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Comparison of sensitivities of 0.5Fλ, 1.0Fλ and 2.0Fλ arrays for the BOL

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Date: 15 December 1997

Note no.: BOL/QMW/N/0026.10

1. Introduction

Following the workshop at QMW in October 1997, it was decided to change the BOL base-line for the BOL detectors to large format filled absorber arrays rather than feed-horn arrays. The reasons for this decision were:

(i) it was judged that, for deep surveys especially, filled arrays offer an increase in sensitivity (mapping speed) and simpler operating modes (no "jiggling" of the field of view needed to get a fully sampled map);

(ii) it would be better at this stage to specify interfaces and bid for spacecraft resources based on the more ambitious design as it is generally increasingly difficult to negotiate on such issues with ESA as time goes on;

(iii) the feed-horn array design can be retained as a fall-back option which we can use in the event of filled arrays not proving to be greatly superior or not being developed and proven in time.

This note presents an analysis of the relative performance of the nominal options:

(i) 2.0Fλ feedhorns as in SCUBA (the original base-line);

(ii) 1.0Fλ feedhorns (similar to BOLOCAM);

(iii) 0.5Fλ filled absorber arrays (new base-line).

We consider the possible improvement in sensitivity which could be achieved ***in principle*** by using the filled arrays, taking into account the actual observing modes that would be used in each case. The analysis is based on work originally done by Jamie and Jason Glenn, and subsequently checked and extended by Matt, Walter and Jamie. It is based on an analysis of the photometer, but similar considerations should apply to the FTS. We consider observations of a point source, either by on-axis photometry or by extraction from a fully sampled map. This is probably the most important case for the BOL. We consider only the 250- and 500-μm bands - results for the 350-μm case will be intermediate between these two cases.

2. Assumptions

In estimating the relative performance of the different options, we assume the following:

(i) the optical efficiency of the instrument and the absorption efficiency of the detectors are the same for all cases;

(ii) there is no stray light or out-of-band radiation;

(iii) no excess noise is added by pixel co-addition, multiplexing, on-board data processing, or anything else;

(iv) the detectors are photon noise limited by the telescope thermal background (the more realistic case of a detector NEP of around 3×10^{-17} W $Hz^{-1/2}$ is also considered);

(v) in the case of feed-horn arrays, overheads due to re-pointing are assumed to be negligible;

(vi) in the case of 0.5Fλ arrays, focal plane dead area due to gaps between pixels is assumed to be negligible.

3. Summary of results

The analysis is presented in detail in the appendix. The main results are summarised in the tables below. In all instances, figures are normalised to the results for the 2.0Fλ case.

3.1 Observations of a point source on-axis (see A.4)

- No telescope jiggling in any of the cases
- Telescope pointing accuracy and source position must be good enough for blind pointing
- Nine pixels (3 x 3) co-added for the filled arrays
- No pixel co-addition in the feed-horn cases
- Results are the same for all wavelengths

S/N achieved for a given integration time	2.0Fλ	1
	1.0Fλ	0.69
	0.5Fλ	0.70
Mapping speed [$\propto$ $(S/N)^2$]	2.0Fλ	1
	1.0Fλ	0.48
	0.5Fλ	0.49

3.2 Observations of a point source on axis with seven-point jiggle in the case of 2.0Fλ arrays (see A.5.3)

- Nine-pixel co-addition for the filled arrays (no jiggling: same observing mode as above)
- Seven-point jiggle pattern for the feed-horn case to allow for pointing error and/or uncertainty in source co-ordinates (central pixel + six nearest neighbours)
- Jiggle separation $\approx$ 8" = 0.25Fλ at 500 μm and 0.5Fλ at 250 μm

- Seven pixels co-added in the feed-horn cases
- Results are given below for the 2.0Fλ and 0.5Fλ cases: the 1.0Fλ case will be intermediate.

		250 μm	500 μm
S/N achieved for a given integration time	2.0Fλ	1	1
	0.5Fλ	1.03	0.77
Mapping speed [∝ $(S/N)^2$]	2.0Fλ	1	1
	0.5Fλ	1.05	0.60

Note: Five- and nine-point options are also analysed in the appendix (A.5.3). The results are not substantially different (the 0.5Fλ does a bit better compared to a nine-point and a bit worse compared to a five-point).

3.3 Extraction of a point source from a fully-sampled map (see A.5.2)

- The point spread function is fully sampled in each case.
- A jiggle map is needed to do this for the feed-horn arrays, with 64 points or 16 point for 2.0Fλ and 1.0Fλ, respectively.
- Neighbouring pixels are co-added to improve S/N.

		250 μm	500 μm
S/N achieved for a given integration time	2.0Fλ	1	1
	1.0Fλ	1.25	1.28
	0.5Fλ	1.56	1.70
Mapping speed [∝ $(S/N)^2$]	2.0Fλ	1	1
	1.0Fλ	1.54	1.64
	0.5Fλ	2.42	2.92

Note:

The above assumes background-limited sensitivity (i.e., $NEP_{det} << NEP_{background}$). If we adopt 3×10^{-17} W $Hz^{-1/2}$ as a standard figure for the detector optical NEP, then the advantage in performance for the 1.0Fλand 2.0Fλ pixels is less:

		250 μm	500 μm
S/N achieved for a given integration time	2.0Fλ	1	1
	1.0Fλ	1.22	1.25
	0.5Fλ	1.41	1.55
Mapping speed [∝ $(S/N)^2$]	2.0Fλ	1	1
	1.0Fλ	1.48	1.57
	0.5Fλ	1.99	2.40

4. Conclusions and comments

(a) In principle, the 2.0Fλ option is best for 3-band photometry of an isolated point source (assuming no jiggle pattern is needed). Even if it is necessary to do a seven-point map, the feed-horn array is still as good or better than the filled array. However, in practice, the filled arrays have some operational advantages.

(i) the point spread function is completely sampled and the background around the source is completely characterised in the case of the filled array;

(ii) there is no need to co-align accurately three separate pixels on the three arrays in order to do simultaneous point source photometry;

(iii) if a small map (e.g., a seven-point) is needed with feed-horn arrays (depending on the telescope pointing accuracy and the accuracy of the source co-ordinates), there will be some time overhead associated with this (but this is likely to be small).

(b) The maximum potential improvement in mapping speed from having filled arrays is a factor of approx. 2.5 at 250 μm and approx. 3 at 500 μm. This assumes that:

(i) the bolometer absorption efficiency is as high as for feed-horn-coupled bolometers;
(ii) the instrument optical efficiency can be as high as for the feed-horn arrays;
(iii) stray light does not degrade the sensitivity of the bare array detectors;
(iv) the multiplexing and complex on-board data processing and compression requirements (de-spiking and frame co-addition) can be met, and that multiplexing and data processing do not add any extra noise;
(v) the bolometers are still background limited even with the lower backgrounds per pixel (AΩ is lower by a factor of ~ 4, so the required detector NEP is a factor of two lower).

Assumption (i) may be valid, but it still needs to be demonstrated experimentally. Assumptions (ii), (iii) and (iv) should be regarded as optimistic. Assumption (v) requires a more stringent specification for the bolometer sensitivity. If a realistic figure of 3×10^{-17} W $Hz^{-1/2}$ is used for the NEP, the theoretical advantage in mapping speed for the filled arrays is reduced by 15-20%.

(c) There are no telescope motion overheads when mapping with filled arrays, and the observing modes (and chopper design) are simpler - this would translate to an additional advantage in mapping speed.

(d) The 1.0Fλ option represents a compromise between the other two options.

(e) The attractiveness of the filled arrays is greater for spectroscopy than for photometry:
- simpler operating modes (no need to jiggle and take long spectral scans at multiple array pointings);
- lower susceptibility to 1/f noise in the overall system;
- out-of-band radiation and stray light control are not so critical.

(f) In the event of the telescope emissivity being less than the 4% currently assumed, it will be easier to take advantage of this with 2.0Fλ feed-horn pixels as the lower background limit can more easily be met.

(g) **It will be a considerable technical challenge to achieve the theoretical improvement in performance offered by filled arrays. The theoretical advantage will be degraded, or could even become negative, if one or more of the assumptions in (b) above is not true.**

(h) **It is essential that we develop in parallel an instrument design based on feed-horns, and that we regard it as most likely option unless and until filled arrays are clearly demonstrated in practice to be significantly better.**

Appendix: Detailed calculations

A.1 Definitions

P_s	Total power collected by telescope from a point source
P_{sig}	Source power incident on the on-axis pixel
$A\Omega_{tel}$	Throughput for telescope emission at detector
T_{tel}	Telescope temperature
ε_{tel}	Telescope emissivity
η_{opt}	Telescope to detector transmission efficiency:
η_{pix}	Fraction of P_s incident on a pixel
η_{bol}	Bolometer quantum efficiency
t	Integration time per pixel
NEP_{ph}	Photon noise limited NEP
P_B	Telescope background power incident on a pixel
σ	Signal-to-noise ratio

A.2 Estimation of signal-to-noise ratio

The telescope background power incident on each pixel is $P_B = A\Omega_{tel}B(\nu,T_{tel})\Delta\nu\varepsilon_{tel}\eta_{opt}$.

The background-limited NEP is $$NEP_{ph} = \left[\frac{2A\Omega_{tel}B(\nu,T_{tel})\Delta\nu\varepsilon_{tel}\eta_{opt}}{\eta_{bol}}h\nu\right]^{0.5}.$$

The signal power incident on the on-axis pixel is

$$P_{sig} = P_s\eta_{opt}\eta_{pix}.$$

After an integration time, t, the signal-to-noise ratio for the central pixel is

$$\sigma = \frac{P_{sig}(2t)^{0.5}}{NEP_{ph}}.$$

Therefore $$\sigma = \frac{P_{sig}\eta_{pix}\left(\eta_{opt}\eta_{bol}t\right)^{0.5}}{\left(A\Omega_{tel}B(\nu,T_{telI})\Delta\nu\varepsilon_{tel}h\nu\right)^{0.5}}.$$

For simplicity, put those quantities that are independent of the pixel size equal to one:

$$\sigma = \frac{\eta_{pix}\left(\eta_{opt}\eta_{bol}t\right)^{0.5}}{\left(A\Omega_{tel}\right)^{0.5}}.$$

We are also assuming that η_{opt} and η_{bol} are the same for all options, so

$$\sigma = \frac{\eta_{pix}(t)^{0.5}}{(A\Omega_{tel})^{0.5}}.$$

A.3 Values of η_{pix} and $A\Omega_{tel}$ for the various options

2.0Fλ: $\eta_{pix} = 0.77$
[Aperture efficiency of a single-moded 2.0Fλ feed-horn - see Jamie's presentation from Bolometer Arrays Workshop]

1.0Fλ: $\eta_{pix} = 0.40$
[Aperture efficiency of a single-moded 1.0Fλ feed-horn]

0.5Fλ: $\eta_{pix} = 0.18$
[Fraction of Airy disc power contained in central 0.5Fλ square pixel (calculated by integrating under the Airy pattern].

2.0Fλ: $A\Omega_{tel} = 0.8\lambda^2$
[0.8 accounts for illumination of telescope through Lyot stop]

1.0Fλ: $A\Omega_{tel} = 0.45\lambda^2$
[0.45 accounts for illumination of telescope through Lyot stop]

0.5Fλ: $A\Omega_{tel} = (\pi/16)\lambda^2 = 0.196\lambda^2$
[$A = (0.5F\lambda)^2$; $\Omega = \pi/(4F^2)$]

A.4 Observation of a known point source

2.0Fλ pixels: Assume that for observation of a point source, the telescope is pointed with a pixel centred on the source. Only the signal from that pixel is used since neighbouring pixels are too far away on the sky to contain any useful signal. No jiggle pattern is needed, so the total integration time is used on-source.

1.0Fλ pixels: Again, the signals from neighbouring pixels cannot be co-added because the signal levels are too low (separation = 1.0Fλ). No jiggle pattern is needed, so the total integration time is used on-source.

0.5Fλ pixels: In this case, the signals from neighbouring pixels can be co-added to improve the S/N. Assume that the 8 nearest neighbour pixels are co-added to the central one.

- The signal is therefore increased by a factor of

$$\frac{\text{Fraction of Airy disc power contained in central square of side 1.5F}\lambda}{\text{Fraction of Airy disc power contained in central square of side 0.5F}\lambda} = 4.44$$

  (calculated by integrating under the Airy pattern).
- The noise is increased by a factor of (Total no. of pixels being co-added)$^{0.5}$ = $9^{0.5}$ = 3.
- The S/N is therefore enhanced by a factor of 4.44/3 = 1.48.

Normalising to the S/N for the 2.0Fλ case, we have:

0.5Fλ: $$\frac{\sigma_{0.5}}{\sigma_2} = (1.48)\left[\frac{[A\Omega_{tel}]_2}{[A\Omega_{tel}]_{0.5}}\right]^{0.5}\frac{[\eta_{pix}]_{0.5}}{[\eta_{pix}]_2} = (1.48)\left[\frac{0.8}{0.196}\right]^{0.5}\left[\frac{0.18}{0.77}\right] = \mathbf{0.70}$$

1.0Fλ: $$\frac{\sigma_1}{\sigma_2} = \left[\frac{[A\Omega_{tel}]_2}{[A\Omega_{tel}]_1}\right]^{0.5}\frac{[\eta_{pix}]_1}{[\eta_{pix}]_2} = \left[\frac{0.8}{0.45}\right]^{0.5}\left[\frac{0.40}{0.77}\right] = \mathbf{0.69}$$

So, in principle, the 2.0Fλ option therefore gives somewhat better S/N. However, in practice the 0.5Fλ option has a number of significant operational advantages (see below).

A.5 Mapping observation: extraction of point sources

A.5.1 Integration time per pixel

Let t be the total integration time available to make a map of one field. *Ideally*, the integration time per pixel is

0.5Fλ:	t	
1.0Fλ:	t/4	4 telescope pointings needed for full sampling
2.0Fλ:	t/16	16 telescope pointings needed for full sampling

However, we want to observe simultaneously in the three bands, and get fully sampled images in each of them, the jiggle-patterns need to be tailored accordingly.

2.0Fλ case: A 64-point jiggle is needed, with spacing equal to 0.5λ/D at the shortest wavelength (250 μm) and number of steps in each dimension equal to 2λ/D at the longest wavelength (500 μm). The number of telescope pointings is therefore 8 x 8 = 64. (Note: in practice, the jiggle map is not executed using a square grid, but a grid with hexagonal symmetry: this is a detail which does not affect the main conclusions here).

1.0Fλ case: A 16-point jiggle is needed, with spacing equal to 0.5(250 μm)/D at the shortest wavelength (250 μm) and number of steps in each dimension equal to (500 μm)/D. The number of telescope pointings is therefore 4 x 4 = 16.

The actual integration times per pixel are therefore

0.5Fλ:	t
1.0Fλ:	t/16
2.0Fλ:	t/64

A.5.2 Relative signals in adjacent pixels and S/N enhancement on co-adding

For extraction of a point source signal, we can co-add neighbouring pixels. Assume, for simplicity, that the source is on-axis in a particular pixel (the final result will not be too different even if this is not so). The same co-adding method is assumed for the filled arrays and the feed-horn arrays. In the case of 2.0Fλ feed-horns, we assume a Gaussian beam on the sky of FWHM 1.22λ/D. For 1.0Fλ horns, the beam is somewhat narrower as the outer parts of the dish are more strongly illuminated: we have assumed a FWHM of 1.07λ/D.

Note: Simple co-addition is assumed here: the total signal is the sum of the signals in all the pixels being combined; and the noise increases as the square root of number of pixels. Weighting the pixels in some suitable manner might increase the S/N somewhat, but not by a large factor.

2.0Fλ; 500 μm band

- 64-point jiggle with 0.25λ/D spacing
- Integration time per pixel = t/64
- Co-addition of central pixel plus first ring of 6 (0.25λ/D offset) plus next ring of 12 (0.5λ/D spacing)
- Let signal in central pixel = 1
- Separation to each of 6 nearest neighbours = 0.25λ/D
- Signal in each of these = 0.89
- Separation to 6 of 12 pixels in next ring = 0.5λ/D
  Signal in each of these = 0.63
- Separation to other 6 of 12 pixels in this ring = $(3^{0.5}/2)$ (0.5λ/D) = 0.43λ/D
  Signal in each of these = 0.71
- Total signal in 18 adjacent pixels = (6)(0.89) + (6)(0.71) + (6)(0.63) = 13.38
- Total signal in 19 pixels to be co-added = 14.8
- Enhancement in S/N compared to central pixel = $14.38/19^{0.5}$ = 3.30
- Enhancement in S/N compared to observation of known point source = $3.30/64^{0.5}$ = **0.41**.

2.0Fλ; 250 μm band

- 64-point jiggle with 0.5λ/D spacing
- This is equivalent to four separate 16-point jiggles which overlap exactly
- Integration time per pixel = t/64
- Let signal in central pixel = 1
- Co-addition of (4 x central pixel) plus (4 x first ring of 6) (0.5λ/D offset)
- Signal in each of these = 0.63
- Total signal in 4 x 7 pixels to be co-added = (4)(1) + (4)(6)(0.63) = 19.12
- Enhancement in S/N compared to central pixel = $19.12/28^{0.5}$ = 3.61
- Enhancement in S/N compared to observation of known point source = $3.61/64^{0.5}$ = **0.45.**

0.5Fλ; any band

- No telescope jiggling needed
- Let signal in central pixel = 1
- Total signal in 8 adjacent pixels = 3.44
- Total signal in 9 pixels to be co-added = 4.44
- Enhancement in S/N compared to central pixel = 4.44/3 = 1.48 (as before)
- Enhancement in S/N compared to observation of known point source = **1**.

Comparing the S/N for the extraction of a point source from the 0.5Fλ and 2.0Fλ maps we therefore have:

500 μm: $$\frac{\sigma_{0.5}}{\sigma_2} = (0.70)\frac{1}{0.41} = 1.71$$ (factor of **2.9** faster in mapping speed).

250 μm: $$\frac{\sigma_{0.5}}{\sigma_2} = (0.70)\frac{1}{0.45} = 1.55$$ (factor of **2.4** faster in mapping speed).

1.0Fλ; 500 μm band

- 16-point jiggle with 0.25λ/D spacing
- Co-addition of central 7 pixels plus next ring of 12
- Integration time per pixel = t/16
- Let signal in central pixel = 1
- Separation to each of 6 nearest neighbours = 0.25λ/D
- Signal in each of these = 0.86
- Separation to 6 of 12 next nearest neighbours = 0.5λ/D
  Signal in each of these = 0.55
- Separation to 6 of 12 next nearest neighbours = $(3^{0.5}/2)$ (0.5λ/D) = 0.43λ/D
  Signal in each of these = 0.64
- Total signal in 18 adjacent pixels = (6)(0.86) + (6)(0.64) + (6)(0.55) = 12.30
- Total signal in 19 pixels to be co-added = 13.3
- Enhancement in S/N compared to central pixel = $13.3/19^{0.5}$ = 3.05
- Enhancement in S/N compared to observation of known point source = $3.05/16^{0.5}$ = **0.76**.

1.0Fλ; 250 μm band

- 16-point jiggle with 0.5λ/D spacing
- This is equivalent to four separate 4-point jiggles which overlap exactly
- Integration time per pixel = t/16
- Co-addition of (4 x central pixel) plus (4 x first ring of 6) (0.5λ/D offset)
- Let signal in central pixel = 1
- Separation to each of 6 nearest neighbours = 0.5λ/D
- Signal in each of these = 0.55
- Total signal in 6 adjacent pixels = (6)(0.55) = 3.30
- Total signal in 4 x 7 = 28 pixels to be co-added = (4)(1) + (4)(3.30) = 17.20
- Enhancement in S/N compared to central pixel = $17.20/28^{0.5}$ = 3.25
- Enhancement in S/N compared to observation of known point source = $3.25/16^{0.5}$ = **0.81**.

Comparing the S/N for the extraction of a point source from the 1.0Fλ and 2.0Fλ maps we therefore have:

500 μm: $\frac{\sigma_1}{\sigma_2} = (0.69)\frac{0.76}{0.41} = 1.28$ (factor of **1.64** faster in mapping speed).

250 μm: $\frac{\sigma_1}{\sigma_2} = (0.69)\frac{0.81}{0.45} = 1.24$ (factor of **1.54** faster in mapping speed).

A.5.3 Small maps of point sources with feed-horn arrays

In the case feedhorns, if the source position is uncertain or if the telescope pointing is a worry, then a small map can be done with separation of say 8" (≡ 0.5λ/D at 250 μm or 0.25λ/D at 500 μm). Three options are examined below to compare the performance with that of the bare 0.5Fλ pixel array:

(i) a five-point cross;
(ii) a seven-point (central pixel + 6 nearest neighbours);
(iii) a nine point (3 x 3) map.

A.5.3.1 Five-point

250 μm

- Total signal is increased by (1 + 4(0.63)) = 3.52
- Noise is increased by $(5^{0.5})(5^{0.5}) = 5$
- S/N reduced by **0.70**
- Relative S/N of the 0.5Fλ array compared to this mode = (0.70)/(0.70) = **1.0**.
- Relative S/N of the 0.5Fλ array compared to this mode = $(1.0)^2$ = **1.0**

500 μm

- Total signal is increased by (1 + 4(0.89)) = 4.56
- Noise is increased by $(5^{0.5})(5^{0.5}) = 5$
- S/N reduced by 0.91
- Relative S/N of the 0.5Fλ array compared to this mode = (0.70)/(0.91) = **0.77**.
- Relative speed of the 0.5Fλ array compared to this mode = $(0.77)^2$ = **0.59**.

A.5.3.2 Seven-point

250 μm

- Signal is (central pixel) + (6 pixels at 0.5λ/D spacing)
  Total signal is increased by 1 + 6(0.63) = 4.78
- Noise is increased by $(7^{0.5})(7^{0.5}) = 7$
- S/N reduced by (4.78)/7 = 0.68
- Relative S/N of the 0.5Fλ array compared to this mode = (0.70)/(0.68) = **1.03**.
- Relative S/N of the 0.5Fλ array compared to this mode = $(1.03)^2$ = **1.05**

500 μm

- Signal is (central pixel) + (6 pixels at 0.25λ/D spacing)
  Total signal is increased by 1 + 6(0.89) + 4 (0.79) = 6.34
- Noise is increased by $(7^{0.5})(7^{0.5})$ = 7
- S/N reduced by (6.34)/7 = 0.91
- Relative S/N of the 0.5Fλ array compared to this mode = (0.70)/(0.91) = **0.77**.
- Relative speed of the 0.5Fλ array compared to this mode = $(0.77)^2$ = **0.60**.

A.5.3.3 Nine-point

250 μm

- Signal is (central pixel) + (4 pixels at 0.5λ/D spacing) + (4 pixels at $0.5(2)^{0.5}$λ/D spacing)
  Total signal is increased by 1 + 4(0.63)+ 4(0.39) = 5.08
- Noise is increased by $(9^{0.5})(9^{0.5})$ = 9
- S/N reduced by (5.08)/9 = 0.56
- Relative S/N of the 0.5Fλ array compared to this mode = (0.70)/(0.56) = **1.25**.
- Relative S/N of the 0.5Fλ array compared to this mode = $(1.25)^2$ = **1.56**

500 μm

- Signal is (central pixel) + (4 pixels at 0.25λ/D spacing) + (4 pixels at $0.25(2)^{0.5}$λ/D spacing)
  Total signal is increased by 1 + 4(0.89) + 4 (0.79) = 7.72
- Noise is increased by $(9^{0.5})(9^{0.5})$ = 9
- S/N reduced by (7.72)/9 = 0.86
- Relative S/N of the 0.5Fλ array compared to this mode = (0.70)/(0.86) = **0.81**.
- Relative speed of the 0.5Fλ array compared to this mode = $(0.81)^2$ = **0.66**.