = 0.17$ and the image motion is 0.3″, corresponding to $\sigma \sim 0.03$ in normalized units.

With the above assumptions, Table 1 gives Strehl ratio at 200 μm for the FIRST telescope according to Eq. 2 and detectivity according to Eq. 6. Since image motion is very small, the difference between S and D is negligible. It also shows relative detectivity given by:

$$D' = D/D(\omega_D = 0)$$

Table 1: Strehl ratio (S) and detectivity (D) and relative detectivity (D′) as functions of RMS wavefront error due to defocus (ω_D).

ω_D (μm)	S at 200 μm	D at 200 μm	D′ at 200 μm
0	.937	.939	1.0
10	.847	.849	.904
20	.630	.632	.673
30	.378	.379	.403

2.2 Defocus detection

When observing a point source much stronger than the noise with a filled focal plane array, one may detect fine changes in the PSF structure and hence, by phase retrieval, determine the amount of defocus. Calculating the PSF profile for the FIRST telescope taking into account 6 μm RMS spherical aberration and various amounts of defocus gives an idea of the possibility of realizing such a phase retrieval.

A simple model based upon the circular symmetry of a wavefront aberrated by defocus and spherical aberration has been built. The model accounts for central obstruction but not for image motion. Figure 1 shows a comparison between the PSF for an unaberrated, unobstructed wavefront calculated at 200 μm by the model (broken line) and the theoretical PSF calculated by the classical Airy disk formula (solid line). The difference (dotted line) is everywhere less than 1/1000 of the central peak and about 1/200 of the maximum of the first ring.

Figure 2 shows aberrated PSF profiles for $\lambda = 200$ μm. The curves are normalized to unit peak amplitude. For a signal-to-noise ratio (SRN) of 1000, one may detect changes in the second ring where effects of 10 μm RMS defocus is clearly visible. If the SNR is of the order of 100, the detectable defocus is about 20 μm RMS. Note that the presence of spherical aberration leaves the changes in PSF asymmetrical with respect to the best-focus position. From a single PSF image one may therefore determine not only the amount of defocus but also the direction of defocus.

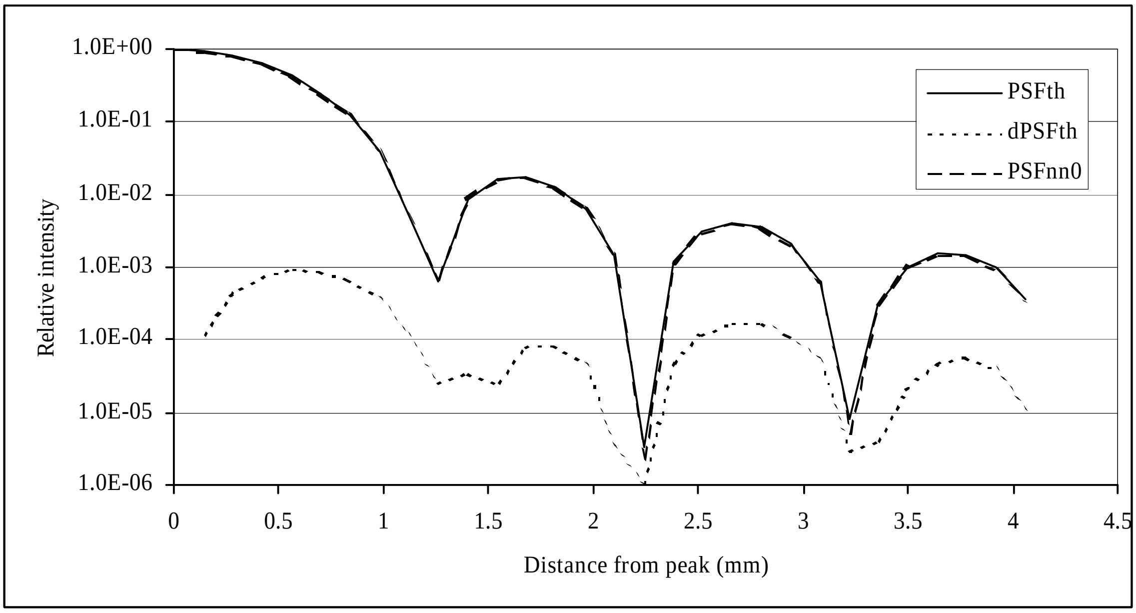

Figure 1. Verification of the model for an unabberated, unobstructed wavefront. Theoretical PSF (solid line) compared with the modelized PSF (broken line). The dotted line shows the absolute value of the difference between the two.

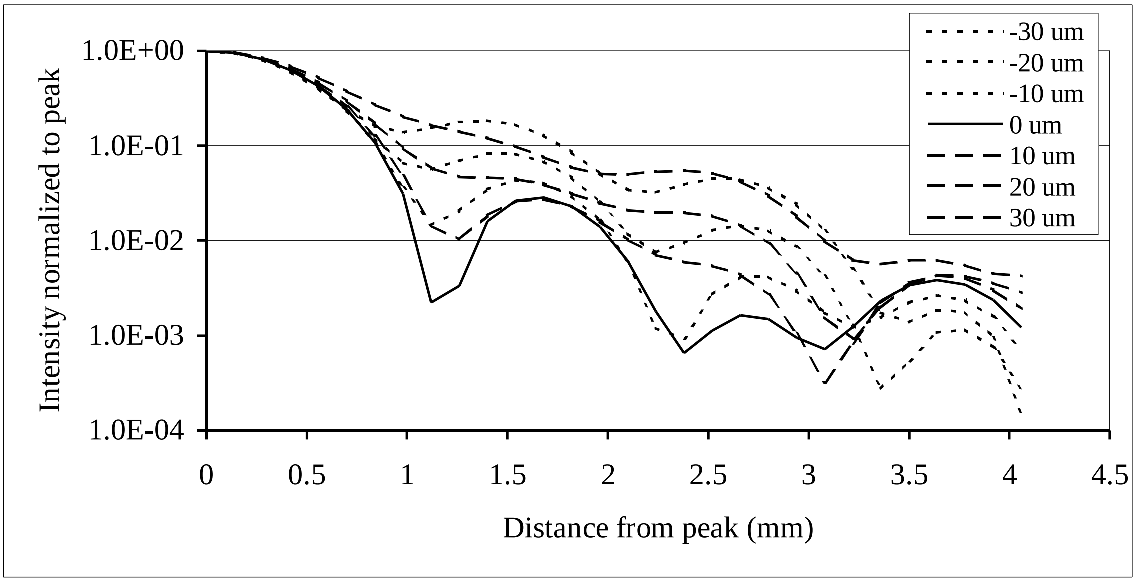

Figure 2. Normalized PSF profiles for the FIRST telescope with zero defocus (solid line) and increasing amounts of positive (broken lines) and negative (dotted lines) defocus. Defocus is given in microns of RMS wavefront error.

3. Conclusion

The above indicates that even at 200 µm we are quite sensitive to defocus of the FIRST telescope. Taking into account spherical aberration, central obstruction, and image motion in addition to defocus, indicates that a detectivity of 0.8 times the ideal detectivity is reached for an RMS wavefront error due to defocus of 13 µm. Letting the 0.8 criterion be relative to the detectivity of the optimally focussed telescope, the amount of defocus may be increased to 15 µm RMS. These numbers assume an ideal instrument, allowing no tolerances for aberrations in the instrument.

Modelling the PSF for a defocussed telescope indicated the possibility to detect quantitatively defocus down to 10-20 µm RMS. The presence of spherical aberration offers the possibility to estimate the direction of defocus from a single image.

4. References

[1] Wetherell, W. B., "The calculation of image quality", in: *Applied Optics and Optical Engineering, vol. VIII*, Ed. R. S. Shannon,J. C. Wyant, Academic Press, London, 1980.